A Study of Linguistic GMCDM Based on Different Aggregation Processes of Decision Information

Chen-Tung Chen
Department of Information Management, National United University, Miao-Li, Taiwan
ctchen@nuu.edu.tw

Ping-Feng Pai
Department of Information Management, National Chi Nan University, Nan-Tou, Taiwan
paipf@ncnu.edu.tw

Wei-Zhan Hung
Department of International Business Studies, National Chi Nan University, Nan-Tou, Taiwan
steady_2006@hotmail.com

Abstract

Group multi-criteria decision-making (GMCDM) is the main methodology to make a decision in real situation. There are two decision processes for aggregating experts' opinions such as first aggregation and last aggregation to determine the ranking order of decision alternatives. In general, the linguistic variables are suitable used to represent the subjective opinions of experts. The purpose of this study is to present a linguistic TOPSIS method based on different aggregation processes of decision information. In this paper, three last aggregation methods such as weighted ranking value, elite selection and elimination method are presented to determine the final ranking order of all alternatives by aggregating the experts' judgment about the order of each alternative. And then, a numerical example is implemented to illustrate the procedure of the proposed method. Finally, the conclusions are discussed at the end of this paper.

Keywords: GMCDM, First aggregation, Last aggregation, Linguistic TOPSIS.

I. INTRODUCTION

Decision-making is the main enterprise activity which will happen everywhere in every time. From product selection, investment decision to supplier selection, no activity can execute without decision making. Decision-Making is the procedure to find the best action among a set of feasible actions (Figueira et al. 2005). Multi criteria decision making is a rational technology can be used to efficiently and effectively deal with decision making problem by explicitly improve the quality of decision process (Wanga and Triantaphylloub 2008). In order to avoid individual persons' subjective opinion and reduce the judgment loss by single person, group multi criteria decision making (GMCDM) is the main situation for making decision, especially in some important investment decision (such as factory location selection, product develop decision and high level employee selection etc).

In GMCDM, there are two decision processes for aggregating experts' opinions including performance rating of each alternative with respect to each criterion and relative importance of each evaluation criterion with respect to the overall objective (1) First aggregation, (2) Last aggregation (Roghanian and Rahimi 2010). In first aggregation decision process, experts' opinions are aggregated first, and then the process of MCDM method is executed for ranking alternatives. First aggregation decision process can be considered as **"group opinion aggregation"** decision process. On the other hand, the ranking of each alternative (each expert's judgment) is determined by MCDM method according to each expert's individual opinions (performance of each alternative respect to each criterion and the importance of each criterion). Every expert possesses his/her opinion about the ranking of each alternative. Each expert's judgment about the ranking of each alternative can be aggregated by different aggregation method. Last aggregation decision process can be considered as **"alternative ranking aggregation"** decision process.

The Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS) is first developed by Hwang and Yoon (Hwang and Yoon 1981) is one kind of MCDM method for making decision. TOPSIS is already used in many management fields such as human resources management, factory location analysis, supplier selection, water management and. quality control etc. The concept of TOPSIS is to choose the best alternative by simultaneously consider the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS) in each alternative. The advantage of TOPSIS includes

(1) TOPSIS is an effective method to acquire the total ranking order of each alternative (Chen and Hung 2009).

(2) TOPSIS can avoid rank reversal problem (Roghanian and Rahimi, 2010).

(3) TOPSIS is a simple computation process that is easy to be programmed (Kim et al. 1997).

(4) In TOPSIS method, the performance of each alternatives respect to each criterion can be visualized on a polyhedron for any two dimensions (Kim et al. 1997).

In reality, crisp value is not suitable to formulate real-life situations. Because experts' subject opinion, preference and judgment are usually vague and uncertainty, it is hard to express them by exact numerical value. A more practically solution is to use linguistic assessments instead of numerical values. The 2-tuple linguistic representation model is one kind of linguistic variable and is based on the concept of symbolic translation (Xu 2005). Experts can apply 2-tuple linguistic variables to express their opinions and obtain the final evaluation result with appropriate linguistic variable. The advantage of 2-tuple linguistic variable is that it can reduce the mistakes of information translation and avoid information loss through computing with words (Herrera-Viedma et al. 2003).

In fact, using first aggregation decision process (group opinion aggregation) to make decision is the main research topic in MCDM research. Last aggregation decision process (alternative ranking aggregation) is usually took place in management field (such as new singer selection, new sportsman player selection and high level employee selection etc).

However, a few of literature use last aggregation concept to discuss decision making problem (Roghanian and Rahimi, 2010).

The goal of this research is to present one kind of last aggregation model named-alternative ranking aggregation and to develop three aggregation methods (weighted ranking value, elite selection and elimination method) to aggregate experts' judgment about the ranking of each alternative.

II. PRELIMINARIES

Definition 2.1. Let $S=\{s_0, s_1, s_2,..., s_g\}$ be a finite and totally ordered linguistic term set. A 2-tuple linguistic variable can be expressed as $(s_i,\ \alpha_i)$, where s_i is the central value of i-th linguistic term in S and α_i is a numerical value representing the difference between calculated linguistic term and the closest index label in the initial linguistic term set.

Definition 2.2. The symbolic translation function Δ is used to translate a crisp value β into a 2-tuple linguistic variable (Herrera, and Martinez 2001). Then, the symbolic translation process is applied to translate β (β∈ [0, 1]) into a 2-tuple linguistic variable. The generalized translation function can be represented as $\Delta(\beta)=(s_i,\alpha_i)$ where $i=round(\beta\times g)$, $\alpha_i=\beta-\frac{i}{g}$ and $\alpha_i\in[-\frac{1}{2g},\frac{1}{2g})$ (Tai and Chen 2009).

Definition 2.3. A reverse function Δ^{-1} is defined to return an equivalent numerical value β from 2-tuple linguistic information $(s_i,\ \alpha_i)$. According to the symbolic translation, an equivalent numerical value β is obtained as $\Delta^{-1}(s_i,\alpha_i)=\frac{i}{g}+\alpha_i=\beta$ (Tai and Chen 2009).

Definition 2.4. Let x = {(r_1, α_1), (r_2, α_2),…, (r_n, α_n)} be a 2-tuple linguistic variable set. The arithmetic mean is computed as $\overline{X}=\Delta\left(\frac{1}{n}\sum_{i=1}^{n}\Delta^{-1}(r_i,\alpha_i)\right)=(s_m,\alpha_m)$ (Herrera-Viedma et al. 2004).

Definition 2.5. The linguistic variable sets with different types will be defined by partitioning the interval [0, 1]. Transforming a crisp number β (β∈ [0, 1]) into i-th linguistic term $(s_i^{n(t)},\alpha_i^{n(t)})$ of type t as $\Delta_t(\beta)=(s_i^{n(t)},\alpha_i^{n(t)})$ where $i=round(\beta\times g_t)$, $\alpha_i^{n(t)}=\beta-\frac{i}{g_t}$ $g_t=n(t)-1$ and $n(t)$ is the number of linguistic variable of type t.

Definition 2.6. Transforming i-th linguistic term of type t into a crisp number β (β∈ [0, 1]) as Transforming i-th linguistic term of type t into a crisp number β (β∈ [0, 1]) as $\Delta_t^{-1}(s_i^{n(t)},\alpha_i^{n(t)})=\frac{i}{g_t}+\alpha_i^{n(t)}=\beta$ where $g_t=n(t)-1$ and $\alpha_i^{n(t)}\in[-\frac{1}{2g_t},\frac{1}{2g_t})$.

Definition 2.7. The transformation from i-th linguistic term $(s_i^{n(t)},\alpha_i^{n(t)})$ of type t to k-th linguistic term $(s_k^{n(t+1)},\alpha_k^{n(t+1)})$ of type t+1 at interval [0, 1] can be expressed as $\Delta_{t+1}(\Delta_t^{-1}(s_i^{n(t)},\alpha_i^{n(t)}))=(s_k^{n(t+1)},\alpha_k^{n(t+1)})$ where $g_{t+1}=n(t+1)-1$ and $\alpha_k^{n(t+1)}\in[-\frac{1}{2g_{t+1}},\frac{1}{2g_{t+1}})$.

III. GROUP OPINION AGGREGATION BASED ON LINGUISTIC TOPSIS

3.1 Basic representation of GMCDM

General speaking, GMCDM problem can be described by means of the following sets:

(i) A set of alternatives is called $A = \{A_1, A_2, \cdots, A_m\}$;

(ii) A set of criteria $C = \{C_1, C_2, \cdots, C_n\}$ with which alternatives' performances are measured;

(iii) A set of decision-makers is called $D = \{D_1, D_2, \cdots, D_p\}$;

(iv) A set of performance ratings of alternatives with respect to criteria from experts is called $\tilde{x}_{ijk}$. i=1,2,…,m; j=1,2,…,n; k=1,2,…,p.

$$\tilde{D}^k = \left[\tilde{x}_{ij}^k\right]_{mnp} = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11}^k & \tilde{x}_{12}^k & \cdots & \tilde{x}_{1n}^k \\ \tilde{x}_{21}^k & \tilde{x}_{22}^k & \cdots & \tilde{x}_{2n}^k \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{x}_{m1}^k & \tilde{x}_{m2}^k & \cdots & \tilde{x}_{mn}^k \end{bmatrix} \end{array} \tag{1}$$

$\tilde{D}^k$ represents decision matrix of expert k.

$\tilde{x}_{ij}^k$ represents the opinion of expert k about the performance rating of alternative i respect to criterion j. $\tilde{x}_{ij}^j$ can be described as 2-tuple linguistic variable (S_{ijk}, α_{ijk}).

(V) A set of importance ratings of criteria from experts is called $\tilde{w}_{jk}$. j=1,2,…,n; k=1,2,…,p.

$$\tilde{W} = \left[\tilde{w}_{jk}\right]_{np} = \begin{array}{c} \\ D_1 \\ D_2 \\ \cdots \\ D_p \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} \tilde{w}_{11} & \tilde{w}_{21} & \cdots & \tilde{w}_{n1} \\ \tilde{w}_{12} & \tilde{w}_{22} & \cdots & \tilde{w}_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{w}_{1p} & \tilde{w}_{2p} & \cdots & \tilde{w}_{np} \end{bmatrix} \end{array} \tag{2}$$

$\tilde{w}_{jk}$ can be represented as the opinion of expert k about the importance of j-th criterion.

$\tilde{w}_{jk}$ can be described as 2-tuple linguistic variable $(S_{jk}^w, \alpha_{jk}^w)$.

3.2 Linguistic TOPSIS

In traditional linguistic TOPSIS method, experts' opinions which are including performance of each alternative respect to each criterion and the importance of each criterion are aggregated first, and then the process of TOPSIS method is executed.

The step of traditional linguistic TOPSIS is as follows.

Step 1. Experts express their opinions about the performance rating of each alternative respect to each criterion and the importance rating of each criterion.

Step 2. Aggregate the opinion of each expert about the performance rating of each alternative respect to each criterion and acquire an aggregated decision matrix.

$\tilde{x}_{ij}^k = (S_{ijk}, \alpha_{ijk})$ represents the performance rating of i-th alternative respect to j-th criterion which is expressed by k-th expert.

The formula of aggregating all of the opinions about the performance rating of i-th alternative with respect to j-th criterion can be handled as follows:

$$\tilde{x}_{ij} = \Delta\left(\frac{1}{K}\sum_{k=1}^{K}\Delta^{-1}(S_{ij}^k, \alpha_{ij}^k)\right) = (S_{ij}, \alpha_{ij}) \tag{3}$$

And then, an aggregated decision matrix ($\tilde{D}$) is acquired.

$$\tilde{D} = \left[\tilde{x}_{ij}\right]_{mn} = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} \end{array} \tag{4}$$

Step 3. Aggregate the opinion of each expert about the importance rating of each criterion and acquire an aggregated weight of each criterion.

$(S^w_{jk}, \alpha^w_{jk})$ represents the importance rating of j-th criterion which is expressed by k-th expert.

The formula of aggregating all of the opinions about the importance rating of j-th criterion can be handled as follows:

$$\tilde{w}_j = \Delta\ \left(\frac{1}{K}\sum_{k=1}^{K}\Delta^{-1}(S^w_{jk}, \alpha^w_{jk})\right) = (S^w_j, \alpha^w_j) \tag{5}$$

And then, an aggregated weight of each criterion is acquired. $\tilde{\mathrm{W}} = \{\tilde{\mathrm{w}}_1, \tilde{\mathrm{w}}_2, \cdots, \tilde{\mathrm{w}}_n\}$

Step 4. Construct weighted decision matrix

$$V = \left[v_{ij}\right]_{mn} = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{bmatrix} \end{array} \tag{6}$$

where $v_{ij} = \Delta^{-1}\left(\tilde{x}_{ij}\right) * \Delta^{-1}\left(\tilde{w}_j\right)$.

Step 5. Calculate positive ideal solution (PIS) $A^* = \left(v_1^*, v_2^*, \ldots, v_n^*\right)$ and negative ideal solution (NIS) $A^- = \left(v_1^-, v_2^-, \ldots, v_n^-\right)$.

where $v_j^* = \max_i\left(v_{ij}\right)$ and $v_j^- = \min_i\left(v_{ij}\right)$.

Step 6. Calculate the distance of each alternative from PIS and NIS.

$$d_i^+ = \sqrt{\sum_{j=1}^{n}\left(v_j^* - v_{ij}\right)^2} \tag{7}$$

$$d_i^- = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_j^-\right)^2} \tag{8}$$

Step 7. Calculate the relative closeness to the ideal solution of each alternative.

$$c_i = d_i^- / \left(d_i^+ + d_i^-\right) \tag{9}$$

where c_i is between 0 and 1. The higher c_i, the better alternative i.

Step 8. Calculate the rank of each alternative.

3.3 Last aggregation process based on linguistic TOPSIS

This investigation develops a new decision making process. First of all, the ranking of each alternative is determined by linguistic TOPSIS method individually according to each expert's opinions about performance of each alternative respect to each criterion and the importance of each criterion. So, every expert possesses his/her opinion about the ranking of each alternative. This study develops three aggregation methods (weighted ranking value, elite selection and elimination method) to aggregate experts' judgment about the ranking of each alternative.

The step of new decision making process based on linguistic TOPSIS method is as follows.

Step 1. Experts express their opinions about the performance rating of each alternative respect to each criterion and the importance rating of each criterion.

$\tilde{x}_{ij}^{k} = (S_{ijk}, \alpha_{ijk})$ represents the performance rating of i-th alternative respect to j-th criterion which is expressed by k-th expert.

$\tilde{w}_{j}^{k} = (S_{jk}^{w}, \alpha_{jk}^{w})$ represents the importance rating of j-th criterion which is expressed by k-th expert.

Step 2. Construct decision matrix of each expert.

$$\tilde{D}^{k} = \left[\tilde{x}_{ij}^{k}\right]_{mnp} = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11}^{k} & \tilde{x}_{12}^{k} & \cdots & \tilde{x}_{1n}^{k} \\ \tilde{x}_{21}^{k} & \tilde{x}_{22}^{k} & \cdots & \tilde{x}_{2n}^{k} \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{x}_{m1}^{k} & \tilde{x}_{m2}^{k} & \cdots & \tilde{x}_{mn}^{k} \end{bmatrix} \end{array} \tag{10}$$

$\tilde{D}^{k}$ represents decision matrix of expert k.

Step 3. Construct weighted decision matrix of each expert.

$$V^{k} = \left[v_{ij}^{k}\right]_{mn} = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} v_{11}^{k} & v_{12}^{k} & \cdots & v_{1n}^{k} \\ v_{21}^{k} & v_{22}^{k} & \cdots & v_{2n}^{k} \\ \cdots & \cdots & \cdots & \cdots \\ v_{m1}^{k} & v_{m2}^{k} & \cdots & v_{mn}^{k} \end{bmatrix} \end{array} \tag{11}$$

where $v_{ij}^{k} = \Delta^{-1}\left(\tilde{x}_{ij}^{k}\right) * \Delta^{-1}\left(\tilde{w}_{j}^{k}\right)$.

Step 4. Calculate positive ideal solution (PIS) $A^{k*} = \left(v_1^{k*}, v_2^{k*}, \ldots, v_n^{k*}\right)$ and negative ideal solution (NIS) $A^{k-} = \left(v_1^{k-}, v_2^{k-}, \ldots, v_n^{k-}\right)$ of each expert.

where $v_j^{k*} = \max_i \left(v_{ij}^{k}\right)$ and $v_j^{k-} = \min_i \left(v_{ij}^{k}\right)$.

Step 5. Calculate the distance of each alternative from PIS and NIS of each expert.

$$d_i^{k+} = \sqrt{\sum_{j=1}^{n} \left(v_j^{k*} - v_{ij}^{k}\right)^2} \tag{12}$$

$$d_i^{k-} = \sqrt{\sum_{j=1}^{n} \left(v_{ij}^{k} - v_j^{k-}\right)^2} \tag{13}$$

Step 6. Calculate the relative closeness to the ideal solution of each alternative of each expert.

$$c_i^{k} = d_i^{k-} / \left(d_i^{k+} + d_i^{k-}\right) \tag{14}$$

where c_i^k is between 0 and 1. The higher c_i^k, the better alternative i for expert k.

c_i^k represents the opinion of expert k about the performance of alternative i.

Step 7. Calculate the rank of each alternative of each expert and construct alternative rank matrix R.

$$R = [r_{ik}]_{mp} = \begin{array}{c} \\ A_1 \\ A_2 \\ \dots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} D_1 & D_2 & \dots & D_p \end{array} \\ \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1p} \\ r_{21} & r_{22} & \dots & r_{2p} \\ \dots & \dots & \dots & \dots \\ r_{m1} & r_{m2} & \dots & r_{mp} \end{bmatrix} \end{array} \tag{15}$$

where r_{ik} represents the rank of alternative i about the opinion of expert k.

The smaller r_{ik}, the better alternative i for the opinion of expert k.

This research develops three aggregation methods to aggregate experts' ranking of each alternative.

1. Ordering according to weighted ranking value

Suppose that the importance of expert k can be expressed as w_k. The aggregated rank of each alternative can be calculated as follows.

$$\bar{r}_i = \sum_{k=1}^{p} w_k * r_{ik} \tag{16}$$

The smaller $\bar{r}_i$, the better alternative i for aggregative experts' opinions.

2. Ordering according to elite selection

As implied by the name, elite selection means select alternative according to experts' consensus opinions about the best alternative. First of all, we can calculate the best alternative count (BAC) of each alternative. BAC means the volume of experts consider this alternative is best. And then, pick up this alternative who possesses the maximum best alternative count. We add it into sort list and delete it from alternative set. Repeat above process until all of alternative is ranked. The flow chart of elite selection can refer to Figure. 1.

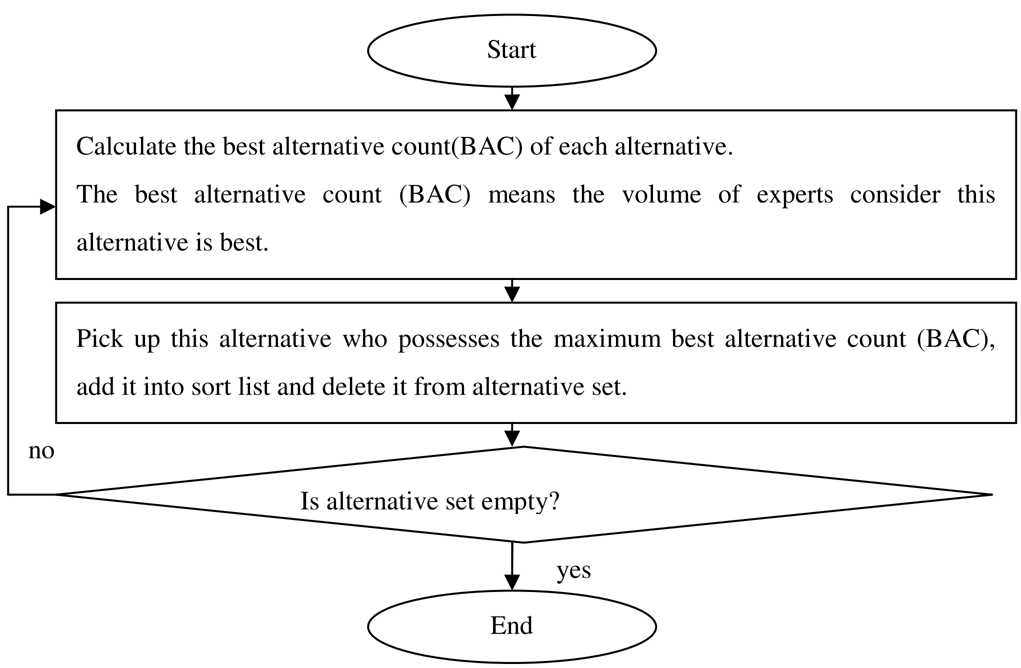

Figure 1. The process of elite selection

3. Ordering according to elimination method

On the other way, elimination method means select alternative according to experts' consensus opinions about the worst alternative. At First, The worst alternative count (WAC) of each alternative is calculated. WAC means the volume of experts consider this alternative is worst. And then, pick up this alternative that possesses the maximum worst alternative count. We add it into sort list and delete it from alternative set. Repeat above process until all of alternative is ranked. The flow chart of elimination method can refer to Figure. 2.

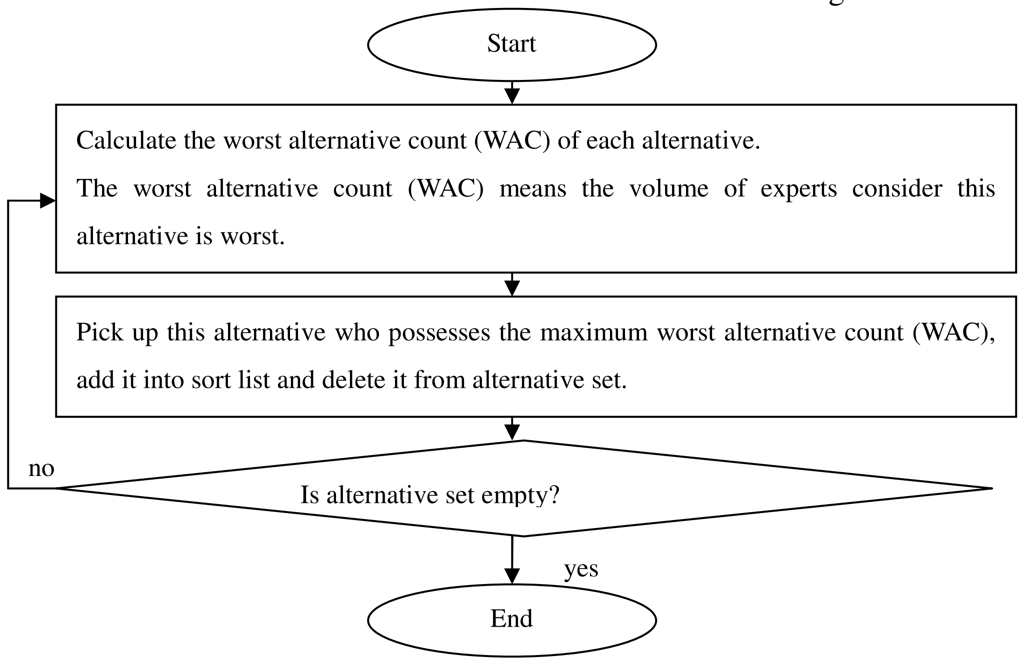

Figure 2. The process of elimination method

IV. NUMERICAL EXAMPLE

For letting reader understands our method, this investigation makes an example about a dining service enterprise wants to select a manager. In a council of restaurant, the restaurant boss assigns five experts to choose the best candidate from five applicants according to five criteria. The criteria is including Service related skill (C_1), Communication skill (C_2), Work experience (C_3), Emotional steadiness (C_4) and English ability (C_5).

According to linguistic TOPSIS method, the computational procedures of the problem are summarized as follows.

Step 1. Each expert chooses linguistic variable type to express his/her opinion. Expert D_1, D_2 choices type 1, D_3, D_4 choices type 2, D_5 choices type 3 (See Table 1). And then, each expert uses linguistic variables to express his/her opinions about the performance ratings of each alternative with respect to criteria as Table 2 and importance rating of each criterion as Table 3.

Step 2. Transform experts' opinions (the linguistic ratings of each alternative with respect to criteria) into the linguistic variables of type 2 and aggregate the linguistic ratings of each alternative with respect to criteria.

Step 3. Transform experts' opinions (the linguistic weight of each criterion) into the linguistic variables of type 2 and aggregate the linguistic weight of each criterion.

Step 4. Construct weighted decision matrix as Table 4.

Step 5. Calculate positive ideal solution (PIS) and negative ideal solution (NIS) as Table 5.

Step 6. Calculate the distance of each alternative from PIS and NIS as Table 6.

Step 7. Calculate the relative closeness to the ideal solution of each alternative as Table 6.

Step 8. The rank of each alternative based on traditional linguistic TOPSIS is $A_3 > A_4 > A_2 > A_1 > A_5$.

According to new decision making process based on linguistic TOPSIS method, the computational procedures of the problem are summarized as follows.

Step 1. Each expert chooses linguistic variable type to express his/her opinion. Expert D_1, D_2 choices type 1, D_3, D_4 choices type 2, D_5 choices type 3 (See Table 1). And then, each expert uses linguistic variables to express his/her opinions about the performance ratings of each alternative with respect to criteria as Table 2 and importance rating of each criterion as Table 3.

Step 2. Construct decision matrix of each expert.

Step 3. Construct weighted decision matrix of each expert as Table 7.

Step 4. Calculate positive ideal solution (PIS) and negative ideal solution (NIS) of each expert as Table 8.

Step 5. Calculate the distance of each alternative from PIS and NIS of each expert as Table 9.

Step 6. Calculate the relative closeness to the ideal solution of each alternative of each expert as Table 9.

Step 7. Calculate the rank of each alternative of each expert and construct alternative rank matrix as Table 10.

Suppose that the importance of each expert is the same. So, the weight of expert k is $w_e^k = \frac{1}{5} = 0.2$.

If decision maker use weighted ranking value as aggregation method, the aggregated rank of each alternative can be calculated as Table 11. The rank of each alternative based on weighted ranking value is $A_3 > A_2 > A_4 > A_1 > A_5$.

If decision maker use elite selection as aggregation method, the elite selection analysis can be executed as Table 12. The rank of each alternative based on elite selection is $A_3 > A_2 > A_4 > A_1 > A_5$.

If decision maker use elimination method as aggregation method, the elimination method analysis can be executed as Table 13. The rank of each alternative based on elimination method is $A_3 > A_2 > A_4 > A_1 > A_5$.

V. CONCLUSIONS

In this study, we present a last aggregation decision making model. In proposed model, individual ranking of alternatives is determined by linguistic TOPSIS method according to each expert's opinions. Experts' judgments about the ranking of each alternative are aggregated by three aggregation methods (weighted ranking value, elite selection and elimination method). In the future, some comparison between first aggregation decision process and last aggregation decision process will be simulated and other kind of MCDM methods (such as ELECTRE, PROMETHEE, VIKOR, ANP and AHP) can use last aggregation model for making decision in special kind of management situation.

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Table 1. Different types of linguistic variables

Type	Linguistic variable		Figure
1	performance	Extremely Poor (s_0^5), Poor (s_1^5), Fair (s_2^5), Good (s_3^5), Extremely Good (s_4^5)	Fig. 3
	weight	Extremely Low (s_0^5), Low (s_1^5), Fair (s_2^5), High (s_3^5), Extremely High (s_4^5)	
2	performance	Extremely Poor (s_0^7), Poor (s_1^7), Medium Poor (s_2^7), Fair (s_3^7), Medium Good (s_4^7), Good (s_5^7), Extremely Good (s_6^7)	Fig. 4
	weight	Extremely Low (s_0^7), Low (s_1^7), Medium Low (s_2^7), Fair (s_3^7), Medium High (s_4^7), High (s_5^7), Extremely High (s_6^7)	
3	performance	Extremely Poor (s_0^9), Very Poor (s_1^9), Poor (s_2^9), Medium Poor (s_3^9), Fair (s_4^9), Medium Good (s_5^9), Good (s_6^9), Very Good (s_7^9), Extremely Good (s_8^9)	Fig. 5
	weight	Extremely Low (s_0^9), Very Low (s_1^9), Low (s_2^9), Medium Low (s_3^9), Fair (s_4^9), Medium High (s_5^9), High (s_6^9), Very High (s_7^9), Extremely High (s_8^9)	

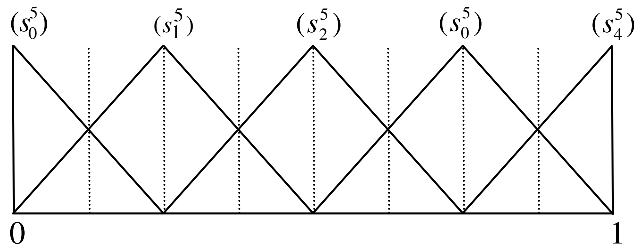

Figure. 3. Membership Functions of Linguistic Variables at Type 1

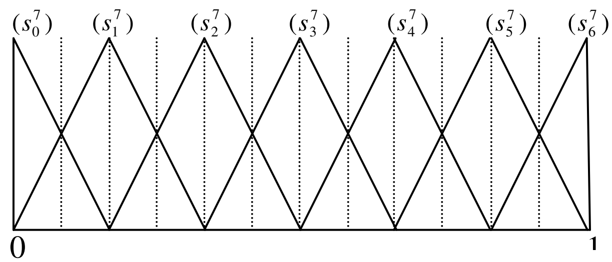

Figure. 4. Membership Functions of Linguistic Variables at Type 2

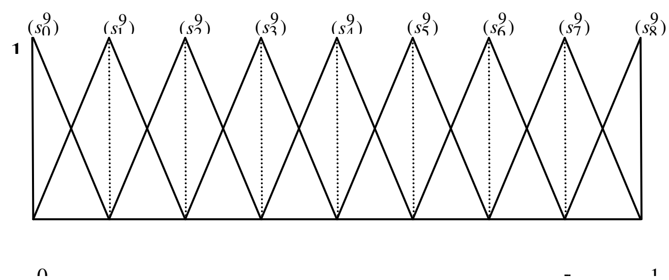

Figure. 5. Membership Functions of Linguistic Variables at Type 3

Table 2. The performance ratings of each alternative

Criterion	applicant	D_1	D_2	D_3	D_4	D_5
C_1	A_1	$(s_2^5,0)$	$(s_4^5,0)$	$(s_5^7,0)$	$(s_5^7,0)$	$(s_8^9,0)$
	A_2	$(s_3^5,0)$	$(s_1^5,0)$	$(s_5^7,0)$	$(s_5^7,0)$	$(s_6^9,0)$
	A_3	$(s_4^5,0)$	$(s_3^5,0)$	$(s_4^7,0)$	$(s_6^7,0)$	$(s_4^9,0)$
	A_4	$(s_4^5,0)$	$(s_1^5,0)$	$(s_4^7,0)$	$(s_5^7,0)$	$(s_7^9,0)$
	A_5	$(s_0^5,0)$	$(s_1^5,0)$	$(s_4^7,0)$	$(s_3^7,0)$	$(s_2^9,0)$
C_2	A_1	$(s_3^5,0)$	$(s_1^5,0)$	$(s_6^7,0)$	$(s_4^7,0)$	$(s_4^9,0)$
	A_2	$(s_3^5,0)$	$(s_4^5,0)$	$(s_1^7,0)$	$(s_4^7,0)$	$(s_6^9,0)$
	A_3	$(s_4^5,0)$	$(s_4^5,0)$	$(s_3^7,0)$	$(s_1^7,0)$	$(s_4^9,0)$
	A_4	$(s_4^5,0)$	$(s_4^5,0)$	$(s_1^7,0)$	$(s_5^7,0)$	$(s_8^9,0)$
	A_5	$(s_1^5,0)$	$(s_0^5,0)$	$(s_2^7,0)$	$(s_3^7,0)$	$(s_4^9,0)$
C_3	A_1	$(s_1^5,0)$	$(s_0^5,0)$	$(s_1^7,0)$	$(s_5^7,0)$	$(s_6^9,0)$
	A_2	$(s_0^5,0)$	$(s_3^5,0)$	$(s_4^7,0)$	$(s_4^7,0)$	$(s_0^9,0)$

	A_3	$(s_4^5,0)$	$(s_1^5,0)$	$(s_2^7,0)$	$(s_4^7,0)$	$(s_7^9,0)$
	A_4	$(s_0^5,0)$	$(s_4^5,0)$	$(s_5^7,0)$	$(s_2^7,0)$	$(s_7^9,0)$
	A_5	$(s_0^5,0)$	$(s_2^5,0)$	$(s_2^7,0)$	$(s_2^7,0)$	$(s_3^9,0)$
C_4	A_1	$(s_0^5,0)$	$(s_4^5,0)$	$(s_3^7,0)$	$(s_5^7,0)$	$(s_4^9,0)$
	A_2	$(s_3^5,0)$	$(s_0^5,0)$	$(s_3^7,0)$	$(s_5^7,0)$	$(s_8^9,0)$
	A_3	$(s_3^5,0)$	$(s_4^5,0)$	$(s_4^7,0)$	$(s_6^7,0)$	$(s_8^9,0)$
	A_4	$(s_3^5,0)$	$(s_3^5,0)$	$(s_2^7,0)$	$(s_6^7,0)$	$(s_2^9,0)$
	A_5	$(s_0^5,0)$	$(s_0^5,0)$	$(s_3^7,0)$	$(s_2^7,0)$	$(s_4^9,0)$
C_5	A_1	$(s_0^5,0)$	$(s_0^5,0)$	$(s_4^7,0)$	$(s_3^7,0)$	$(s_4^9,0)$
	A_2	$(s_4^5,0)$	$(s_3^5,0)$	$(s_6^7,0)$	$(s_3^7,0)$	$(s_7^9,0)$
	A_3	$(s_2^5,0)$	$(s_2^5,0)$	$(s_6^7,0)$	$(s_3^7,0)$	$(s_4^9,0)$
	A_4	$(s_2^5,0)$	$(s_1^5,0)$	$(s_2^7,0)$	$(s_6^7,0)$	$(s_7^9,0)$
	A_5	$(s_2^5,0)$	$(s_1^5,0)$	$(s_0^7,0)$	$(s_4^7,0)$	$(s_6^9,0)$

Table 3. The importance rating of each criterion

Criterion	D_1	D_2	D_3	D_4	D_5
C_1	$(s_4^5,0)$	$(s_4^5,0)$	$(s_4^7,0)$	$(s_6^7,0)$	$(s_5^9,0)$
C_2	$(s_2^5,0)$	$(s_3^5,0)$	$(s_6^7,0)$	$(s_3^7,0)$	$(s_7^9,0)$
C_3	$(s_2^5,0)$	$(s_1^5,0)$	$(s_5^7,0)$	$(s_4^7,0)$	$(s_6^9,0)$
C_4	$(s_1^5,0)$	$(s_2^5,0)$	$(s_3^7,0)$	$(s_4^7,0)$	$(s_8^9,0)$
C_5	$(s_3^5,0)$	$(s_4^5,0)$	$(s_3^7,0)$	$(s_5^7,0)$	$(s_6^9,0)$

Table 4. Weighted decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	0.7153	0.4592	0.2400	0.3306	0.2556
A_2	0.5865	0.4833	0.2500	0.3597	0.6325
A_3	0.6724	0.4592	0.3750	0.5153	0.4600
A_4	0.6223	0.5800	0.3650	0.3597	0.4536
A_5	0.2861	0.2296	0.1850	0.1556	0.3322

Table 5. PIS and NIS

	C_1	C_2	C_3	C_4	C_5
LPIS	0.7153	0.5800	0.3750	0.5153	0.6325
LNIS	0.2861	0.2296	0.1850	0.1556	0.2556

Table 6. Distance from PIS, distance from NIS and relative closeness

	A_1	A_2	A_3	A_4	A_5
Distance from PIS	0.4572	0.2564	0.2149	0.2548	0.7501
Distance from NIS	0.9427	1.0956	1.1705	1.1264	0.2769
Relative closeness	0.6734	0.8103	0.8449	0.8155	0.2696

Table 7. Weighted decision matrix based on each expert's opinion

		C_1	C_2	C_3	C_4	C_5
D_1	A_1	0.5000	0.3750	0.1250	0.0000	0.0000
	A_2	0.7500	0.3750	0.0000	0.1875	0.7500
	A_3	1.0000	0.5000	0.5000	0.1875	0.3750
	A_4	1.0000	0.5000	0.0000	0.1875	0.3750
	A_5	0.0000	0.1250	0.0000	0.0000	0.3750
D_2	A_1	1.0000	0.1875	0.0000	0.5000	0.0000
	A_2	0.2500	0.7500	0.1875	0.0000	0.7500
	A_3	0.7500	0.7500	0.0625	0.5000	0.5000
	A_4	0.2500	0.7500	0.2500	0.3750	0.2500
	A_5	0.2500	0.0000	0.1250	0.0000	0.2500
D_3	A_1	0.5556	1.0000	0.1389	0.2500	0.3333
	A_2	0.5556	0.1667	0.5556	0.2500	0.5000
	A_3	0.4444	0.5000	0.2778	0.3333	0.5000
	A_4	0.4444	0.1667	0.6944	0.1667	0.1667
	A_5	0.4444	0.3333	0.2778	0.2500	0.0000
D_4	A_1	0.8333	0.3333	0.5556	0.5556	0.4167
	A_2	0.8333	0.3333	0.4444	0.5556	0.4167
	A_3	1.0000	0.0833	0.4444	0.6667	0.4167
	A_4	0.8333	0.4167	0.2222	0.6667	0.8333
	A_5	0.5000	0.2500	0.2222	0.2222	0.5556
D_5	A_1	0.6250	0.4375	0.5625	0.5000	0.3750
	A_2	0.4688	0.6563	0.0000	1.0000	0.6563
	A_3	0.3125	0.4375	0.6563	1.0000	0.3750
	A_4	0.5469	0.8750	0.6563	0.2500	0.6563
	A_5	0.1563	0.4375	0.2813	0.5000	0.5625

Table 8. PIS and NIS based on each expert's opinion

		C_1	C_2	C_3	C_4	C_5
D_1	PIS	1.0000	0.5000	0.5000	0.1875	0.7500
	NIS	0.0000	0.1250	0.0000	0.0000	0.0000
D_2	PIS	1.0000	0.7500	0.2500	0.5000	0.7500
	NIS	0.2500	0.0000	0.0000	0.0000	0.0000
D_3	PIS	0.5556	1.0000	0.6944	0.3333	0.5000
	NIS	0.4444	0.1667	0.1389	0.1667	0.0000
D_4	PIS	1.0000	0.4167	0.5556	0.6667	0.8333
	NIS	0.5000	0.0833	0.2222	0.2222	0.4167
D_5	PIS	0.6250	0.8750	0.6563	1.0000	0.6563
	NIS	0.1563	0.4375	0.0000	0.2500	0.3750

Table 9. Distance from PIS ,NIS and relative closeness based on experts' opinion

		A_1	A_2	A_3	A_4	A_5

D_1	Distance from PIS	1.0020	0.5728	0.3750	0.6250	1.2516
	Distance from NIS	0.9354	1.3919	1.5612	1.3919	0.6124
	Relative closeness	0.4828	0.7085	0.8063	0.6901	0.3285
D_2	Distance from PIS	0.9703	0.9036	0.4002	0.9100	1.2809
	Distance from NIS	1.1990	1.2990	1.5207	1.2748	0.6124
	Relative closeness	0.5527	0.5898	0.7917	0.5835	0.3235
D_3	Distance from PIS	0.5860	0.8489	0.6603	0.9196	0.9420
	Distance from NIS	1.1667	1.0541	1.0672	0.8498	0.6236
	Relative closeness	0.6657	0.5539	0.6178	0.4803	0.3983
D_4	Distance from PIS	0.4698	0.4827	0.5450	0.3727	0.8146
	Distance from NIS	1.1180	1.0672	1.0801	1.2360	0.5528
	Relative closeness	0.7041	0.6885	0.6646	0.7683	0.4043
D_5	Distance from PIS	0.7275	0.7092	0.6068	0.7541	0.9003
	Distance from NIS	1.1319	1.2500	1.2500	1.3288	0.8478
	Relative closeness	0.6087	0.63802	0.6732	0.63796	0.4850

Table 10. Alternative rank matrix.

	A_1	A_2	A_3	A_4	A_5
D_1	4	2	1	3	5
D_2	4	2	1	3	5
D_3	1	3	2	4	5
D_4	2	3	4	1	5
D_5	4	2	1	3	5

Table 11. The aggregated rank of each alternative

	A_1	A_2	A_3	A_4	A_5
Average rank	3	2.4	1.8	2.8	5
Overall ranking	4	2	1	3	5

Table 12. The elite selection analysis

	select	A_1	A_2	A_3	A_4	A_5
Round 1	A_3	1	0	3	1	0
Round 2	A_2	1	3	-	1	0
Round 3	A_4	1	-	-	4	0
Round4	A_1	5	-	-	-	0
Round 5	A_5	-	-	-	-	5

Table 13. The elimination method analysis

	select	A_1	A_2	A_3	A_4	A_5
Round 1	A_5	0	0	0	0	5
Round 2	A_1	3	0	1	1	-
Round 3	A_4	-	0	1	4	-
Round 4	A_2	-	4	1	-	-
Round 5	A_3	-	-	5	-	-

基於不同決策資訊整合流程的語意群體多準則決策方法之研究

陳振東
國立聯合大學資訊管理學系
ctchen@nuu.edu.tw

白炳豐
國立暨南國際大學資訊管理學系
paipf@ncnu.edu.tw

洪偉展
國立暨南國際大學國際企業學系
steady_2006@hotmail.com

摘要

群體多準則決策是實際生活上進行決策的主要方法。整合專家意見以進行決策方案的排序有前整合與後整合兩種主要決策流程。一般來說，語意變數適合用來表達專家的主觀意見。本研究目的是介紹不同決策資訊整合流程基礎下的語意理想解類似度偏好順序評估法。在本研究中，三種後整合方法(平均加權法、菁英篩選選法、淘汰法)被用來整合專家各自的決策方案排序結果以產生決策方案的最終排序。最後，提供一個範例說明本研究所提出之方法的決策整合流程，並提出結論。

關鍵詞：群體多準則決策分析、前整合、後整合、語意 TOPSIS。