Quasi-static Channel Assignment Algorithms for Wireless Communications Networks

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Outline

- Introduction
- Problem formulation
- Solution procedures
- Computational experiments
- Summary

Introduction

- Problem Description
  - Given:
    - number of available channels
    - locations of the cells and co-channel interference relations among the cells and
    - channel requirement of each cell (to ensure that the call blocking probability associated with each cell not exceed a given bound)
  - To determine: feasible channel assignment for each cell
  - Subject to:
    - call blocking probability constraint for each cell and
    - co-channel interference constraints between adjacent cells

Introduction (cont'd)

- Multi-coloring problem
- NP-complete
- Using the concept of bisecting search to minimize the number of channels required
- Design criteria
  - quasi-static
  - generic for arbitrary network configurations

Problem Formulation

- Notation
  - $N = \{ 1, 2, \ldots , n \}$: the set of cells
  - $F = \{ 1, 2, \ldots , f \}$: the set of available channels
  - c_j: the channel requirement of Cell j
  - A_{jk}: the indicator function which is 1 if Cell j and Cell k shall not use the same channel (due to co-channel interference) and 0 otherwise
  - x_{ij}: a decision variable which is 1 if channel i is assigned to Cell j and 0 otherwise
  - y_i: a decision variable which is 1 if channel i is assigned to any cell and 0 otherwise

Problem Formulation (cont'd)

- Integer programming formulation

$$Z_{IP} = \min \sum_{i \in F} y_i \qquad \text{(IP)}$$

subject to :

$$(x_{ij} + x_{ij})\, A_{jk} \leq 1 \qquad \forall i \in F,\ j \in L, k \in L \quad (1)$$

$$\sum_{i \in F} x_{ij} \geq c_j \qquad \forall j \in L \quad (2)$$

$$x_{ij} = 0 \text{ or } 1 \qquad \forall i \in F,\ j \in L \quad (3)$$

$$x_{ij} \leq y_i \qquad \forall i \in F,\ j \in L \quad (4)$$

$$y_i = 0 \text{ or } 1 \qquad \forall i \in F. \quad (5)$$

Solution Procedures

- Solution approaches
  - Dual approach
    - The basic approach in the algorithm development is Lagrangean relaxation.
    - A simple but effective procedure was developed to calculate good feasible channel assignment policies.
    - Lower bounds on the optimal objective function values were obtained to evaluate the quality of the heuristic solutions.
  - Primal approach
    - Linear programming relaxation
    - cost-based heuristic

Dual Solution Approach (cont'd)

- Lagrangean relaxation

$$Z_D(u,v) = \min \sum_{i\in F} y_i + \sum_{i\in F}\sum_{j\in L}\sum_{k\in L} u_{ijk}[(x_{ij}+x_{ik})A_{jk}-1] + \sum_{j\in L}\sum_{i\in F} v_{ij}(x_{ij}-y_i)$$

$$= \min \sum_{i\in F}(1-\sum_{j\in L} v_{ij})y_i + \sum_{j\in L}\sum_{i\in F} x_{ij}[\sum_{k\in L}(u_{ijk}+u_{ikj})A_{jk}+v_{ij}] \qquad \text{(LR)}$$

subject to

$$\sum_{i\in F} x_{ij} \ge c_j \qquad \forall j\in L \qquad (6)$$

$$x_{ij} = 0 \text{ or } 1 \qquad \forall i\in F, \forall j\in L \qquad (7)$$

$$y_i = 0 \text{ or } 1 \qquad \forall i\in F. \qquad (8)$$

Dual Solution Approach (cont'd)

- (LR) can be decomposed into two independent and easily solvable subproblems.
- The dual problem (D) is constructed below

$$Z_D = \max_{u, v \geq 0} Z_D(u, v) \qquad \text{(D)}$$

- Due to the nondifferentiable nature of the dual problem (D), the subgradient method is applied as a solution procedure.

Primal Solution Approach

- Algorithm description
  - A cost function $p(i,j,k)$ associated with the assignment of each channel i to a cell j with respect to another cell k is introduced.
  - In each round of the solution procedure, an upper limit of the channels available is specified.
  - The proposed algorithm then iteratively adjusts the cost functions based upon which the channels are assigned.
  - This process is repeated until either a feasible solution is found or a pre-specified number of iterations is exceeded.

Primal Solution Approach (cont'd)

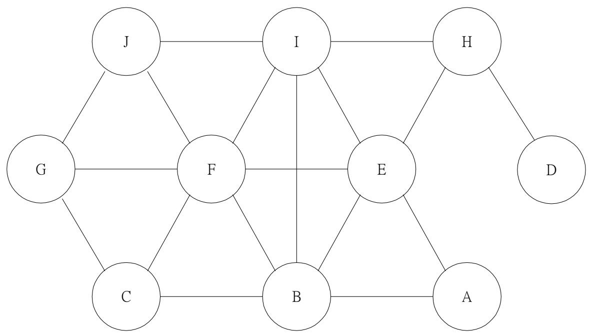
- Algorithm description (cont'd)
  - In the former case, the upper limit of the channels available is reduced, while in the latter case, the upper limit of the channels available is augmented.
  - Then a new round is initiated until no improvement of the total number of channels required is possible.
- Algorithm characteristics
  - Unbalanced cost adjustment for oscillation avoidance
  - Randomization of cost augmentation for stability
  - Step size control for fine tuning and stall avoidance
  - Explicit enumeration for a low degree of violation

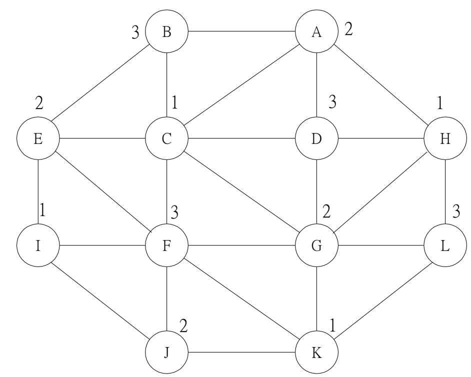
Computation Experiments

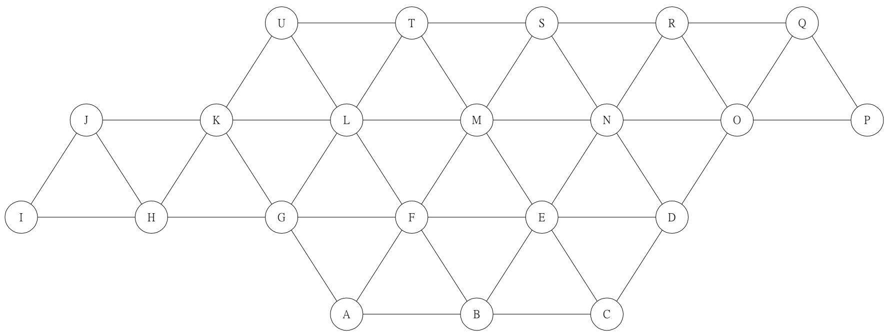
- v is set to 8, q is set to 0.1 and M is set to 10000.
- Two sets of experiments are performed on a PC to test the efficiency and effectiveness of Algorithm A.
  - The first set of experiments are performed on a number of irregular networks.
  - The second set of experiments are performed on a number of n times n regular networks (of hexagonal structures) where n ranges from 4 (16 cells) to 10 (100 cells).

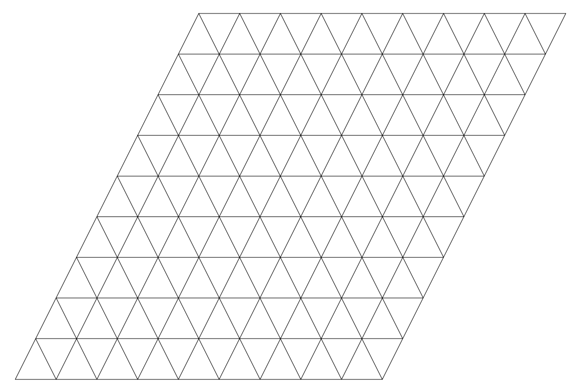
Computation Experiments (cont’d)

- Test network topologies









Computation Experiments (cont'd)

- The lower bounds calculated by linear programming relaxation and Lagrangean relaxation are typically loose.
- The concept of generalized maximum clique can be applied to calculate legitimate lower bounds.
- The approaches of linear programming relaxation and Lagrangean relaxation are shown to be ineffective.
- The proposed primal heuristic calculates optimal solutions for all test examples in minutes of CPU time on a PC.

Summary

- The channel assignment problem in wireless communications networks is considered.
- A mathematical problem formulation is presented.
- The total number of channels required is minimized subject to demand and interference constraints.
- A dual and a primal solution procedure are proposed.
- The proposed algorithms calculate optimal solutions for test networks with up to 100 cells in minutes of CPU time on a PC.