Minimax Open Shortest Path First (OSPF) Routing Algorithms in Networks Supporting the SMDS Service

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Outline

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- The default Inter-Switching System Interface (ISSI) routing algorithm
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Introduction to the SMDS Service

- Switched Multi-megabit Data Service (SMDS) is a public, high-speed, connectionless (datagram), packet switched data service that the Regional Bell Operating companies (RBOCs) have offered.
- Provides LAN-like performance and features over a wide area.
- Regarded as the first phase of B-ISDN
- High-speed access (1.5 Mbps to 45 Mbps)
- Multicast capability

The Default ISSI Routing Algorithm

- Open Shortest Path First (OSPF) routing algorithms
  - Each Switching System (SS) has identical information about (i) the network topology and (ii) the link set metrics.
  - Each SS uses the link set metrics (arc weights) to calculate a shortest path spanning tree (by applying the Dijkstra's algorithm) for each root to transmit individually addressed and group addressed (multicast) traffic.
  - OSPF routing protocols are also widely applied in the Internet and other high-speed networks.

The Default ISSI Routing Algorithm

- The default link set metrics: inversely proportional to the link set capacities
- Advantages
  - Simplicity (static)
  - Minimizing the **total** link set utilization factors
- Disadvantages
  - Does not respond to the network load fluctuation
  - Does not impose link set capacity constraints

Minimax Criteria

- The maximum link set utilization is minimized.
- Advantages:
  - Respond to and balance the network load
  - Remain optimal if network load grows uniformly
  - Robust to demand fluctuation
  - The difficulty of non-linearity is circumvented
  - Perform well with respect to other performance measures, e.g. packet loss rate and average packet delay
  - Conform to the default routing algorithm (OSPF)

Problem Formulation

Notation

- The network is modeled as a graph $G(V, L)$.
- $V = \{1, 2, \ldots, N\}$: the set of nodes in the graph.
- L: the set of links in the graph (network).
- W: the set of O-D pairs (with individually addressed traffic demand) in the network.
- γ_w: the mean arrival rate of new traffic for each O-D pair $w \in W$.
- α_r: the mean arrival rate of multicast traffic for each multicast root $r \in V$.

Problem Formulation (cont'd)

Notation (cont'd)

- P_w: the set of all possible elementary directed paths form the origin to the destination for O-D pair w.
- P: the set of all elementary directed paths in the network, that is, $P = \cup_{w \in W} P_w$.
- O_w: the origin of O-D pair w.
- T_r : the set of all possible spanning trees rooted at r for multicast root r.
- T : the set of all spanning trees in the network, that is, $T = \cup_{r \in V} T_r$.

Problem Formulation (cont'd)

Notation (cont'd)

- C_l : the capacity of link $l \in L$.
- a_l: the link set metric for link $l \in L$ (a decision variable).
- x_p: the routing decision variable which is 1 if path p is used to transmit the packets for O-D pair w and 0 otherwise.
- δ_{pl}: the indicator function which is 1 if link l is on path p and 0 otherwise.

Problem Formulation (cont'd)

Notation (cont'd)

- y_t: the routing decision variable which is 1 if tree $t \in T_r$ is used to transmit the multicast traffic originated at root r and 0 otherwise.
- σ_{tl}: the indicator function which is 1 if link l is on tree t and 0 otherwise.

Problem Formulation (cont'd)

$$Z_{IP'} = \min \max \frac{\sum_{w \in W} \sum_{p \in P_w} x_p \gamma_w \delta_{pl} + \sum_{r \in V} \sum_{t \in T_r} y_t \alpha_r \sigma_{tl}}{C_l} \qquad \text{(IP')}$$

subject to:

$$\sum_{w \in W} \sum_{p \in P_w} x_p \gamma_w \delta_{pl} + \sum_{r \in V} \sum_{t \in T_r} y_t \alpha_r \sigma_{tl} \le C_l \quad \forall l \in L \qquad (1)$$

$$\sum_{p \in P_w} x_p = 1 \quad \forall w \in W \qquad (2)$$

$$\sum_{t \in T_r} y_t = 1 \quad \forall r \in V \qquad (3)$$

$$x_p = 0 \; or \; 1 \quad \forall p \in P_w, w \in W \qquad (4)$$

$$y_l = 0 \; or \; 1 \quad \forall l \in L, r \in V \qquad (5)$$

$$\sum_{\substack{w \in W \\ O_w = r}} \sum_{p \in P_w} x_p \delta_{pl} \le (N-1) \sum_{t \in T_r} y_t \sigma_{tl} \quad \forall l \in L, r \in V \qquad (6)$$

$$\sum_{q \in P_w} \sum_{l \in L} a_l x_q \delta_{ql} \le \sum_{l \in L} a_l \delta_{pl} \quad \forall p \in P_w, w \in W \qquad (7)$$

$$a_l \ge 0 \quad \forall l \in L. \qquad (8)$$

Problem Formulation (cont'd)

Define the following notation $s = \max_{l \in L} \frac{\sum_{w \in W} \sum_{p \in P_w} x_p \gamma_w \delta_{pl} + \sum_{r \in V} \sum_{t \in T_r} y_t \alpha_r \sigma_{tl}}{C_l}$

An equivalent formulation of IP':

$$Z_{IP} = \min s$$

subject to:

$$\sum_{w \in W} \sum_{p \in P_w} x_p \gamma_w \delta_{pl} + \sum_{r \in V} \sum_{t \in T_r} y_t \alpha_r \sigma_{tl} \le C_l \quad \forall l \in L \tag{9}$$

$$\sum_{p \in P_w} x_p = 1 \quad \forall w \in W \tag{10}$$

$$\sum_{t \in T_r} y_t = 1 \quad \forall r \in V \tag{11}$$

$$x_p = 0 \; or \; 1 \quad \forall p \in P_w, w \in W \tag{12}$$

$$y_l = 0 \; or \; 1 \quad \forall l \in L, r \in V \tag{13}$$

$$\sum_{\substack{w \in W \\ O_w = r}} \sum_{p \in P_w} x_p \delta_{pl} \le (N-1) \sum_{t \in T_r} y_t \sigma_{tl} \quad \forall l \in L, r \in V \tag{14}$$

$$\sum_{q \in P_w} \sum_{l \in L} a_l x_q \delta_{ql} \le \sum_{l \in L} a_l \delta_{pl} \quad \forall p \in P_w, w \in W \tag{15}$$

$$a_l \ge 0 \quad \forall l \in L. \tag{16}$$

(17)

Solution Approach

- A dual approach based on Lagrangean relaxation

$$Z_D(u,b)=\min\left\{s+\sum_{l\in L}u_l\Big(\sum_{w\in W}\sum_{p\in P_w}x_p\gamma_w\delta_{pl}+\sum_{r\in V}\sum_{t\in T_r}y_t\alpha_r\sigma_{tl}-C_l s\Big)\right.$$

$$\left.+\sum_{r\in V}\sum_{l\in L}b_{rl}\Big(\sum_{\substack{w\in W\\ O_w=r}}\sum_{p\in P_w}x_p\delta_{pl}-(N-1)\sum_{t\in T_r}y_t\sigma_{tl}\Big)\right\}$$

subject to:

$$\sum_{p\in P_w}x_p=1 \quad \forall w\in W$$

$$\sum_{t\in T_r}y_t=1 \quad \forall r\in V$$

$$x_p=0\ or\ 1 \quad \forall p\in P_w, w\in W$$

$$y_l=0\ or\ 1 \quad \forall l\in L, r\in V$$

$$\sum_{q\in P_w}\sum_{l\in L}a_l x_q\delta_{ql}\le\sum_{l\in L}a_l\delta_{pl} \quad \forall p\in P_w, w\in W$$

$$a_l\ge 0 \quad \forall l\in L.$$

Solution Approach (cont'd)

A dual approach (cont'd)

- (LR) and be decomposed into three independent sub-problems.
  - A trivial problem for *S*
  - A shortest path problem for each O-D pair *w*
  - A minimum cost spanning tree problem for each root *r*
- The dual problem is $Z_D = \max_{u,b \geq 0} Z_D(u,b)$.
- The subgradient method is applied to solve the dual problem.
- A heuristic for determining the link set metrics is to let a_l be u_l.

Solution Approach (cont'd)

A primal approach

(1) Assign an initial value to each a_l. Set the iteration counter k to be 1.

(2) If k is greater than a pre-specified counter limit, stop.

(3) Apply Dijkstra's shortest path algorithm to calculate a shortest path spanning tree for each origin.

(4) Calculate the aggregate flow for each link.

(5) Identify the set of link(s) with the highest utilization, denoted by S.

(6) For each $l \in S$, increase a_l by a positive value t^k.

(7) Increase k by 1 and go to Step 2.

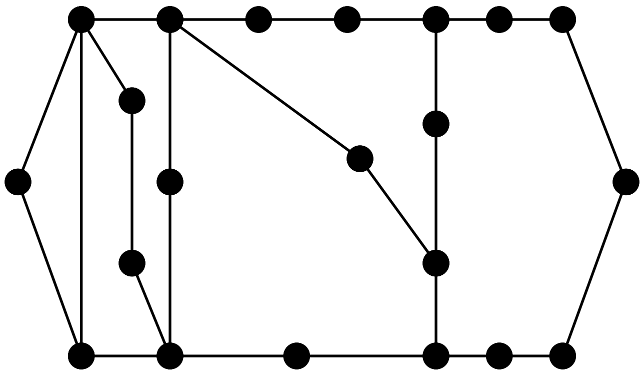
Solution Approach (cont'd)

- The following two properties of $\{t^k\}$ are suggested
  - $\sum_{k=1}^{\infty} t^k$ approaches infinity and
  - t^k approaches 0 as k approaches infinity.
- Advantages of the primal approach
  - The algorithm is simple.
  - Both types of traffic are considered in a uniform way.

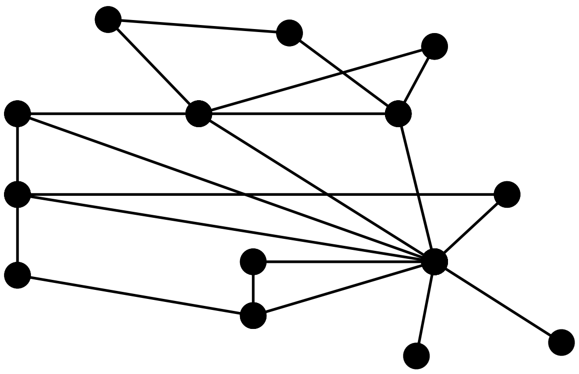
Computational Results

- The dual approach provides lower bounds on Z_{IP} so that the quality of the heuristic solutions can be evaluated.
- The dual approach is expected to perform well when $|L| / |W|$ is small.
- Compared with the default ISSI routing, the minimax routing algorithm based upon the dual approach results in a 7% to 53% improvement in the maximum link utilization.

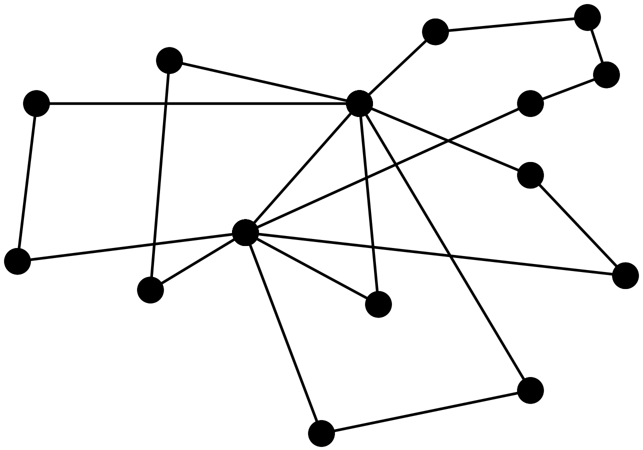
Computational Results (cont'd)



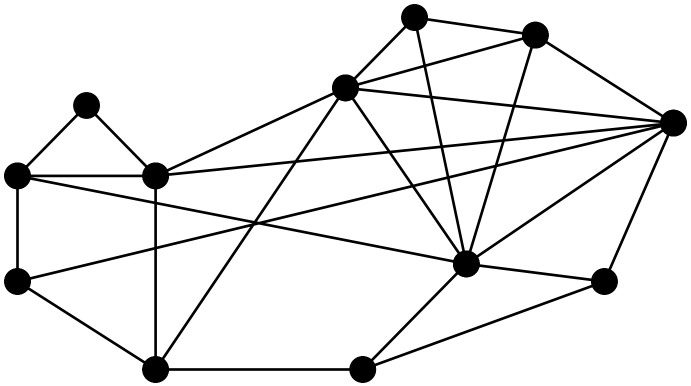
21-node 52-link ARPA2 network



14-node 42-link PSS network



15-node 38-link SWIFT network



12-node 50-link GTE network

Computational Results (cont’d)

- The primal approach is in general (but not uniformly) superior to the dual approach in terms of computation time and quality of solutions.
- It is then suggested that the dual and the primal approaches be applied in a joint fashion to achieve better performance.
- Compared with the default ISSI routing, the joint (combining the primal and the dual approach) minimax routing algorithm results in a 13% to 133% improvement in the maximum link utilization.

Summary

- Investigate more responsive routing algorithms than the ISSI routing scheme (OSPF routing with default link set metrics) for SMDS networks.
- Find a new set of link set metrics such that the maximum link set utilization is minimized.
- Formulate the problem as a nonlinear mixed integer programming problem.
- Propose two solution procedures.
- Compared with the default ISSI routing, the proposed minimax routing algorithm results in a 13% to 133% improvement in the maximum link utilization.