Multiuser OFDM with Adaptive Subcarrier, Bit, and Power Allocation

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***Abstract*—Multiuser orthogonal frequency division multiplexing (OFDM) with adaptive multiuser subcarrier allocation and adaptive modulation is considered. Assuming knowledge of the instantaneous channel gains for all users, we propose a multiuser OFDM subcarrier, bit, and power allocation algorithm to minimize the total transmit power. This is done by assigning each user a set of subcarriers and by determining the number of bits and the transmit power level for each subcarrier. We obtain the performance of our proposed algorithm in a multiuser frequency selective fading environment for various time delay spread values and various numbers of users. The results show that our proposed algorithm outperforms multiuser OFDM systems with static time-division multiple access (TDMA) or frequency-division multiple access (FDMA) techniques which employ fixed and predetermined time-slot or subcarrier allocation schemes. We have also quantified the improvement in terms of the overall required transmit power, the bit-error rate (BER), or the area of coverage for a given outage probability.**

***Index Terms*—Adaptive modulation, frequency selective fading channel, multiaccess communication, multiuser channel, orthogonal frequency division multiplexing (OFDM), resource management.**

I. INTRODUCTION

RECENTLY, intense interest has focused on modulation techniques which can provide broadband transmission over wireless channels for applications including wireless multimedia, wireless Internet access, and future-generation mobile communication systems. One of the main requirements on the modulation technique is the ability to combat intersymbol interference (ISI), a major problem in wideband transmission over multipath fading channels. There are many methods proposed to combat the ISI, e.g., [1]–[3]. Multicarrier modulation techniques, including orthogonal frequency division multiplex (OFDM), (e.g., [4]) are among the more promising solutions to this problem.

Assuming that the transmitter knows the instantaneous channel transfer functions of all users, many papers [5]–[7] have demonstrated that significant performance improvement can be achieved if adaptive modulation is used with OFDM. In particular, subcarriers with large channel gains employ higher order modulation to carry more bits/OFDM symbol, while subcarriers in deep fade carry one or even zero bits/symbol. Integrated design of forward error correcting code and adaptive modulation has also been studied using BCH code and trellis coded modulation (TCM) in [8] and [9], respectively. Although both references considered only time-varying flat fading channels, the same coded adaptive modulation design can be easily applied to OFDM systems. As different subcarriers experience different fades and transmit different numbers of bits, the transmit power levels must be changed accordingly. The problem of optimal power allocation has also been studied in [10].

In this paper, we consider extending OFDM with adaptive modulation to multiuser frequency selective fading environments. When OFDM with adaptive modulation is applied in a frequency selective fading channel, a significant portion of the subcarriers may not be used. These are typically subcarriers which experience deep fade and are not power efficient to carry any information bit. In multiuser systems using static time-division multiple access (TDMA) or frequency-division multiple access (FDMA) as multiaccess schemes, each user is allocated a predetermined time slot or frequency band to apply OFDM with adaptive modulation. Consequently, these unused subcarriers (as a result of adaptive modulation) within the allocated time slot or frequency band of a user are wasted and are not used by other users. However, the subcarriers which appear in deep fade to one user may not be in deep fade for other users. In fact, it is quite unlikely that a subcarrier will be in deep fade for all users, as the fading parameters for different users are mutually independent. This motivates us to consider an adaptive multiuser subcarrier allocation scheme where the subcarriers are assigned to the users based on instantaneous channel information. This approach will allow all the subcarriers to be used more effectively because a subcarrier will be left unused only if it appears to be in deep fade to all users.

We consider a multiuser subcarrier, bit, and power allocation scheme where all users transmit in all the time slots. Our objective is to minimize the overall transmit power by allocating the subcarriers to the users and by determining the number of bits and the power level transmitted on each subcarrier based on the instantaneous fading characteristics of *all* users. In this paper, we formulate the multiuser subcarrier, bit, and power allocation problem and propose an iterative algorithm to perform the multiuser subcarrier allocation. Once

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the subcarrier allocation is determined, the bit and power allocation algorithm can be applied to each user on its allocated subcarriers. We also compare the performance of our proposed solution to various other static subcarrier allocation schemes.

The results of the work can be applied, for instance, to the downlink transmission in a time division duplex (TDD) wireless communication system to improve the downlink capacity. In such a system, the base station (BS) can estimate the instantaneous channel characteristics of all the BS-to-mobile links based on the received uplink transmissions. The multiuser subcarrier, bit, and power allocation can then be used. It is clear that there is a certain amount of transmission overhead as the BS has to inform the mobiles about their allocated subcarriers and the number of bits assigned to each subcarrier.[1] However, this overhead can be relatively small, especially if the channels vary slowly (e.g., in an indoor low mobility environment), and the assignment is done once every many OFDM symbols. To further reduce the overhead, we can assign a contiguous band of subcarriers with similar fading characteristics as a group, instead of assigning each individual subcarrier. In this paper, we will not focus on how the subcarrier allocation information is transmitted. Instead, we will focus on how— and by how much—this new strategy can reduce the required transmit power; or how and by how much this new scheme can improve the bit-error rate (BER) for a fixed transmit power. Alternately, we also consider how and by how much this new scheme can increase the area of coverage for a given transmit power and target BER.

While the bit allocation algorithm can be viewed as a practical implementation of the water-pouring interpretation for achieving the Shannon capacity of an ISI channel [13], the multiuser subcarrier and bit allocation algorithm presented in this paper is the counterpart of the multiuser water-pouring solution given in [14]. In information theoretic studies, the usual approach is to maximize the capacity (or information rate) under the power constraint. In this study, we focus on deriving practical algorithms that can support real-time multimedia data whose bit rates are generally fixed by the compression algorithms. Hence, we assume a given set of user data rates and attempt to minimize the total transmit power under a fixed performance requirement.

The organization of this paper is as follows. In Section II, we will first give the system model and formulate the minimum overall transmit power problem. The optimization problem seeks to minimize the overall transmit power using combined subcarrier, bit, and power allocation schemes for multiuser OFDM systems. The bit and power allocation algorithm for a single-user system is studied in Section III. In Section IV, we derive a lower bound to the minimum overall transmit power by relaxing some of the constraints in the original problem. We also derive a suboptimal subcarrier allocation algorithm. In Section V, we compare the performance between our proposed method and other static approaches via Monte Carlo simulations. Finally, we conclude in Section VI.

II. SYSTEM MODEL

The configuration of our multiuser adaptive OFDM system is shown in Fig. 1. We assume that the system has K users and the kth user has a data rate equal to R_k bit per OFDM symbol. In the transmitter, the serial data from the K users are fed into the subcarrier and bit allocation block which allocates bits from different users to different subcarriers. We assume that each subcarrier has a bandwidth that is much smaller than the coherence bandwidth of the channel and that the instantaneous channel gains on all the subcarriers of all the users are known to the transmitter. Using the channel information, the transmitter applies the combined subcarrier, bit, and power allocation algorithm to assign different subcarriers to different users and the number of bits/OFDM symbol to be transmitted on each subcarrier. Depending on the number of bits assigned to a subcarrier, the adaptive modulator will use a corresponding modulation scheme, and the transmit power level will be adjusted according to the combined subcarrier, bit, and power allocation algorithm. We define $c_{k,n}$ to be the number of bits of the kth user that are assigned to the nth subcarrier. As we do not allow more than one user to share a subcarrier, it follows that for each n, if $c_{k',n} \neq 0$, $c_{k,n} = 0$ for all $k \neq k'$. We also assume that the adaptive modulator allows $c_{k,n}$ to take values in the set $\mathbf{D} = \{0, 1, 2, \ldots, M\}$ where M is the maximum number of information bits/OFDM symbol that can be transmitted by each subcarrier.

The complex symbols at the output of the modulators are transformed into the time domain samples by inverse fast Fourier transform (IFFT). Cyclic extension of the time domain samples, known as the guard interval, is then added to ensure orthogonality between the subcarriers, provided that the maximum time dispersion is less than the guard interval. The transmit signal is then passed through different frequency selective fading channels to different users.

We assume that the subcarrier and bit allocation information is sent to the receivers via a separate control channel. At the receiver, the guard interval is removed to eliminate the ISI, and the time samples of the kth user are transformed by the FFT block into modulated symbols. The bit allocation information is used to configure the demodulators while the subcarrier allocation information is used to extract the demodulated bits from the subcarriers assigned to the kth user.

In the frequency selective fading channel, different subcarriers will experience different channel gains. We denote by $\alpha_{k,n}$ the magnitude of the channel gain (assuming coherent reception) of the nth subcarrier as seen by the kth user. We assume that the single-sided noise power spectral density (PSD) level N_0 is equal to unity (i.e., $N_0 = 1$), for all

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[1] Note that the power level used does not need to be transmitted to the receiver in such a TDD system. As the subcarrier gain is known to the transmitter, it can adjust the transmit power level to achieve a predetermined receiver power level based on the number of bits allocated to that subcarrier. However, in FDD systems, the transmit power levels determined by the receiver have to be sent back to the transmitter. In such systems, the additional performance gain achieved by power allocation may not justify the cost of sending the transmit power level information to the transmitter.

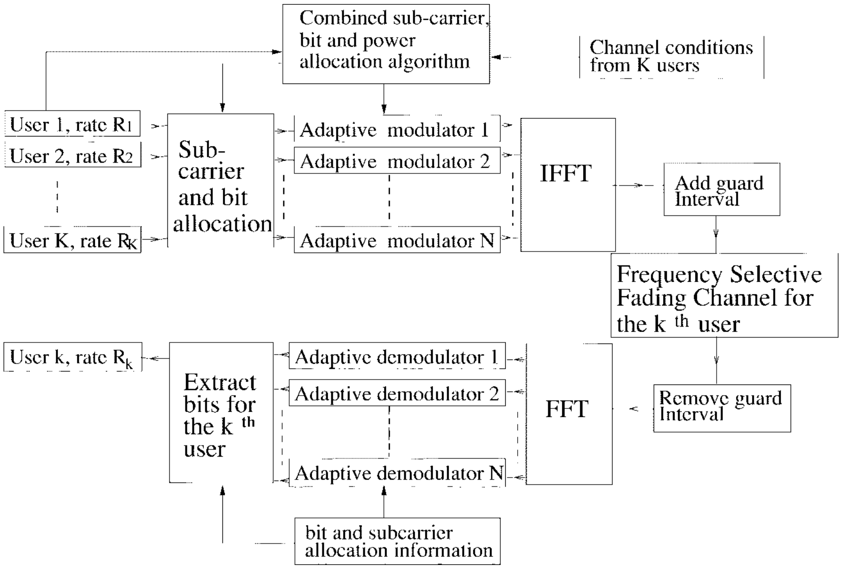

Fig. 1. Block diagram of a multiuser OFDM system with subcarrier, bit, and power allocation.

subcarriers and is the same for all users. Furthermore, we denote by $f_k(c)$ the required received power (in energy per symbol) in a subcarrier for reliable reception of c information bits/symbol when the channel gain is equal to unity. Note that the function $f_k(c)$ depends on k, and this allows different users to have different quality-of-service (QoS) requirements and/or different coding and modulation schemes. In order to maintain the required QoS at the receiver, the transmit power, allocated to the nth subcarrier by the kth user must equal

$$P_{k,n} = \frac{f_k(c_{k,n})}{\alpha_{k,n}^2}. \tag{1}$$

Using these transmit power levels, the receiver can demodulate the modulated symbols at the output of the FFT processor and achieve the desired QoS's of all users.

The goal of the combined subcarrier, bit, and power allocation algorithm is then to find the best assignment of $c_{k,n}$ so that the overall transmit power, the sum of $P_{k,n}$ over all subcarriers and all users, is minimized for given transmission rates of the users and given QoS requirements specified through $f_k()$, $k = 1, \ldots, K$. In order to make the problem tractable, we further require that $f_k(c)$ is a convex and increasing function with $f_k(0) = 0$. This condition essentially means that no power is needed when no bit is transmitted and that the required additional power to transmit an additional bit increases with c [i.e., $f_k(c+1) - f_k(c)$ is increasing in c]. Almost all popular coding and modulation schemes satisfy this condition.

It is important to note that even though the problem is formulated to minimize the overall transmit power for given QoS requirements, the same solution can be applied to improve the QoS's of the users for a given overall transmit power. The latter can simply be achieved by increasing the power proportionally for all the subcarriers, while using the same set of $c_{k,n}$.

Mathematically, we can formulate the problem as

$$P_T^* = \min_{c_{k,n} \in \mathbf{D}} \sum_{n=1}^{N} \sum_{k=1}^{K} \frac{1}{\alpha_{k,n}^2} f_k(c_{k,n}) \tag{2}$$

and the minimization is subjected to the constraints

$$\mathbf{C1}\text{: For all } k \in \{1, \ldots, K\},\ R_k = \sum_{n=1}^{N} c_{k,n} \tag{3}$$

and

$$\begin{aligned}&\mathbf{C2}\text{: For all } n \in \{1, \ldots, N\},\\ &\quad \text{if there exists } k' \text{ with } c_{k',n} \neq 0, \text{ then } c_{k,n} = 0,\\ &\quad \forall k \neq k'.\end{aligned} \tag{4}$$

Note that constraint (3) is the data rate requirement and constraint (4) ensures that each subcarrier can only be used by one user. Moreover, $\mathbf{D} = \{0, 1, 2, \ldots, M\}$ is the set of all possible values for $c_{k,n}$, and $c_{k,n} = 0$ means that the kth user does not use the nth subcarrier to transmit any information.

III. BIT ALLOCATION ALGORITHM FOR SINGLE USER CHANNEL

Before we try to solve the multiuser allocation problem, we will first derive the bit allocation algorithm for the single-user environment. The single-user problem not only gives better understanding of the issues involved, but also provides a bit allocation algorithm that we will use in our multiuser solution.

We can rewrite the optimization problem in (2) for the single-user case as

$$P_T^* = \min_{c_n \in \mathbf{D}} \sum_{n=1}^{N} \frac{1}{\alpha_n^2} f(c_n) \tag{5}$$

and the minimization is under the constraint

$$R = \sum_{n=1}^{N} c_n. \tag{6}$$

Note that we have dropped the subscript k, which denotes the user in all notations.

As the power needed to transmit a certain number of bits in a subcarrier is independent of the numbers of bits allocated to other subcarriers, it turns out that a greedy approach is optimal. A greedy algorithm assigns bits to the subcarriers one bit at a time, and in each assignment, the subcarrier that requires the least additional power is selected. The bit allocation process will be completed when all R bits are assigned. Several papers (e.g., [15] and [16]) have provided various algorithms for this problem, and the basic structure of most algorithms are similar and can be described as follows:

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Initialization:
    For all $n$, let $c_n = 0$ and $\Delta P_n = [f(1) - f(0)]/\alpha_n^2$;
Bit Assignment Iterations:
    Repeat the following $R$ times:
        $\hat{n} = \arg\min_n \Delta P_n$;
        $c_{\hat{n}} = c_{\hat{n}} + 1$;
        $\Delta P_{\hat{n}} = [f(c_{\hat{n}} + 1) - f(c_{\hat{n}})]/\alpha_{\hat{n}}^2$;
    End;
Finish:
    $\{c_n\}_{n=1}^N$ is the final bit allocation solution.
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The initialization stage computes, for each subcarrier, the additional power needed to transmit an additional bit. For each bit assignment iteration, the subcarrier that needs the minimum additional power is assigned one more bit, and the new additional power for that subcarrier is updated. After R iterations, the final bit assignment gives the optimal bit allocation for each subcarrier. It is important to note that the bit allocation is optimal only for the given function $f(c)$, which depends on the selected modulation scheme. Different modulation schemes will lead to different $f(c)$, different bit allocation, and possibly lower transmit power P_T^*.

The concept of this algorithm is fairly simple, and many similar algorithms based on the same principle have been obtained before. In particular, there exist faster and less complex algorithms which can speed up the bit allocation process significantly (e.g., [15] and [16]). In our simulations, we use the algorithm given in [16].

IV. MULTIUSER SUBCARRIER AND BIT ALLOCATION

We have observed that, in the single-user case, a greedy approach which assigns one bit at a time to the subcarrier that requires the least additional power gives the optimal allocation in the sense of minimizing the overall transmit power. Unfortunately, the problem becomes more difficult in the multiuser environment. As users cannot share the same subcarrier, allocating bits to a subcarrier essentially prevents other users from using that subcarrier. This dependency makes any greedy algorithm a nonoptimal solution. It turns out that the optimal solution may not assign any of a user's bits to the best subcarrier seen by that user. This may happen when the best subcarrier of a user is also the best subcarrier of another user who happens to have no other good subcarriers. Hence, the multiuser subcarrier and bit allocation problem is much more complicated to solve than that of the single-user case.

It turns out that the optimization problem in (2) is a combinatorial optimization problem. To make the problem tractable, we consider a different but similar optimization problem. We relax the requirement $c_{k,n} \in \mathbf{D}$ to allow $c_{k,n}$ to be a real number within the interval $[0, M]$. Moreover, in order to deal with constraint (4), K variables, $\rho_{k,n}$, $k = 1, \ldots, K$, with values within the interval [0, 1], are introduced to the cost function as sharing factors of the nth subcarrier. The new optimization problem becomes

$$\underline{P}_T = \min_{\substack{c_{k,n} \in [0, M] \\ \rho_{k,n} \in [0, 1]}} \sum_{n=1}^{N} \sum_{k=1}^{K} \frac{\rho_{k,n}}{\alpha_{k,n}^2} f_k(c_{k,n}) \tag{7}$$

where $c_{k,n}$ and $\rho_{k,n}$ have to satisfy

$$R_k = \sum_{n=1}^{N} \rho_{k,n} c_{k,n}, \qquad \text{for all } k \in \{1, \ldots, K\} \tag{8}$$

and

$$1 = \sum_{k=1}^{K} \rho_{k,n}, \qquad \text{for all } n \in \{1, \ldots, N\}. \tag{9}$$

For any valid set of $c_{k,n} \in \mathbf{D}$ satisfying the constraints (3) and (4) in the original optimization problem, we can let

$$\rho_{k,n} = \begin{cases} 1, & \text{if } c_{k,n} \neq 0, \\ 0, & \text{if } c_{k,n} = 0. \end{cases} \tag{10}$$

Then, it is easy to show that the same set of $c_{k,n}$ and the corresponding $\rho_{k,n}$ defined in (10) satisfy the constraints (8) and (9) in the new optimization problem. Moreover, with $\rho_{k,n}$ defined in (10), the new cost function in (7) is equal to the cost function in (2). Hence, the minimization problem in (7) is the same as the original optimization problem, except that the minimization is done over a larger set. Consequently, the minimum power obtained in (7) $\underline{P}_T$ is a lower bound to the minimum power obtained in (2), P_T^*.

Another way to interpret the optimization in (7) is to consider $\rho_{k,n}$ as the time-sharing factor for the kth user of the nth subcarrier. For example, in every L OFDM symbol (L being a very large number), user k uses the nth subcarrier in $L\rho_{k,n}$ symbols. Clearly, the average (over L symbols) information data rate and the average transmit power has to be scaled by the same factor $\rho_{k,n}$. Hence, we can consider (7) as the optimization problem when the users are allowed to time-share each subcarrier over a large number of OFDM symbols. However, most wireless communication channels are time varying, and the channels may not stay unchanged long enough for timesharing to be feasible. Hence, in this paper, we will continue to consider the original problem in (2) and use the optimization problem in (7) as a lower bound, even though it has its own physical interpretation.

The modified optimization problem in (7) is more tractable. However, even though the function $f_k(c)$ is convex in c, the terms in the cost function have the form $\rho f_k(c)$, and as a function of (ρ, c), $\rho f_k(c)$ is not convex in (ρ, c). To proceed further, we let $r_{k,n} = c_{k,n}\rho_{k,n}$ and rewrite the cost function in terms of $r_{k,n}$ and $\rho_{k,n}$. The constraint on $r_{k,n}$

becomes $r_{k,n} \in [0, M\rho_{k,n}]$, and it can be easily shown that $\rho f_k(c) = \rho f_k(r/\rho)$ is convex in (ρ, r) within the triangular region specified by $\rho \in [0, 1]$ and $r \in [0, M\rho]$. In particular, the Hessian evaluated at any point within this region is a positive semidefinite matrix. Hence, we can reformulate the optimization problem in (7) as a convex minimization problem over a convex set. That is

$$\underline{P}_T = \min_{\substack{r_{k,n}\in[0,\, M\rho_{k,n}]\\ \rho_{k,n}\in[0,\, 1]}} \sum_{n=1}^{N}\sum_{k=1}^{K} \frac{\rho_{k,n}}{\alpha_{k,n}^2} f_k\left(\frac{r_{k,n}}{\rho_{k,n}}\right) \tag{11}$$

where $r_{k,n}$ and $\rho_{k,n}$ have to satisfy

$$R_k = \sum_{n=1}^{N} r_{k,n}, \qquad \text{for all } k \in \{1, \ldots, K\} \tag{12}$$

and

$$1 = \sum_{k=1}^{K} \rho_{k,n}, \qquad \text{for all } n \in \{1, \ldots, N\}. \tag{13}$$

Using standard optimization techniques in [17], we obtain the Lagrangian

$$\begin{aligned} L = &\sum_{n=1}^{N}\sum_{k=1}^{K} \frac{\rho_{k,n}}{\alpha_{k,n}^2} f_k\left(\frac{r_{k,n}}{\rho_{k,n}}\right) - \sum_{k=1}^{K} \lambda_k \left(\sum_{n=1}^{N} r_{k,n} - R_k\right)\\ &- \sum_{n=1}^{N} \beta_n \left(\sum_{k=1}^{K} \rho_{k,n} - 1\right) \end{aligned} \tag{14}$$

where λ_k and β_n are the Lagrangian multipliers for the constraints (12) and (13), respectively.

After differentiating L with respect to $r_{k,n}$ and $\rho_{k,n}$, respectively, we obtain the necessary conditions for the optimal solution, $r_{k,n}^*$ and $\rho_{k,n}^*$. Specifically, if $\rho_{k,n}^* \neq 0$, we have

$$\begin{aligned} &\left.\frac{\partial L}{\partial r_{k,n}}\right|_{(r_{k,n},\, \rho_{k,n}) = (r_{k,n}^*,\, \rho_{k,n}^*)}\\ &\quad = \frac{1}{\alpha_{k,n}^2} f_k'\left(\frac{r_{k,n}^*}{\rho_{k,n}^*}\right) - \lambda_k \begin{cases} > 0, & \text{if } r_{k,n}^* = 0\\ = 0, & \text{if } r_{k,n}^* \in (0, M\rho_{k,n}^*)\\ < 0, & \text{if } r_{k,n}^* = M\rho_{k,n}^* \end{cases} \end{aligned} \tag{15}$$

and

$$\begin{aligned} &\left.\frac{\partial L}{\partial \rho_{k,n}}\right|_{(r_{k,n},\, \rho_{k,n}) = (r_{k,n}^*,\, \rho_{k,n}^*)}\\ &\quad = \frac{1}{\alpha_{k,n}^2}\left[f_k\left(\frac{r_{k,n}^*}{\rho_{k,n}^*}\right) - f_k'\left(\frac{r_{k,n}^*}{\rho_{k,n}^*}\right)\frac{r_{k,n}^*}{\rho_{k,n}^*}\right]\\ &\qquad - \beta_n \begin{cases} = 0, & \text{if } \rho_{k,n}^* \in (0, 1)\\ < 0, & \text{if } \rho_{k,n}^* = 1. \end{cases} \end{aligned} \tag{16}$$

On the other hand, if $\rho_{k,n}^* = 0$, then $r_{k,n}^* = 0$, and we have

$$\begin{aligned} &r_{k,n}\frac{\partial L}{\partial r_{k,n}} + \rho_{k,n}\frac{\partial L}{\partial \rho_{k,n}} \geq 0,\\ &\qquad \text{for all } \rho_{k,n} \in (0, 1] \text{ and } r_{k,n} \in (0, M\rho_{k,n}]. \end{aligned} \tag{17}$$

These necessary conditions can be interpreted by the fact that if the minimum occurs within the constrained region [(0, 1) for $\rho_{k,n}$ and $(0, M\rho_{k,n})$ for $r_{k,n}$], then the derivative evaluated at the minimum point must be zero. On the other hand, if the optimal solution occurs at a boundary point, then the derivative must be positive along all directions pointing toward the interior of the constraint set. Then, (17) follows from considering the boundary point at $(r_{k,n}^*, \rho_{k,n}^*) = (0, 0)$.

From (15) and (17), we can conclude that

$$r_{k,n}^* = \rho_{k,n}^* f_k'^{-1}(\lambda_{q,k}\alpha_{k,n}^2) \tag{18}$$

where

$$\lambda_{q,k} = \begin{cases} f_k'(0)/\alpha_{k,n}^2, & \text{if } f_k'^{-1}(\lambda_k\alpha_{k,n}^2) < 0;\\ \lambda_k, & \text{if } 0 \leq f_k'^{-1}(\lambda_k\alpha_{k,n}^2) \leq M;\\ f_k'(M)/\alpha_{k,n}^2, & \text{if } f_k'^{-1}(\lambda_k\alpha_{k,n}^2) > M. \end{cases}$$

Moreover, from (16) and (17), it follows that

$$\rho_{k,n}^* = \begin{cases} 0, & \text{if } \beta_n < H_{k,n}(\lambda_{q,k})\\ 1, & \text{if } \beta_n > H_{k,n}(\lambda_{q,k}) \end{cases} \tag{19}$$

where

$$H_{k,n}(\lambda) = \frac{1}{\alpha_{k,n}^2}\left[f_k\left(f_k'^{-1}(\lambda\alpha_{k,n}^2)\right) - \lambda\alpha_{k,n}^2 f_k'^{-1}(\lambda\alpha_{k,n}^2)\right]. \tag{20}$$

Since constraint (13) must be satisfied, we find from (19) that for each n, if $H_{k,n}(\lambda_{q,k})$ for $k = 1, \ldots, K$ are all different, then only the user with the smallest $H_{k,n}(\lambda_{q,k})$ can use that subcarrier. In other words, for the nth subcarrier, if $H_{k,n}(\lambda_{q,k})$ are different for all k, then

$$\rho_{k',n}^* = 1, \ \rho_{k,n}^* = 0, \qquad \text{for all } k \neq k' \tag{21}$$

where

$$k' = \arg\min_{k} H_{k,n}(\lambda_{q,k}). \tag{22}$$

Hence, it follows that for a fixed set of Lagrange multipliers λ_k, $k = 1, \ldots, K$, we can use them to determine k' for each n using (22). The $r_{k,n}^*$ and $\rho_{k,n}^*$ obtained will then form an optimal solution for the optimization problem; however, the individual rate constraint (12) may not be satisfied.

In order to find the set of λ_k such that the individual rate constraints are satisfied, we have obtained an iterative searching algorithm. Starting with some small values for all λ_k, this iterative procedure increases one of the λ_k until the data rate constraint (12) for user k is satisfied. Then, we switch to another user and go through the users one at a time. This process repeats for all users until the data rate constraint for all users are satisfied. This algorithm converges because for a given k, as λ_k increases, $H_{k,n}(\lambda_{q,k})$ for all n decreases, and more $\rho_{k,n}^*$ in (19) become one while $r_{k,n}^*$ in (18) increases for those n where $\rho_{k,n}^* > 0$. Hence, $\sum_{n=1}^{N} r_{k,n}^*$ increases. During this process, some of the other $\rho_{k',n}^*$ may change from one to zero and consequently decrease the total data rate for other users. However, as all the λ_k increase, $r_{k,n}^*$ increases accordingly. As long as the total data rate is less than MN bits/symbol, which is the total number of bits possibly transmitted within an OFDM symbol, the algorithm will converge to a solution that satisfies all the constraints. Since the optimization problem is a convex optimization problem over a convex set, the set of necessary conditions is also sufficient, and the solution that satisfies all the necessary conditions is the unique optimal solution.

In the process of adjusting λ_k for $k = 1, \ldots, K$, the situation where, for a fixed n, more than one $H_{k,n}(\lambda_{q,k})$ has the same values cannot be ignored. In that case, $\rho^*_{k,n}$ has to take values within the interval (0, 1). This solution suggests that the subcarrier should be shared by multiple users. In practice, this can be done by having these users with $\rho^*_{k,n} > 0$ time share the nth subcarrier, and the ratio of the symbols used by different users are set proportionally to $\rho^*_{k,n}$. The detailed flow chart of the algorithm is given in the Appendix.

Now, we have an algorithm to obtain the optimal values of $\rho^*_{k,n}$ and

$$c^*_{k,n} = \begin{cases} r^*_{k,n}/\rho^*_{k,n}, & \text{if } \rho^*_{k,n} \neq 0 \\ 0, & \text{otherwise.} \end{cases} \tag{23}$$

This solution, when substituted in (7), gives a lower bound to the minimum overall transmit power. However, we cannot use these results immediately in (2). One problem is that $c^*_{k,n}$ may not be in $\mathbf{D}$, and the other is that some $\rho^*_{k,n}$ may be within (0, 1), indicating a time-sharing solution. Furthermore, simply quantizing $c^*_{k,n}$ and $\rho^*_{k,n}$ will not satisfy the individual rate constraints in (3).

To solve this problem, we propose a multiuser adaptive OFDM (MAO) scheme where the subcarrier allocation follows essentially the solution to the lower bound in (7), and then the single-user bit allocation algorithm given in Section III is applied to each user on the allocated subcarriers. Specifically, we modify $\rho^*_{k,n}$ for the optimization problem in (7) by letting for each n $\rho^*_{k',n} = 1$ where $k' = \arg\max_k \rho^*_{k,n}$, and $\rho^*_{k,n} = 0$ for $k \neq k'$. Then, we apply the single-user bit allocation algorithm on each user using the assigned subcarriers. We denote the total transmit power (in energy/symbol) obtained using this MAO scheme by P_T. It is easy to see that $\underline{P_T} \leq P^*_T \leq P_T$, where P^*_T is the minimum power in the original problem, and $\underline{P_T}$ is the minimum power for the modified problem with the relaxed constraints. More specifically, the difference between P_T and the minimum $\underline{P_T}$ gives an upper bound to how far away our MAO scheme is from the solution of our original optimization problem.

V. PERFORMANCE COMPARISON

In this section, we obtain and compare the performance of the MAO scheme with other static subcarrier allocation schemes. We consider a system that employs M-ary quadrature amplitude modulation (MQAM) with $\mathbf{D} = \{0, 2, 4, 6\}$. Square signal constellations (4-QAM, 16-QAM, and 64-QAM) are used to carry two, four, or six bits/symbol. The bit-error probability is upper bounded by the symbol error probability, which is tightly approximated by $4Q[\sqrt{d^2/(2N_0)}]$ [12, p. 281], where d is the minimum distance between the points in the signal constellation. Since the average energy of a M-QAM symbol is equal to $(M-1)d^2/6$, it follows that the required power for supporting c bits/symbol at a given BER P_e is

$$f(c) = \frac{N_0}{3}\left[Q^{-1}\left(\frac{P_e}{4}\right)\right]^2 (2^c - 1)$$

where we recall that

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\, dt.$$

It is easy to see that $f(c)$ is convex and increasing in c and that $f(0) = 0$.

To evaluate the performance of our scheme, we have simulated 1000 sets of five-path frequency selective Rayleigh fading channels with an exponential power delay profile. Each set of channels consists of K independent channels, one for each user. We use an OFDM system with 128 subcarriers over a 5 MHz band along with a total (over all users) transmission rate equal to 512 bits/symbol (or equivalently, an average of four bits/subcarrier). Recall that the single-sided power spectral density level N_0 is equal to unity, and we assume that the average subcarrier channel gain $\mathbf{E}|\alpha_{k,n}|^2$ is equal to unity for all k and n.

For comparison purposes, we have also considered three other static multiuser subcarrier allocation methods. Two of them are based on the multiple access methods described in [7]. The methods are presented as follows.

- OFDM-TDMA: each user is assigned a predetermined TDMA time slot and can use all the subcarriers within that time slot exclusively.
- OFDM-FDMA: each user is assigned a predetermined band of subcarriers and can only use those subcarriers exclusively in every OFDM symbol.

  In a frequency selective fading channel, there is a high correlation between the channel gains of adjacent subcarriers. In order to avoid the situation where all subcarriers of a user are in deep fade, we propose an enhanced version of OFDM-FDMA, which we shall refer to as OFDM Interleaved-FDMA.
- OFDM Interleaved-FDMA: this is the same as OFDM-FDMA except that subcarriers assigned to a user are interlaced with other users' subcarriers in the frequency domain.

The time and subcarrier assignment of these three multiuser OFDM schemes are illustrated in Fig. 2. Note that these static schemes have predetermined subcarrier allocations which are independent of the channel gains of the users. The main difference between the proposed MAO scheme and these static schemes is that MAO assigns subcarriers adaptively based on the instantaneous channel gains. To ensure a fair comparison, we use the optimal single-user bit allocation (OBA) for each user on the assigned subcarriers. For comparison purposes, we also show the results when equal bit allocation (EBA) is employed on the assigned subcarriers for these three OFDM schemes. Notice that when using EBA, all three schemes will have the same performance in an uncoded system. This is because the average bit signal-to-noise ration (SNR) needed is a function of only the marginal probability density function of each subcarrier gain.

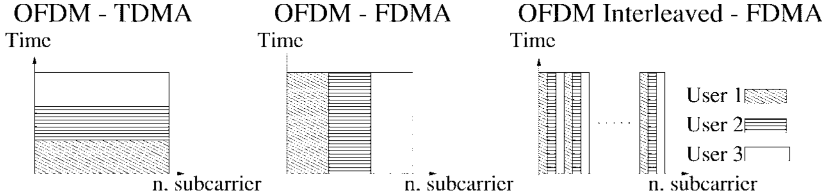

Fig. 2. Subcarrier and time-slot allocations of OFDM-TDMA, OFDM-FDMA, and OFDM interleaved-FDMA schemes.

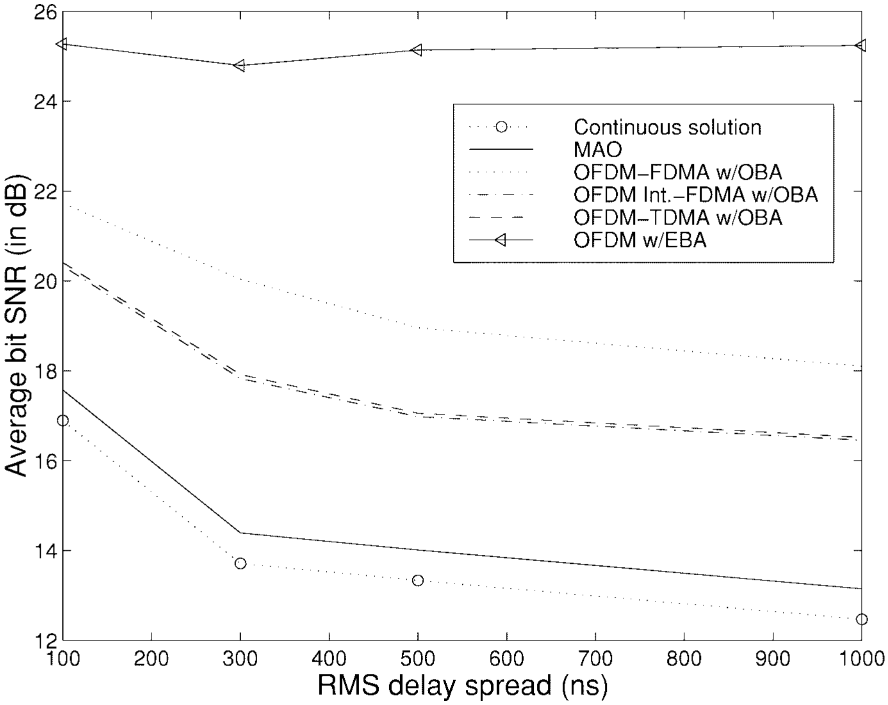

Fig. 3. Average bit signal-to-noise ration (SNR) required by different schemes in various root mean square (RMS) delay spreads in a five-user system with $P_e = 10^{-4}$.

Fig. 3 shows the average bit SNR needed to achieve a BER at $P_e = 10^{-4}$ for a five-user system versus the root mean square (RMS) delay spread (for definition, see for example [18, p. 160]) for different multiuser OFDM schemes. The average required transmit power (in energy per bit) is defined as the ratio of the overall transmit energy per OFDM symbol (including all subcarriers and all users) to the total number of bits transmitted per OFDM symbol. Moreover, we define the average bit SNR as the ratio of the average transmit power to the noise PSD level N_0. As we assume that the data rate is fixed and that N_0 is just a constant, the overall transmit power is proportional to the average bit SNR. For ease of comparison, we have used the average bit SNR for comparison. We find in Fig. 3 that the MAO scheme is never more than 0.6 dB from the lower bound. Since the bit SNR of the optimal combined subcarrier, bit, and power allocation algorithm must lie between the bit SNR's achieved by the lower bound and the MAO scheme, we find that the MAO scheme is never more than 0.6 dB away from the optimal solution. On the other hand, we observe that our proposed MAO scheme is 3–5 dB better than the static subcarrier allocation schemes with OBA, which are in turn 5–10 dB better than that with EBA. We also find that when OBA is used, the OFDM interleaved-FDMA scheme and the OFDM-TDMA scheme have very similar performance, and both of them outperform the OFDM-FDMA scheme.[2] A closer observation of Fig. 3 also indicates that the gains achieved by optimal bit allocation and optimal multiuser subcarrier allocation increase with the RMS delay spread. This is mainly because the larger the RMS delay spread, the more the fading variation and hence higher gains can be obtained when the allocation is performed adaptively.

Fig. 4 shows the average bit SNR (in dB) needed to achieve the same BER versus the number of users when the RMS delay spread is 100 ns. We find that the savings in the required bit SNR achieved by MAO when compared to other schemes are roughly the same, independent of the number of users in the system.

While these two figures show the improvement in the required bit SNR, the results can perhaps be more easily understood using the more familiar BER versus bit SNR curves. For each BER requirement, we compute $f(c)$ for all $c \in \mathbf{D}$ and then use our algorithm to calculate the subcarrier

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[2] OFDM-FDMA refers only to the specific FDMA scheme that assigns to each user a contiguous band of subcarriers as shown in Fig. 2, but not the general FDMA schemes. In fact, both OFDM interleaved-FDMA and MAO can be considered as different forms of FDMA and they are not outperformed by the OFDM-TDMA scheme.

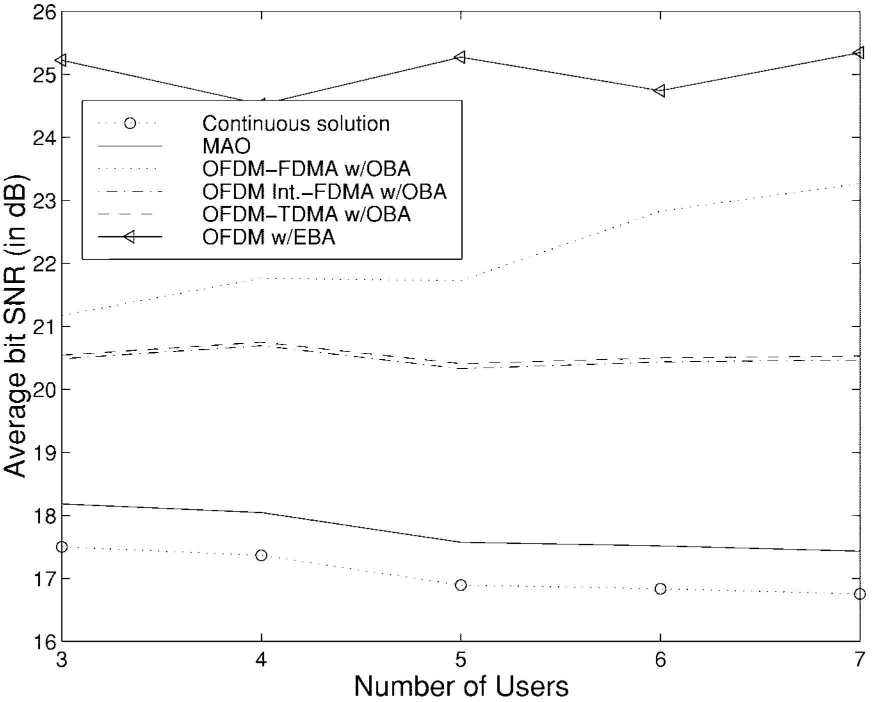

Fig. 4. Average bit SNR required by different schemes versus the number of users in a multiuser OFDM system with 100 ns RMS delay spread, and $P_e = 10^{-4}$.

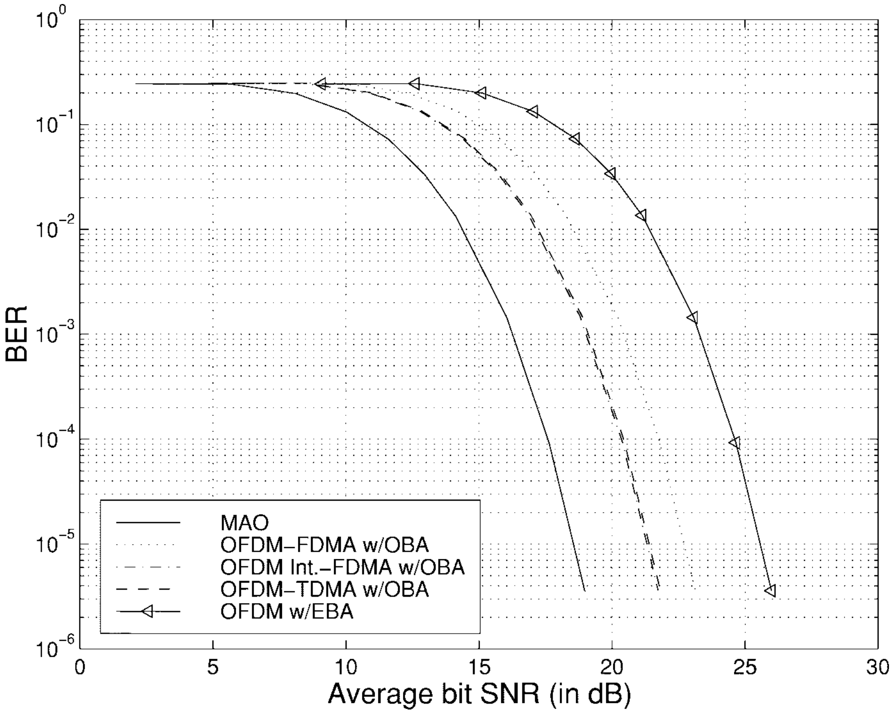

Fig. 5. BER versus average bit SNR for various subcarrier allocation schemes.

allocation for the MAO case. For all other static subcarrier allocation schemes, the allocations are independent of the BER. Once the subcarrier allocation is fixed, we apply the optimal bit and power allocation algorithm to every user. The final average power per bit divided by the noise power spectral density level gives the average bit SNR. We repeat this procedure for different BER values, and the results are plotted in Fig. 5 for a five-user system with an RMS delay spread equal to 100 ns. We find that our proposed MAO has at least 3–4 dB advantage over all other schemes.

Another way to illustrate the impact of the bit and subcarrier allocation is to consider the area of coverage for a given outage probability, assuming that the BS has a maximum transmit power. We consider a circular cell with five users, indepen-

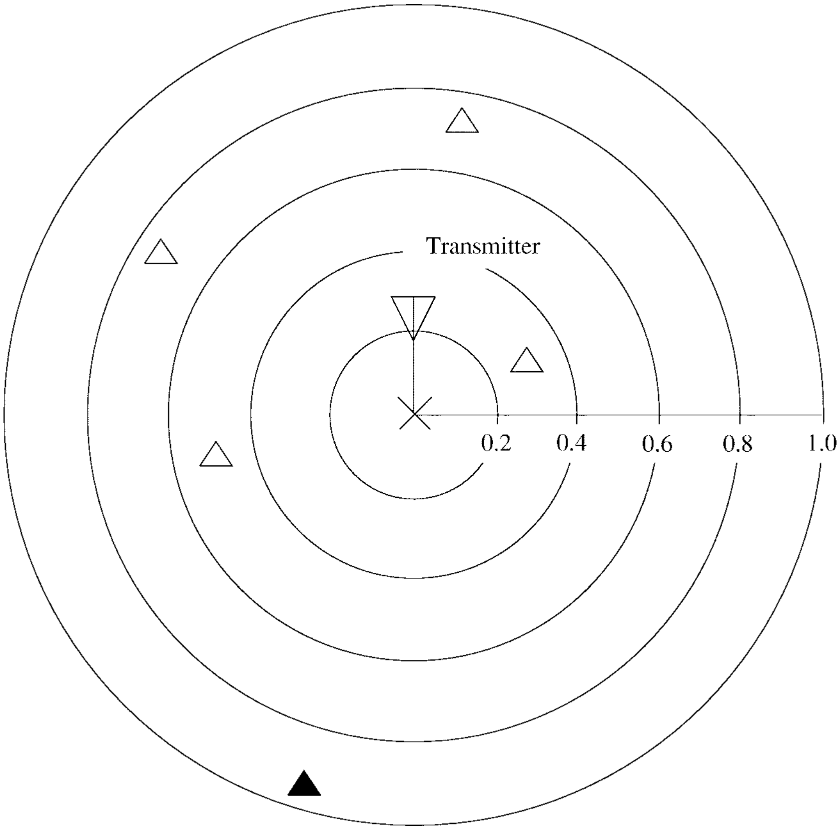

Fig. 6. Cell for analyzing the outage probability.

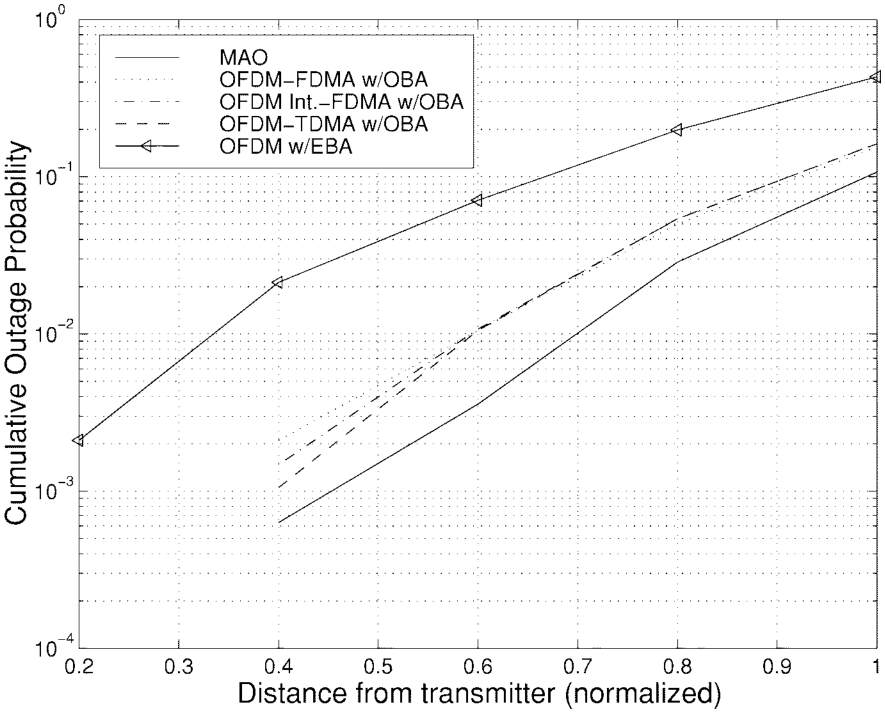

Fig. 7. Outage probability at 17 dB fading margin.

dently and uniformly distributed within the cell. A typical scenario is shown in Fig. 6, where the triangles represent the five users. In addition to frequency selective fading, path loss and log-normal shadowing are also included in simulating the actual channel gains seen by the users. Using these channel gains, subcarriers and bits assigned to each user are determined by the various multiple access schemes and the total required transmit power is calculated. If the total power for all five users

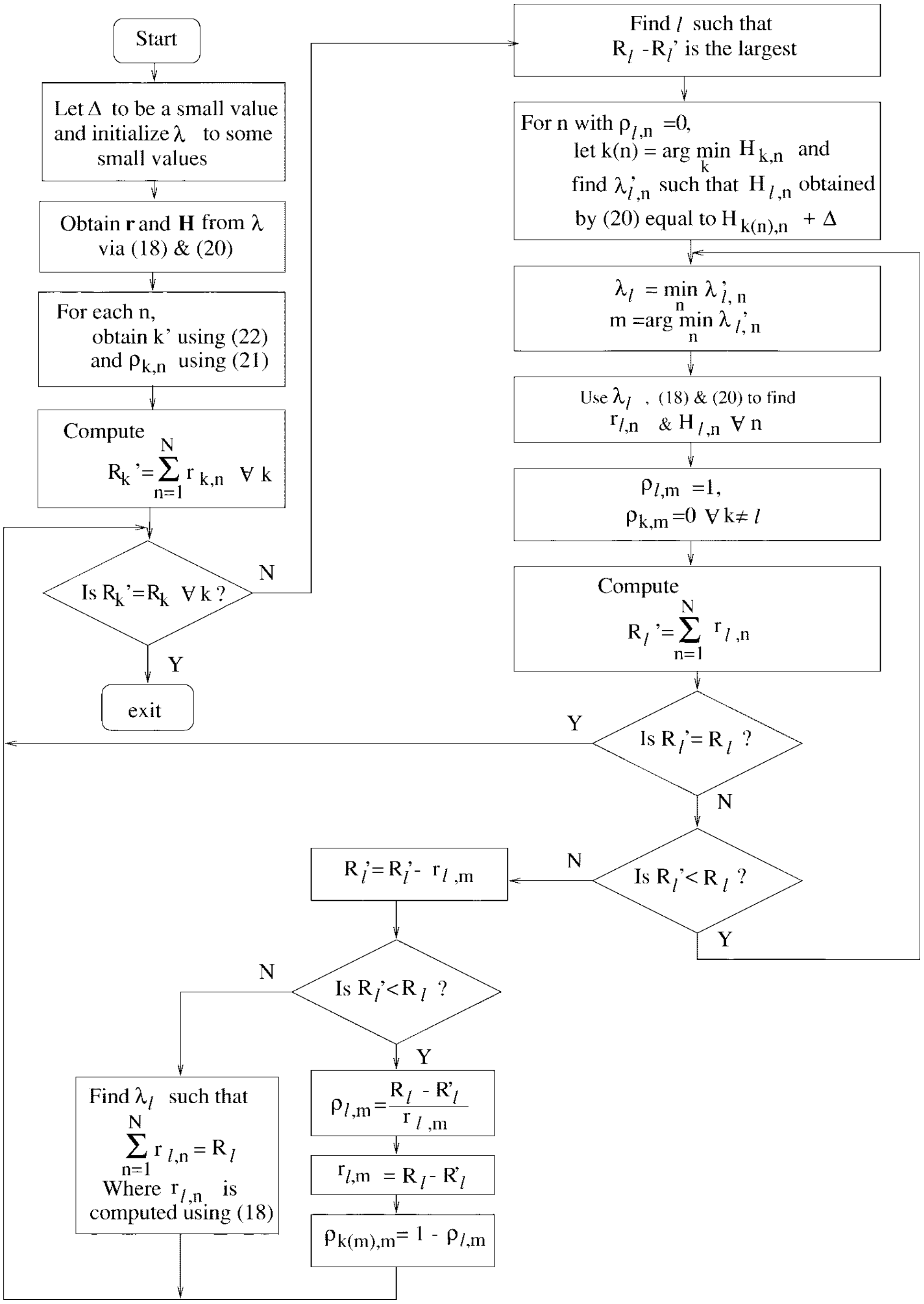

Fig. 8. Flow chart of the multiuser subcarrier allocation algorithm.

exceeds the maximum power of the BS, the user requiring the largest transmit power (in this case, the black one) is dropped and counted as one outage event occurring at a distance equal to the distance between the BS and the dropped user. This process continues until the transmit power is smaller than the maximum power of the BS. In this example, the maximum transmit power is set to the transmit power required for all five users assuming that they are all located at the boundary of the cell, taking into account the path loss effect and a 17 dB fading margin for shadowing.

The cumulative outage probabilities at various normalized distances, normalized to the cell radius, are plotted in Fig. 7. A cumulative outage probability of 5% at a normalized distance of 0.8 means that there is a 5% chance of outage for a mobile located more than $0.8R$ away from the BS where R is the radius of the cell. We observe that MAO outperforms others with a large reduction in the outage probability at all distances. Alternatively, if the same outage probability is maintained, say at 1%, the coverage are provided by MAO is 36% larger than the best of all other schemes.

VI. CONCLUSION

In this paper, we considered OFDM transmission in a multiuser environment and formulated the problem of minimizing the overall transmit power by adaptively assigning subcarriers to the users along with the number of bits and power level to each subcarrier. In particular, we derived a multiuser adaptive subcarrier and bit allocation algorithm. Given the instantaneous channel information, the algorithm obtains a suboptimal subcarrier allocation, and then single-user bit allocation is applied on the allocated subcarriers. Using this scheme, the overall required transmit power can be reduced by about 5–10 dB from the conventional OFDM without adaptive modulation. Likewise, the transmit power can be reduced by about 3–5 dB from the conventional OFDM with adaptive modulation and adaptive bit allocation, but without adaptive subcarrier allocation. The reduction in transmit power can also be translated to a significant reduction in the required bit SNR for a given BER. Moreover, the same improvement can also be translated to a reduction in the outage probability or to an increase in the area of coverage.

The results in this paper assume perfect channel estimation, and we have not considered issues related to imperfect implementation, such as imperfect synchronization. As channel estimation in wireless fading channels is in general not very accurate, the effect of nonideal channel information on the performance of our proposed MAO scheme is a very important issue. We have started looking at this issue, and our preliminary results have indicated that the MAO scheme is not very sensitive to channel estimation errors. Nevertheless, detailed sensitivity studies will be needed before the algorithm can be applied to practical systems.

APPENDIX

A flow chart providing the detailed description of the multiuser subcarrier allocation algorithm is shown in Fig. 8.

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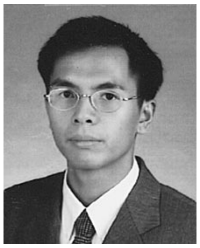

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