Flow Allocation in Multi-hop Wireless Networks: A Cross-Layer Approach

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Outline

- Introduction
- Problem Description
- System Model
- Numerical Studies
- Conclusion
Introduction

- Multi-hop wireless network
  - Pkts are forwards hop-by-hop

- Each flow contending for
  - local resource at each intermediate node in its routing path: local interference
  - the shared wireless medium with those flows located within its interference range: location-dependent interference:

- Due to resource contention from different layers
  - traditional single layer design disciplines lead to inefficient performance
  - calls for cross-layer design manner, to coordinate among the transport, MAC, and physical layer
two model to describe the location-dependent interference:

- Protocol Model
- Physical Model
Introduction (cont.)

- **Protocol Model**
  - **G=(V,E)**
  - $r_i$: transmission radius of node $i$
  - $r_i'$: interference range of node $i$, $r_i' = (1+\triangle) r_i$, $\triangle$ : non-negative number
  - $d_{ii} \leq r_i$
  - for any node $k \in V$, $k \neq i, j$, that is simultaneously transmitting, $d_{k,j} \geq r_j'$

- conflict graph $\rightarrow$ clique $\rightarrow$ NP-complete
- insufficient to guarantee the optimality of link utilization
Physical Model

$\alpha$: pass loss exponent
$\sigma$: thermal background noise

$$\frac{P_t(i)}{d_{i,j}^\alpha} > \beta$$

$$\sum_{k \in K} \frac{P_t(k)}{d_{k,j}^\alpha} + \sigma$$

- Optimize the network capacity while satisfying the power constraint of each node
- Requires selecting the sets of concurrently active communication links
  - Time consuming - each link in a set will interfere with the other link
Introduction (cont.)

- All existing works not explicitly addressed in
  - Relationship between the interference caused by wireless communications
  - Supportable data rate of a node
  - End-to-end flow rate control problem
  - MAC issues caused by the interference due to simultaneous transmissions.
Introduction (cont.)

- **Motivations:**
  - avoiding the enumeration of MAX clique, or the sets of concurrently active links
  - providing a general approach which accounts for the interference constraints in MAC protocol designs in arbitrary network topologies

- **Objective:**
  - Optimize global resource allocation
    - by maximizing the aggregate utilization of wireless resource with coordination between the transport, MAC, and physical layer
New interference model: Node-based Interference Model
- account for MAC protocols
- captures the behavior of local interference and location-dependent interference
- each node locally identify the interference at the physical layer and contentions at the MAC layer only through signal power measurement

The optimal flow allocation problem
- jointly consider physical layer, medium contention at the MAC layer, and end-to-end flow issues at the transport layer
- eliminate the clique or the independent set computation
Fig. 1. Node-based interference model. (a) Location-dependent interference; (b) Local interference.
### Problem Notation (1)

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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</thead>
</table>
| $G=(V, E)$ | $G$: multihop wireless network  
$V$: the set of nodes  
$E$: the set of links |
<p>| $P_t(i)$   | The transmission power of node $i$                                          |
| $d_{ij}$   | The distance between node $i$ and node $j$                                  |
| $RxT$      | The threshold of received signal power. The minimum power level to correctly receive and decode the data from transmitter |
| $L(.)$     | The path gain function                                                      |
| $P_r(j)$   | The received power at node $j$. $P_r(j)=P_t(j)L(d_{ij})$, must exceed $RxT$ |
| $r_i$      | The transmission range of transmitter $i$, the largest distance from $i$ that $i$'s node can be correctly decoded |</p>
<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\sigma$</td>
<td>The thermal background noise</td>
</tr>
<tr>
<td>$\theta$</td>
<td>The SNR threshold for a node to correctly decode the signal</td>
</tr>
<tr>
<td>$(\theta_i)$</td>
<td></td>
</tr>
<tr>
<td>$SNR_{ij}$</td>
<td>$SNR$(Signal to Noise Ratio) of link $(i,j)$, $SNR_{ij} = \frac{P_t(j)L(d_{i,j})}{\sigma}$, $SNR_{ij} \geq \theta$</td>
</tr>
<tr>
<td>$K$</td>
<td>The set of concurrent transmitters</td>
</tr>
<tr>
<td>$SIR_{ij}$</td>
<td>$SIR$(Signal to Interference Ratio) of link $(i,j)$, $SIR_{i,j} = \frac{P_t(i)L(d_{i,j})}{\sum_{k \in K} P_t(k)L(d_{k,j}) + \sigma}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>The $SIR$ threshold determined by the setting of wireless PHY. For node $j$ to receive data from node $i$ correctly, the $SIR_{ij}$ of link $(i,j)$ must exceed $\beta$</td>
</tr>
<tr>
<td>$(\beta_i)$</td>
<td></td>
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<tr>
<td>Notation</td>
<td>Description</td>
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<tr>
<td>$W$</td>
<td>The frequency bandwidth of the communication channel</td>
</tr>
<tr>
<td>$R_i$</td>
<td>The supportable data rate of any communication link incident to node $i$ is at least $R_i = W \times \log_2(1 + \beta_i)$</td>
</tr>
<tr>
<td>$P_{\text{max}}(i)$</td>
<td>The max transmission power of node $i$. Adjust $0 \leq P_i(i) \leq P_{\text{max}}(i)$ such that the signal power of the receiver node $j$ is slightly larger than $\theta_i \cdot \sigma$</td>
</tr>
<tr>
<td>$R_{j,\text{max}}$</td>
<td>Max supportable data rate of a wireless link connecting node $j$. $R_{j,\text{max}} = W \times \log_2(1 + \theta_i)$</td>
</tr>
<tr>
<td>$I$</td>
<td>A set of end-to-end flows</td>
</tr>
<tr>
<td>$f={s,d}$</td>
<td>End-to-end flow traverses the system from source node $s$ to destination node $d$</td>
</tr>
<tr>
<td>$t_{i,j}^f$</td>
<td>The portion of time shared by flow $f$ transmitted from node $i$ to node $j$</td>
</tr>
<tr>
<td>$T$</td>
<td>Each fixed time period</td>
</tr>
<tr>
<td>$C_i$</td>
<td>$C_i = T \times R_i$, the total capacity of node $i \in V$</td>
</tr>
</tbody>
</table>
## Problem Notation (4)

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$B_j$</td>
<td>Max interference budget which node $j$ can sustain to correctly decode the signal from a transmitter, $B_j = (\theta_j \times \sigma_f \beta_j) - \sigma$</td>
</tr>
<tr>
<td>$\omega_{i,k,j}$</td>
<td>For node $k$, the ratio of the interference contributed by the concurrent transmission from node $i$ to node $j$</td>
</tr>
<tr>
<td></td>
<td>$\omega_{i,k,j} = \frac{P_s(i) \times L(d_{i,k})}{B_k} = \frac{L(d_{i,k})\theta_k\beta_k}{L(d_{i,j})(\theta_k - \beta_k)}$</td>
</tr>
<tr>
<td>$\zeta_{i,k,j}$</td>
<td>The interference indicator for the communications performed at the set of contending nodes of node $k$. $\zeta_{i,k,j} = 1$ if node $k$ contends with the transmission from node $i$ to node $j$, otherwise $\zeta_{i,k,j} = 0$</td>
</tr>
<tr>
<td>$r^f_{i,j}$</td>
<td>Indicator, if link$(i,j)$ carries the traffic of flow $f$, then $r^f_{i,j} = 1$, otherwise $r^f_{i,j} = 0$</td>
</tr>
<tr>
<td>$U_f(x_f)$</td>
<td>The utility function. Each end-to-end flow $f \in \Gamma$ is associated with a utility function $U_f(x_f)$, which indicates the degree of satisfaction of its end-user</td>
</tr>
</tbody>
</table>
Node-based interference model. (a) Location-dependent interference; (b) Local interference.
System Model (2)

\[\sum_{f \in \Gamma} \sum_{j \in V} t_{j,i}^f + \sum_{f \in \Gamma} \sum_{j \in V} \sum_{k \in V(i,j)} \xi_{j,i,k} \times t_{j,k}^f \leq T. \quad (3)\]
System Model (3)

\[ \sum_{f \in F} \sum_{j \in V} t_{j, i}^f + \sum_{f \in F} \sum_{j \in V} t_{i, j}^f \leq T. \]
Problem Formulation

\[
P: \text{Maximize } f(x) = \sum_{f \in \Gamma} U_f(x_f), \tag{6}
\]

subject to

\[
\sum_{f \in \Gamma} \sum_{j \in V} r^f_{j,i} x_f + \sum_{f \in \Gamma} \sum_{j \in V} r^f_{i,j} x_f \frac{R_i}{R_j} \leq C_i, \tag{7}
\]

\[
\sum_{f \in \Gamma} \sum_{j \in V} r^f_{j,i} x_f + \sum_{f \in \Gamma} \sum_{j \in V} \sum_{k \in V(i,j)} s^f_{j,i,k} x_f \frac{R_i}{R_j} \leq C_i. \tag{8}
\]
Flow Allocation in Multi-Hop Wireless Networks

Duality

The Lagrangian form of the optimization problem $P$ can be expressed as follows.

$$L(x, \lambda, \mu) = \sum_{f=1}^{\left|\Gamma\right|} U(x_f) + \sum_{i=1}^{\left|V\right|} \lambda_i[C_i - \sum_{f=1}^{\left|\Gamma\right|} a_{i,f}x_f] + \sum_{i=1}^{\left|V\right|} \mu_i[C_i - \sum_{f=1}^{\left|\Gamma\right|} b_{i,f}x_f],$$

$$\lambda_i, \mu_i, i \in V, \text{ are Lagrange multipliers}$$

$$a_{i,f} = \sum_{j=1}^{\left|V\right|} r_{j,i}^f + \sum_{j=1}^{\left|V\right|} r_{i,j}^f \frac{R_i}{R_i}$$

$$b_{i,f} = \sum_{j=1}^{\left|V\right|} r_{j,i}^f + \sum_{j=1}^{\left|V\right|} \sum_{k=1; k \neq i,j}^{\left|V\right|} s_{j,i,k} r_{j,k}^f \frac{R_i}{R_k}.$$
Flow Allocation in Multi-Hop Wireless Networks

Duality (cont.)

\[ L(x, \lambda, \mu) = \sum_{f=1}^{|\Gamma|} L_f(x_f, \lambda^f, \mu^f) + \sum_{i=1}^{|V|} C_i(\lambda_i + \mu_i), \quad (10) \]

where \( \lambda^f = \sum_{i=1}^{|V|} \lambda_i a_{i,f} \) and \( \mu_f = \sum_{i=1}^{|V|} \mu_i b_{i,f} \).

For each flow \( f \in \Gamma \), \( L_f(x_f, \lambda^f, \mu^f) = U_f(x_f) - (\lambda^f + \mu^f)x_f \) and its value is determined by \( x_f \) and flow prices \( \lambda^f \) and \( \mu^f \). Considering the expression \( \lambda^f + \mu^f \), we obtain

\[ \lambda^f + \mu^f = \sum_{i=1}^{|V|} \sum_{i \neq j}^{|V|} r_{j,i}^f (\lambda_i + \lambda_j \frac{R_j}{R_i} + \mu_i + \mu_j + \eta_{j,i}), \quad (11) \]

where \( \eta_{j,i} = \sum_{k=1}^{\mid V \mid} \mu_k s_{j,k,i} \) represents the price of link \((j, i)\) that is the aggregate interference price from the neighborhood of link \((j, i)\).
To determine the Lagrange multipliers, we introduce the dual problem $g$ of the optimization problem $P$, which can be formulated as follows.

$$g: \min_{\lambda \geq 0, \mu \geq 0} \quad g(\lambda, \mu), \quad (12)$$

where $g(\lambda, \mu) = \max_x L(x, \lambda, \mu) = \sum_{f=1}^{[\Gamma]} S_f(\lambda, \mu) + V(\lambda, \mu)$, and

$$S_f = \max_{x_f} (U_f(x_f) - \sum_{i=1, i \neq j}^{[V]} \sum_{j=1}^{[V]} r_{j,i}^f (\lambda_i + \frac{\lambda_j R_j}{R_i} + \mu_i + \mu_j + \kappa_{j,i}) x_f$$

$$V(\lambda, \mu) = \max C_i (\sum_{i=1}^{[V]} (\lambda_i + \mu_i)).$$
Gradient-based Flow Allocation Algorithm

**Input:** A set of nodes $V$, a set of source-destination pairs $\Gamma$, and the routing path of each flow.

**Output:** Flow assignment $x_f$ for each flow $f \in \Gamma$.

1: Initialize flow $x_f(0) \leftarrow 0$, $\forall f \in \Gamma$, and node prices $\lambda_i \leftarrow 0$, $\mu_i \leftarrow 0$, $\forall i \in V$.

2: Update the price at each node $i \in V$.

   $\lambda_i(t+1) = [\lambda_i(t) - \alpha (C_{i} - \sum_{j=1}^{\Gamma} \sum_{j=1}^{V} r_{i,j}^{f} + r_{i,j}^{f} \frac{R_{i}}{R_{j}} x_{f})]^{+}$.

   $\mu_i(t+1) = [\mu_i(t) - \alpha (C_{i} - \sum_{j=1}^{\Gamma} \sum_{j=1}^{V} r_{j,i}^{f} + \sum_{j=1}^{V} \sum_{k=1}^{V} r_{j,k}^{f} \zeta_{j,i,k} \frac{R_{j}}{R_{k}} x_{f})]^{+}$.

3: For each node $i \in V$, send the prices $\lambda_i(t+1)$ and $\mu_i(t+1)$ to the sender of the flow $f \in \Gamma$, for which $r_{i,j}^{f} = 1$ or $r_{j,i}^{f} = 1$ or $r_{j,k}^{f} \zeta_{j,i,k} = 1$.

4: For each flow originator, after receiving node prices $\lambda_i(t+1)$ and $\mu_i(t+1)$ from each node $i \in V$, calculate the gradient by

   $\zeta_{f}(t+1) = \sum_{i=1, i \neq f}^{\Gamma} \sum_{j=1}^{V} r_{i,j}^{f} [\lambda_i(t+1) + \mu_i(t+1) \frac{R_{j}}{R_{i}} + \lambda_j(t+1) + \mu_j(t+1) + \sum_{k=1}^{V} \mu_k(t+1) \zeta_{j,k,i}]$.

5: The flow allocation is adjusted by

   $x_f(t+1) = x_f(\zeta_{f}(t+1))$. 
Numerical Studies (1)

(a) 3- flow scenario

(b) Mutual flow scenario

(c) Aggregate flow scenario

(c) Reverse flow scenario
### Number of Iterations for Convergence

<table>
<thead>
<tr>
<th></th>
<th>3-flow</th>
<th>Mutual flow</th>
<th>Aggregate flow</th>
<th>Reverse flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-node</td>
<td>24</td>
<td>17</td>
<td>19</td>
<td>21</td>
</tr>
<tr>
<td>10-node</td>
<td>30</td>
<td>40</td>
<td>23</td>
<td>37</td>
</tr>
<tr>
<td>15-node</td>
<td>31</td>
<td>63</td>
<td>27</td>
<td>52</td>
</tr>
<tr>
<td>20-node</td>
<td>31</td>
<td>75</td>
<td>37</td>
<td>70</td>
</tr>
</tbody>
</table>
Numerical Studies (3)

(a) 3-flow scenario

Flow throughput comparison under the 3 flow scenario.
Numerical Studies (4)

(b) Mutual flow scenario

(b) Throughput comparison for each flow under the mutual flow scenario.
Numerical Studies (5)

(c) Aggregate flow scenario

Throughput comparison for each flow under the aggregate flow scenario.
Numerical Studies (6)

(c) Reverse flow scenario

(d) Throughput comparison for each flow under reverse flow scenario.
Numerical Studies (7)

(a) Routing paths in the initial network topology  (b) Routing paths after node 16 becomes unavailable.  (c) Routing paths after node 18 becomes unavailable.

Flow4: 10->11
Flow5: 5->14
Flow3: 6->5
Flow1: 3->9
Flow6: 2->3
Flow2: 19->8

Fig. 6. Throughput trajectory of each flow when the network topology changed.
Conclusion

- Node-based Interference Model
  - Consider interference, data rate, signal reception power, contention behavior at MAC, and end-to-end flow at the transport layer

- Objective
  - Maximize network utilization
  - Maintain fairness among flows

- Numerical results
  - Achieve the optimum within a small number of iterations
  - Allocate resource to the end-to-end multi-hop flows
  - Maximize optimal network utilization
  - Maintaining fairness among flows

- The first work which formulate the interference constraints for the flow allocation problem without any global info in multi-hop wireless networks