國立臺灣大學資訊管理研究所碩士論文

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以波長路由為基礎之多波長分工網路上群播樹合併演算法

A Multicast Tree Aggregation Algorithm in Wavelength-Routed WDM Networks

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論文摘要

在光通訊網路技術中，光波長多工(WDM)網路是被廣泛期許的傳輸標準之一，由於技術的持續進步，光波長多工網路未來在廣域骨幹網路中扮演實體傳輸層要角；在光交換的技術上，光波長交換(optical wavelength switching)相較於光封包交換(optical packet switching)是較為成熟且成本較低的選擇；另外，光通訊網路的分時多工(TDM)技術已經非常成熟而且運作了非常久的時間。

本篇論文研究在光交換機(optical cross connect, OXC)具有光群播(optical multicasting/splitting)和分時多工功能且以波長路由(wavelength-routed)為基礎的波分多工網路上，群播服務之路由及光波長指定(routing and wavelength assignment, RWA)的問題，在給定網路容量及沒有波長轉換(no wavelength converter)的限制下，透過共用群播樹的方式將群播群組盡可能地合併，並使路由成本最小化，使總體收益得到最大化。

我們將整個問題仔細地分析並數學模式化為一個最佳化數學模型，這個數學問題在本質上是一個整數線性規劃問題，問題的本身具有高度的複雜性和困難度。我們採用以拉格蘭日鬆弛法為基礎的方法來處理此一複雜問題，並根據所得到的結果發展簡易的演算法。我們設計數項實驗在不同的網路拓撲下測試所提出演算法，實驗結果顯示在效率及效能上都有好的表現。

關鍵詞：光波長多工、分時多工、群播網路、路由、波長分配、最佳化、拉格蘭日鬆弛法、數學規劃。

THESIS ABSTRACT

GRADUATE INSTITUTE OF INFORMATION MANAGEMENT

NATIONAL TAIWAN UNIVERSITY

NAME：CHIH-CHUN KUO **MONTH/YEAR：JUL, 2003**

ADVISER：YEONG-SUNG LIN

A MULTICAST TREE AGGREGATION ALGORITHM IN WAVELENGTH-ROUTED WDM NETWORKS

Wavelength division multiplexing (WDM) has been considered a promising transmission technology in optical communication networks. With the continuous advance in optical technology, WDM network will play an important role in wide area backbone networks. Optical wavelength switching, compared with optical packet switching, is a more mature and more cost-effective choice for optical switching technologies. Besides, the technology of time division multiplexing (TDM) in optical communication networks has been working smoothly for a long time.

In the proposed research, the problem of multicast groups aggregation and multicast routing and wavelength assignment in wavelength-routed WDM network is studied. The optical cross connect switches (OXCs) in the problem are assumed to have limited optical multicast/splitting and TDM functionalities. Given the physical network topology and capacity, the objective is to maximize the total revenue by means of utmost merging multicast groups into larger macro-groups. The groups in the same macro-group will share a multicast tree to conduct data transmission.

The problem is formulated as an optimization problem, where the objective function is to

maximize the total revenue subject to capacity constraints of components in the optical network, wavelength continuity constraints, and tree topology constraints. The decision variables in the formulations include the merging results between groups, multicast tree routing assignment and wavelength assignment.

The basic approach to the algorithm development for this model is Lagrangean relaxation in conjunction with a number of optimization techniques. In computational experiments, the proposed algorithms are evaluated on different network topologies and perform efficiently and effectively according to the experiment results.

Keywords: WDM, TDM, Multicast, Network Planning, Routing, Wavelength Assignment, Optimization, Lagrangean Relaxation Method, Mathematical Programming.

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Chapter 1 Introduction

1.1 Background

It has been widely accepted that optical networks will form the building blocks for the next generation Internet. In the last several years, there has been a growing excitement in the area of optical Dense Wavelength Division Multiplexing – DWDM, or simply, WDM – networks. WDM operates by sending multiple light waves (frequencies) across a single optical fiber. Information is carried by each wavelength, which is called a channel. At the receiving end, an optical prism or a similar device is used to separate the frequencies, and information carried by each channel is extracted separately. Current development activities indicate that WDM technology will be deployed mainly in a backbone network for large regions. WDM also can enhance an optical network's capacity without expensive re-cabling and can reduce the cost of network upgrades. Current optical technology demonstrations have shown the feasibility of up to 160 channels, each operation at 10 Gbps, per fiber [3].

At beginning, telecommunication companies deployed WDM technology for a fixed point-to-point communication, and the deployment was driven by the increasing demands on bandwidth. WDM is a more cost-effective alternative compared to laying more fiber optics [10]. After significant advances in optical component technologies, the switching function of WDM network became possible. The switching/routing mechanisms that have been proposed are the broadcast-and-select routing and the wavelength routing. Under broadcast-and-select routing, a media-access protocol is required to control the transmissions of the various network nodes to avoid collisions and to manage contention for the network bandwidth, so application to large-scale networks is however not feasible due to the lack of graceful scaling [21].

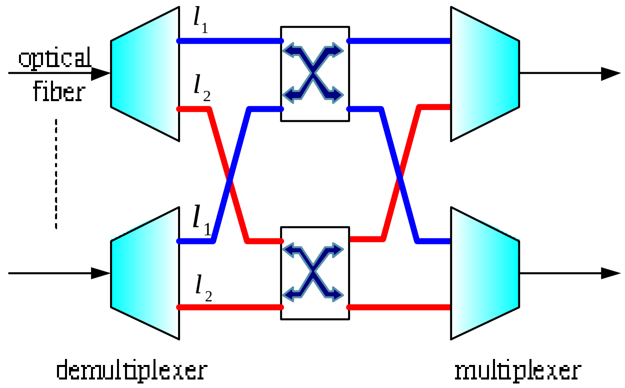


Figure 1-1 The architecture of a wavelength router

Wavelength routing is defined to be the selective routing of optical signals according to their wavelengths as they travel through the network elements between source and destination with or without wavelength converters, as 0 shows. The importance of the reconfigurable optical cross-connect switch (OXCs), and the closely related optical add-drop multiplexer (OADMs), is that they allow the optical network to be reconfigured on a wavelength-by-wavelength basis to optimize traffic, congestion, and network growth [21].

A *lightpath* is an all optical channel which may be used to carry circuit-switched traffic, and it may span multiple fiber links. In the absence of wavelength converters, a lightpath would occupy the same wavelength on all fiber links through which it passes and this is called wavelength continuity constraint. A lightpath can create logical neighbors out of nodes that may be geographically far apart in the network. Using lightpath communications, a large number of lightpaths may be set up on the network in order to embed a logical topology.

The logical topology, whose vertices are the IP routers and whose edges are the lightpaths, is overlayed to the physical topology, made of optical fibers and OXCs.

The Logical (alias Virtual) Topology Design (LTD) problem refers to finding out the logical topology that minimized a target function or cost, given (1) a physical topology comprising nodes equipped with a limited integer number of resources, connected by optical fibers that support a limited number of wavelengths, (2) traffic demand and (3) routing strategy [15]. The usual formulation of logical topology optimization problem is a mixed integer linear program (MILP) problem. Therefore, the LTD problem is NP-hard and thus is numerically intractable, even for networks with a moderate number of nodes. The multicast LTD (MLTD) problem also falls to the class of general MILP problems [15]. In addition, the MILP formulation naturally leads to a combined topology and routing optimization. As a result, the routing strategy is a result of the optimization procedure, together with the logical topology configuration. Several logical topology design heuristics are proposed in literature.

Once the LTD problem has been solved, the resulting logical topology must be overlayed on the physical topology. This procedure must identify the set of physical fibers over which each logical link (lightpath) will be routed from the source node to the destination nodes and the wavelength that will be used without conflict. This problem is referred in the literature as Routing and Wavelength Assignment (RWA) problem and several RWA approaches have been proposed in the literature [27].

The RWA problem for static traffic is known as the Static Lightpath Establishment (SLE) problem. SLE with wavelength continuity constraint can be formulated as an integer linear program (ILP) in which the objective function is to minimize the flow in each link, which, in turn, corresponds to minimizing the number of lightpath passing through a particular link.

The problem is NP-complete [27] and is mathematically intractable. But, it can be simplified by decoupling the problem into two separate subproblems: the routing subproblem and the wavelength assignment subproblem. Several heuristics and approximation algorithms have been proposed to get near optimal solution efficiently.

In addition to wavelength switching, there are optical packet switching (OPS) and optical burst switching (OBS) technologies [11]. In OBS, IP packets are assembled into one burst at the ingress edge node based on destination egress edge router for unicast traffic, multicast group address for multicast traffic, and other attributes such as QoS requirements. But there are two major overheads when using OBS: control packets on out-of-band channel and guarding bands. Therefore, it is important to reduce the overheads for network efficiency.

For optical packet switching, there are three kinds of packet switching scheme in WDM networks: electrical, optic-electronic or full optical. The architecture of optical packet switch able to effectively cope with variable length packet traffic and quality of service management has been proposed, which is able to support IP traffic and to achieve higher bandwidth utilization than wavelength routing. But the optical component technology (e.g., optical memory) is not mature enough. The control and processing overhead at the high speed core node is likely to be very high [11]. Optical packet switch is therefore not available for deployment.

1.2 Motivation

In the wavelength-routed network, the granularity of switching is wavelength. The problem of wasting in bandwidth is arising from not-fully-loaded-lightpaths because the free bandwidth of the wavelengths in these lightpaths cannot be used by others. The bandwidth allocation problem in the wavelength-routed networks has been widely investigated. Several schemes using technologies of Optical Time Division Multiplexing (OTDM) have been proposed in the literature to achieve higher bandwidth utilization.

Many broadband services such as video conferencing and distance learning employ multicasting for data delivery. The support of multicast is therefore essential for these applications. The multicast routing and wavelength assignment (MC-RWA) problem [8] is: given a limited number of wavelengths and a set of multicast calls, maximize the number of multicast calls admitted, or equivalently, minimize the call blocking probability under the constraint that each multicast tree can be assigned only one wavelength. It has been proved that the MC-RWA problem in circuit-switched multi-hop networks is NP-complete [12]. Therefore, the problem is complicated and hard to solve. Obtaining the optimal solution in such kinds of problem is intractable.

If the multicast groups of the same source are merged by means of OTDM technologies, the MC-RWA problem will be more complicated. In ref. [11], a new approach called tree-shared multicast (TS-MCAST) is proposed in optical burst switching networks and a multicast sharing class (MSC) associated with a shared tree is also defined.

Most proposed work assumed that the OXCs in WDM networks are quipped with *full range power splitters* and/or wavelength converters, which may not be true in practice. In this paper, more physical resources constraints are taken into consideration. Besides, the tree topologies are not given in advance and the network capacity is not assumed to be as large as total traffic demands. As a result, some groups may not be admitted in due to the capacity constraint. In addition to maximize the revenue by TDM based groups aggregation, we further try to minimize the needed cost of supporting these groups.

1.3 Literature Survey

In this section, we survey the design and planning problem of optical TDM/WDM networks with multicast traffic demands. Different kinds of mathematical models and formulations to this problem are also studied.

1.3.1 Unicast in TDM/WDM Networks

In [14], a novel concept of "*Super-Lightpath*" is proposed. By the joint use of WDM and OTDM technologies, it offers an interesting opportunity of splitting the bandwidth of a lightpath into a fixed number of subchannels, using TDM scheme directly in the optical domain. The authors assume that the bandwidth available on a wavelength channel is very large with respect to the bandwidth required by each traffic flow. Then a bit level, fixed framing is determined , such that each bit in a given position in the frame, called bit slot, identifies a particular sub-channel. The transmitter multiplexes sub-channels into the frame and transmits the resulting stream into one lightpath with the same wavelength.

This allows designing logical topologies with an increased number of logical links, thus reducing the average distance among nodes, i.e., the number of electronic-to-optical and optical-to-electronic conversions, and the traffic congestion on logical links. At the same time, this reduces the number of wavelength required to solve the RWA problem. The classic RWA is shown in 0 and an illustration of Super-Lightpath RWA is shown in 0.

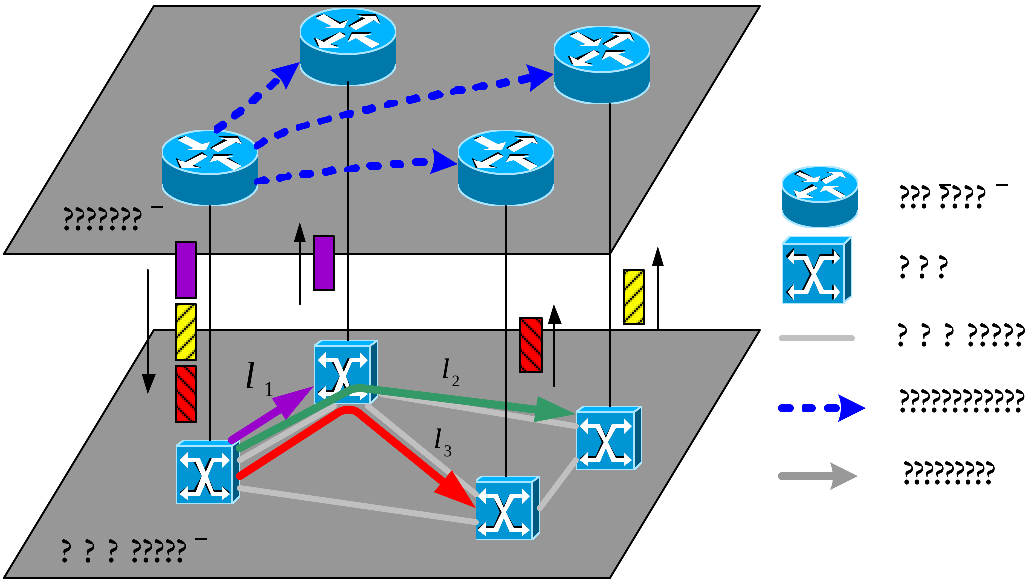


Figure 1-2 Classic RWA

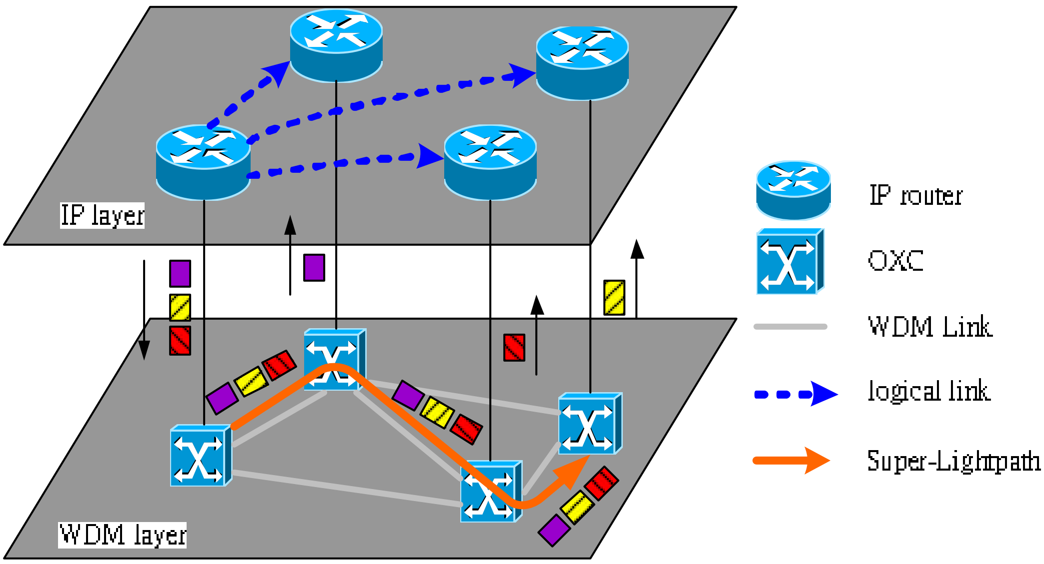


Figure 1-3 Super-Lightpath RWA

A Super-Lightpath can travel through many nodes, and receivers can receive one or more subchannels from it. Each receiver can then synchronize a tunable receiver to particular bit slots, instead of converting the whole bit stream to the electronic domain, while transparently routing the entire frame toward next node. But the authors do not take optical splitting into consideration and suppose that a lightpath cannot be split in a node. In this paper, we assume that OXCs have limited splitting capability, and extend the concept of Super-Lightpath to "Super-Lighttree."

In order to improve bandwidth utilization, another optical TDM-based scheme is proposed in [25] and [10]. The bandwidth of the wavelength is partitioned into fixed-length TDM frames where each frame consists of a certain number of fixed-length time-slots. For each wavelength, the routing node behaves as a traditional TDM circuit switching node. In each TDM frame cycle, the routing node routes the incoming data in each time-slot into the desired output port which is determined at the call-setup time without electronic-to-optical and optical-to-electronic transformation. The granularity of bandwidth allocation is in terms of time-slots instead of the entire channel bandwidth. Therefore, time-slot assignment has to be determined for an incoming request, in addition to the route and wavelength, which is defined as routing, wavelength and time-slot assignment (RWTA) problem.

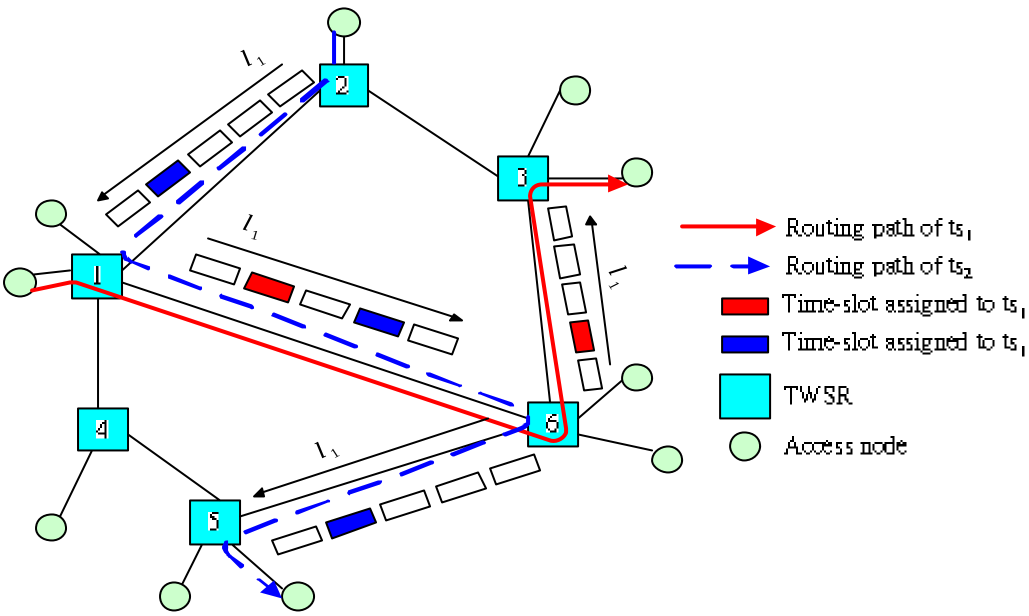


Figure 1-4 TWRN architecture

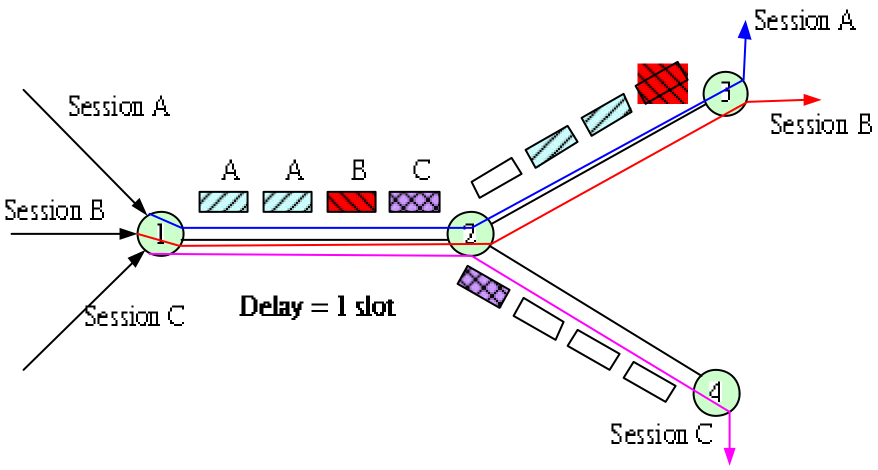


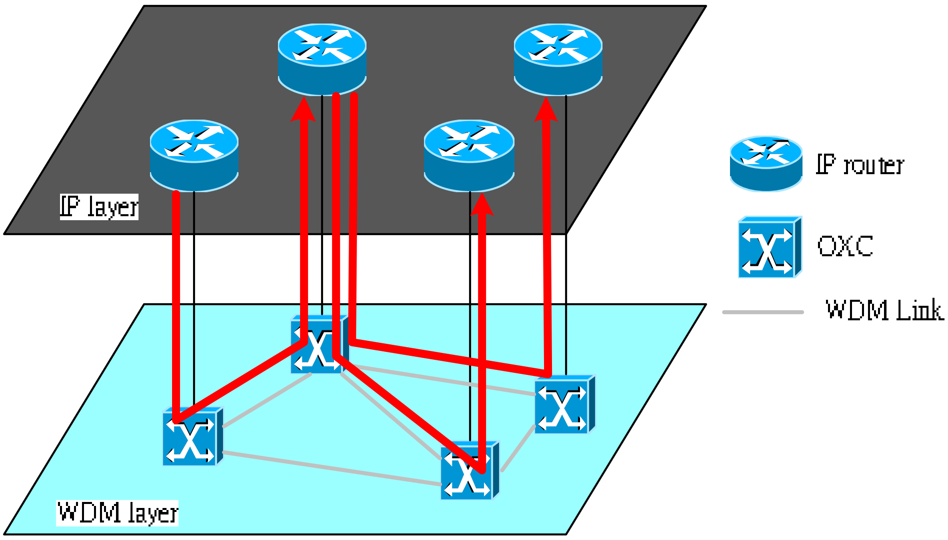
Figure 1-5 TWRN architecture considering processing delay

The proposed architecture is called TDM-based wavelength-routed network (TWRN), while the routing node in TWRN is called *time-wavelength-space router* (TWSR) because it routes the data in the dimensions of time, wavelength, and space. The established connection is called *time-slot-based lightpath* (ts-lightpath). Two different kinds of TWRN are illustrated in Figure 1-1 (from Ref. [10]) and Figure 1-2 (from Ref. [25]). The difference between them is that link propagation and node processing delay are considered in the second architecture, and the combined delay results in a shifted time-slot allocation.

One of the key challenges in realizing TDM/WDM networks is the need for quick reconfiguration of routing nodes. The routing node must configure its switching patterns on a time-slot basis to route each slot's traffic. In this paper, we have no assumption that the OXC is capable of configuring its routing pattern on a time-slot basis because of higher complexity and cost. The TDM capability of OXC is only referred to multiplexing several traffic flows into a TDM frame and then transmitting the resulting frame on a wavelength.

1.3.2 Multicast in TDM/WDM Networks

Since multicast traffic flows are characterized by many destinations, replication or branching of multicast flows somewhere in the network is necessary. Three kinds of possible ways to perform multicasting are described in [11]. The first one lets each IP router on a multicast tree make copies of data and transmit a copy to each downstream router, as depicted in 0. This is inefficient in terms of latency, hardware complexity and cost because the O/E/O conversion is needed at all the routers. Therefore, it is not a desirable approach.



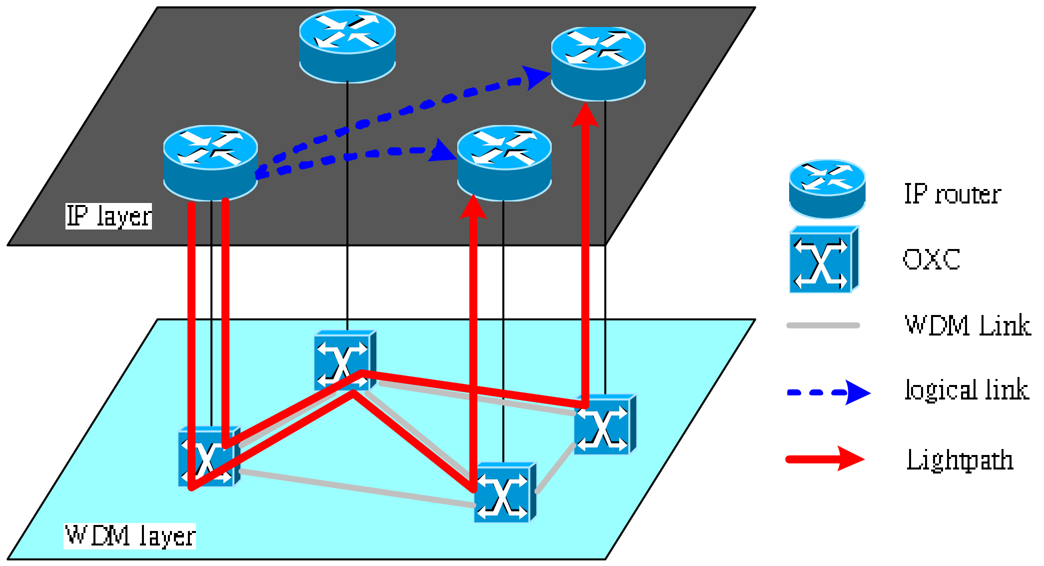


Figure 1-7 Multicasting by Lightpaths

The third approach directly considers multicasting at the WDM layer by taking the advantage of the splitting capability of the WDM switch. In optical networks, the splitting capability of an optical switch will affect the construction of a multicast tree. An example of multicasting at WDM layer is shown in 0, which is the concept of "Light-tree" proposed in [19]. A light-tree is a point-to-multipoint all-optical channel which may span multiple fiber links. Hence, a light-tree enables "single-hop" communication between a source node and a set of destination nodes and a light-tree based virtual topology can significantly reduce the hop distance, there by increasing the network throughput.

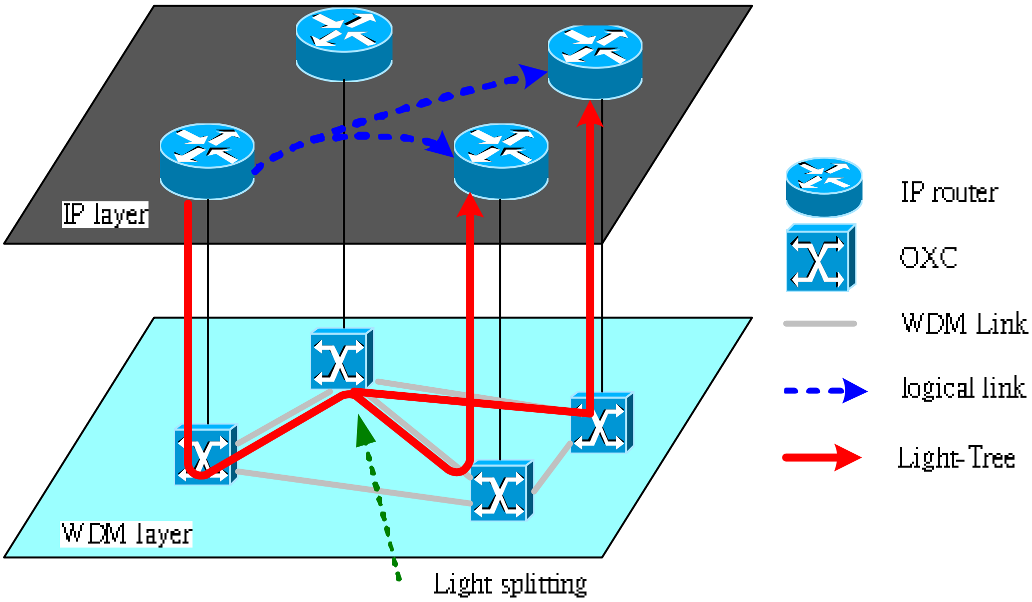


Figure 1-8 Multicasting at the WDM layer

In this paper, we extend the concept of Super-Lightpath and Light-Tree to a "Super-Lighttree" one, which jointly consider the technologies of optical TDM and optical multicasting.

1.3.3 Mathematical Models

Several approaches have been proposed to perform optimal efficient planning and optimization of WDM networks. Heuristic approaches can be categorized into two classes [21]. One is "deterministic heuristic", while the other is called "stochastic heuristic," which uses techniques such as simulated annealing [16] or genetic algorithms [20] to improve heuristic optimization by trying to avoid possible local-minima of the cost function. Integer Linear Programming (ILP) is a widespread technique to solve the optimization problem of network planning besides heuristics. It is because models comprising cost functions of the type we have mentioned above ant topological or wavelength continuity constraints given origin to set of linear equations and inequations; on the other hand, the fact that capacity is expressed in terms of number of WDM channels leads to the integer constraint on the variables.

Two basic methods have been defined to model the RWA problem: *flow formulation* and *route formulation* [24]. In the former, basic decision variables are flows generated through each origin-destination pair on the edge; the later starts with a preprocessed enumeration of all routes or paths between all od-pairs, and then determines how many times each path is used.

Both of the two formulations have been employed to solve various sorts of problems and to investigate different aspects of WDM networks. In [24], both formulations are provided for VWP network and WP network and the problem size in terms of number of variables and constraints are compared. In the route formulation, the number of variables is proportional to the number of possible routes, which increases exponentially with the network size. But, the route formulation has an important advantage that additional limitations, which decrease the problem size significantly, can be imposed during preprocessing time.

A novel *source formulation* for solving multi-fiber network dimensioning [23] in terms of the amounts of fibers per link is proposed in [21]. Source formulation, whose decision variables used for representing flow are transformed to an aggregated form, is equivalent to flow formulation but allows a relevant reduction in the number of variables and of constraints. Additionally, it can avoid nonlinear equation in the formulation without introducing any approximation and can reduce computation time and memory consumption compared to the flow formulation. But, it cannot solve RWA problems because no explicit reference can be inferred regarding lightpaths having the same source and the same destination.

In this paper, because the aggregation of all multicast groups is not known in advance, the candidate trees set could not be determined. However, the membership of each multicast group is given and we can determine the candidate path set between the source node and destination nodes respectively. As a result, we use route formulation in terms of candidate path set [29] to formulate the optimization problem.

1.3.4 Lagrangean Relaxation Method

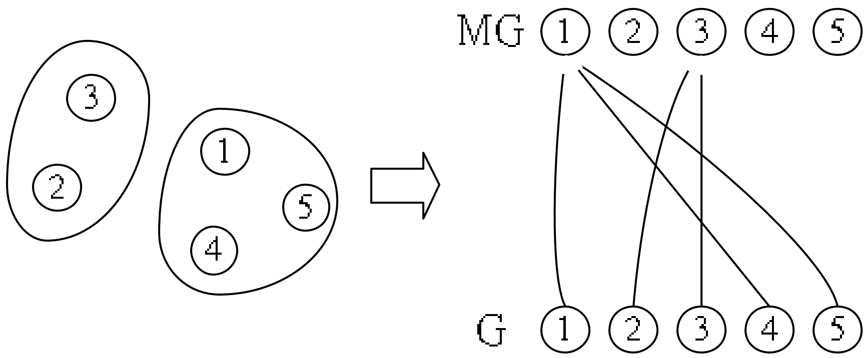
In the 1970s, Lagrangean methods were used in scheduling and the general integer programming problems. Lagrangean relaxation can provide the proper solutions for those problems. In fact, it has become one of the best tools for optimization problems such as integer programming, linear programming, combinatorial optimization, and non-linear programming. Lagrangean relaxation has several advantages, for example, Lagrangean relaxation could decompose mathematical models in many different ways, which is a flexible solution approach. Besides, Lagrangean relaxation solves the sub-problems that we have decomposed as stand-alone problems. From now on, we can optimally solve the sub-problems using any proper algorithm

Lagrangean relaxation method can find out the boundary of the objective function, and it can be used to construct heuristic solutions for getting feasible solutions. Lagrangean relaxation is a flexible solution strategy that permits modelers to exploit the underlying structure in any optimization problem by relaxing complicating constraints. This method permits us to “pull apart” models by removing constraints and instead place them in the objective function with associated Lagrangean multipliers. The optimal value of the relaxed problem is always a lower bound (for minimization problems) on the objective function value of the problem.

To obtain the tightest lower bound, we need to choose the minimization multiplier so that the optimal value of the Lagrangean sub-problem is as large as possible. We can solve the Lagrangean multiplier problem in a variety of ways. The subgradient optimization technique is possibly the most popular technique for solving the Lagrangean multipliers problem.

1.4 Proposed Approach

In this thesis, a multicast tree aggregation algorithm is proposed. It is not easy to represent the mergence between multicast groups in mathematical equations. As a result, “Macro-Group” is introduced. Macro-groups are constructed with the same amount of multicast groups to be considered and the problem of aggregation could be transformed into assignment problem. The transformation is illustrated in Figure 1-10.



problem. This problem is an integer linear programming problem and the Lagrangean relaxation method and the subgradient method will be applied to solve this problem [1] [5].

Chapter 2 Problem Formulation

2.1 Problem Description

The problem to be solved is which multicast group could be admitted in and to which macro-group it should be merged such that the total revenues are maximum. The optimal solution to the constrained multicast RWA should also be answered.

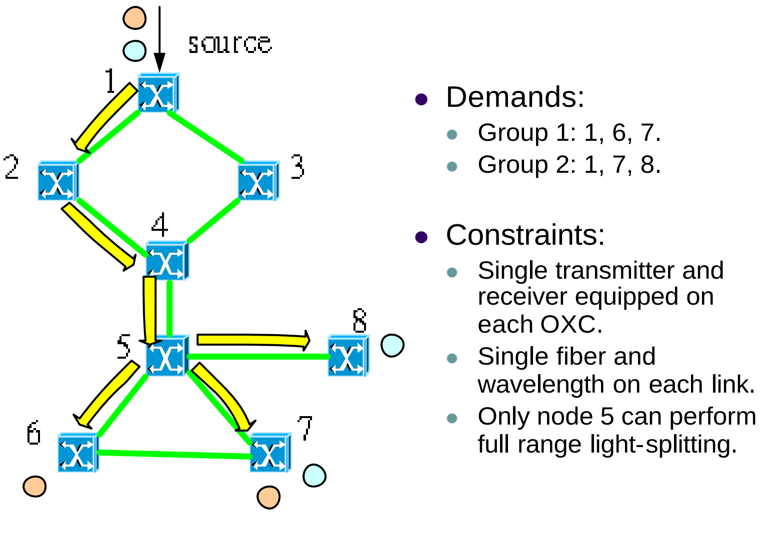


Figure 2-1 A simple scenario with a full range splitter

Take 0 for example. If the node 5 is equipped with a full range splitter, Group 1 and Group 2 can be merged into a macro-group whose destinations are node 6, 7, and 8. The construction of lighttree is simple.

If the splitting compatibility of node 5 is limited to be 1-to-2, the solution is slightly different and is shown in Figure 2-2 and a different placement of splitter is also shown in Figure 2-3.

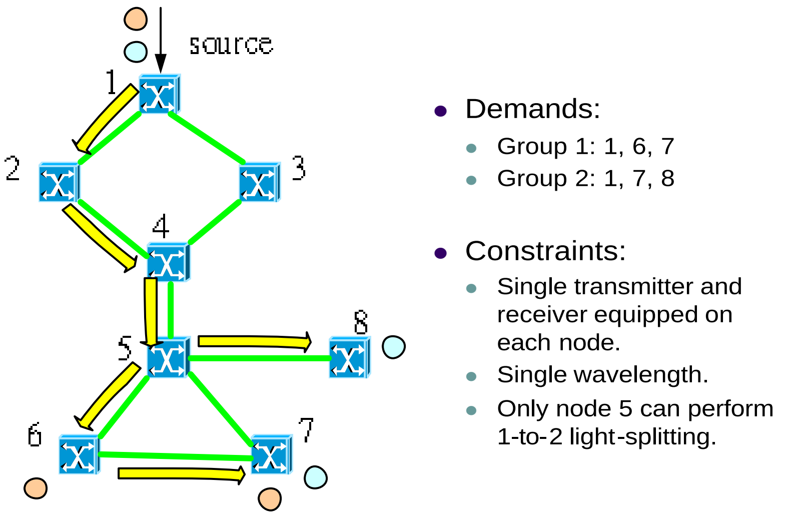


Figure 2-2 A simple scenario with a limited splitting capability

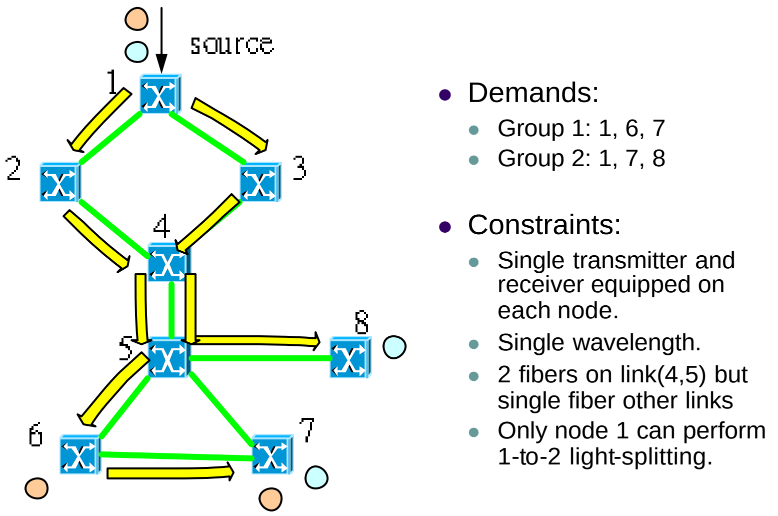


Figure 2-3 A simple scenario with different placement of splitter

The physical topology is modeled as a directed graph $\mathcal{G} = \mathcal{G}(V, L)$. Physical links are represented by the directed edge set L, while the node set V represents the OXCs. Each link is equipped with a certain amount of unidirectional fibers. A multicast group is an application requesting for transmission in the network, which has one origin and one or more destinations. The number of macro-group to be constructed is equivalent to the number of multicast groups to be considered. Each group should be assigned to at most one macro-group. Then, the constrained multicast RWA problems are solved only for those macro-groups having destinations. We now formalize the problem definition.

Assumptions:

1. The basic architecture used is a WDM network.
2. All OXCs used in the optical network have wavelength routing function but lack the capability of wavelength conversion.
3. All OXCs used in the optical network have TDM capability but the routing function is based on optical wavelength switching rather than time-slot or optical burst switching.

Given:

1. The optical network topology.
2. The number of fibers on each link and the wavelength channel cost on it.
3. The number of optical transmitters and receivers equipped on and splitting capability of each OXC.
4. The traffic demand of each multicast group in terms of time-slot and the revenue it can bring in.
5. The set of available wavelengths on each fiber.
6. The number of time-slots supported in a TDM frame.

Objective:

To maximize the total revenue.

Subject to:

1. Only the multicast groups originating at the same source node could be merged

together.

2. Each multicast group should be merged to at most one macro-group.
3. Capacity of components in the network.
4. Splitting Capability of each OXC.
5. Each macro-group is supported by one Super-Lighttree.
6. Wavelength continuity of each Super-Lighttree.

To determine:

1. Which group should be admitted in and the mergence result.
2. Routing and wavelength assignment (Super-Lighttree topology) of each macro-group.

Table 2-1 Problem description

2.2 Notation

Given Parameters	
Notation	**Definition**
$\mathcal{G}=(V,L)$	Directed graph representing an optical network;
V	The set of OXCs;
L	The set of WDM links;
L_v^+	The set of outgoing links of node v;
L_v^-	The set of incoming links of node v;
$Dest(l)$	The destination node of link l;
C_l	The number of unidirectional fibers on link l;
B_l	The cost of link l;
SP_v	The splitting capability of node v;
Tx_v	The number of optical transmitters at node v;

Rx_v	The number of optical receivers at node v;
TS	Number of time-slots in a TDM frame;
G	The set of all multicast groups;
ts_g	Traffic demand of group g in terms of time-slots;
W	The set of available wavelength on each link;
A_g	The revenue of the multicast group g;
MG	The set of macro-groups;
G_v	The set of groups whose source node are v;
T_v	The set of macro-groups whose source node are v, $T_v = G_v$;
D_t	The set of possible destination nodes of macro-group t;
o_g	The source node of group g;
o_t	The source node of macro-group t;
P_{gv}	Candidate path set from the source node of group g to node v;
P_{tv}	Candidate path set from the source of macro-group t to node v, which is identical to P_{gv} if the sources of g and t are the same node;
s_{vg}	1 if node v is a destination of group g, and 0 otherwise;
d_{pl}	1 if link l is on path p, and 0 otherwise.

Table 2-2 Notation of given parameters

2.3 Problem Formulation

2.3.1 Formulation-I

Decision Variables	
Notation	**Descriptions**
m_{gt}	1 if group g is aggregated to macro-group t; otherwise 0;
x_{tvp}	1 if path p is selected for macro-group t to reach node v; otherwise 0;
y_{tl}	The number of fibers on link l used by macro-group t;
z_{tk}	1 if wavelength k is used by macro-group t; otherwise 0.

Table 2-3 Notation of decision variables for Formulation-I

Optimization Problem (IP1):

$$Z_{IP1} = \max \sum_{g\in G} \sum_{t\in MG} A_g m_{gt} - \sum_{l\in L} B_l \sum_{t\in MG} y_{tl} \qquad \textbf{(IP1)}$$

Subject to:

$$m_{gt} = 0 \text{ or } 1 \qquad \forall g \in G, t\in MG \qquad (2.1)$$

$$m_{gt} = 0 \qquad \forall g \in G, t\in MG, o_g \neq o_t \qquad (2.2)$$

$$m_{gt} = 0 \qquad \forall g \in G, t\in MG, t > g \qquad (2.3)$$

$$\sum_{g\in MG} m_{gt} \leq 1 \qquad \forall g \in G \qquad (2.4)$$

$$\sum_{g\in G} ts_g m_{gt} \leq TS \qquad \forall t \in MG \qquad (2.5)$$

$$x_{tvp} = 0 \text{ or } 1 \qquad \forall t\in MG, v\in V, p \in P_{tv} \qquad (2.6)$$

$$\sum_{g\in G} m_{gt} s_{vg} \leq \left|\{G_u \mid u = o_t\}\right| \times \sum_{p\in P_{tv}} x_{tvp} \qquad \forall t\in MG, v\in V \qquad (2.7)$$

$$\sum_{p\in P_{tv}} x_{tvp} \le 1 \qquad \forall t\in MG, v\in V \tag{2.8}$$

$$y_{tl} \in \{0,1,2,...,C_l\} \qquad \forall t\in MG, l\in L \tag{2.9}$$

$$\sum_{v\in V}\sum_{p\in P_{tv}} x_{tvp} d_{pl} \le |D_t| \times y_{tl} \qquad \forall t\in MG, l\in L \tag{2.10}$$

$$\sum_{l\in L_v^+} y_{tl} \le SP_v \times \sum_{l\in L_v^-} y_{tl} \qquad \forall t\in MG, v\in V \tag{2.11}$$

$$z_{tk} = 0 \text{ or } 1 \qquad \forall t\in MG, k\in W \tag{2.12}$$

$$\sum_{k\in W} z_{tk} \le 1 \qquad \forall t\in MG \tag{2.13}$$

$$\sum_{g\in G} m_{gt} \le |\{G_u \mid u=o_t\}| \times \sum_{k\in W} z_{tk} \qquad \forall t\in MG \tag{2.14}$$

$$\sum_{k\in W} z_{tk} \le \sum_{g\in G} m_{gt} \qquad \forall t\in MG \tag{2.15}$$

$$\sum_{t\in MG} y_{tl} z_{tk} \le C_l \qquad \forall l\in L, k\in W \tag{2.16}$$

$$\sum_{t\in T_v}\sum_{k\in W} z_{tk} \le Tx_v \qquad \forall v\in V \tag{2.17}$$

$$\sum_{t\in MG}\sum_{p\in P_{tv}} x_{tvp} \le Rx_v \qquad \forall v\in V\,. \tag{2.18}$$

The objective function of (IP1) is to maximize the total revenue. We can provide different physical meanings of objective function by adjusting the value of A_g. If, for example, all A_g are chosen to be 1, the objective function is to maximize the total amount of admitted multicast groups. If A_g is chosen to be the traffic requirement of group g, then we are maximizing the total system throughput [28].

The set of constraints is explained bellow.

1) Group aggregation constraints:

Constraints (2.1), (2.2), (2.3), (2.4), and (2.5). Each multicast group g should be merged to at most one macro-group, and the aggregated demands can not exceed the number of time slogs supported in a TDM frame. Constraint (2.2) requires that two multicast groups can be merged together only if they originate from the same source node. Equation (2.3) is a redundant constraint which is added to reduce computation time. A more detailed explanation is presented in section 3.1.1.

2) Routing and wavelength channel allocation constraints

Constraints (2.6), (2.7), (2.8), (2.9), (2.10), and (2.11). If a multicast group g is merged to macro-group t, all destination nodes of g should become t's destination. Constraint (2.7) requires that if node v is a destination node of macro-group t, there must be a simple path starting from t's source to it. Constraint (2.9) is a capacity constraint restricting the link usage on l by macro-group t. Constraint (2.10) requires that if link l is on the path used for macro-group t to reach any node v, the link usage of t on that link should be greater than zero. Constraint (2.11) is a splitting constraint which requires that the total outgoing link usage of macro-group t on node v should be less than or equal to the product of splitting capability of v and total incoming link usage of t on node v.

3) Wavelength assignment constraints

Constraints (2.12), (2.13), (2.14), and (2.15). For each macro-group t, it can be assigned to at most one wavelength, which implies wavelength continuity. If any multicast group g is merged to it, Equation (2.14) ensures that it will be assigned a wavelength. On the other hand, Constraint (2.15) requires that macro-group t should not be assigned a wavelength if no group is merged to it.

4) Optical transceiver and capacity constraints

Constraints (2.16), (2.17), and (2.18). Equation (2.16) requires that, for each wavelength, the link usage on link l should not exceed the number of fibers on that link. For each OXC in the network, the total number of super-lighttree rooted at it should not exceed the amount of optical transmitter equipped on it. The number of being destination of any macro-group should not exceed the number of optical receiver equipped on the OXC.

The formulation presented above is not good enough, because the non-linear equation (2.16) makes the problem more complicated and harder to solve. As a result, the problem is reformulated to avoid the non-linearity form in the formulation.

2.3.2 Formulation-II

Decision Variables	
Notation	**Descriptions**
m_{gt}	1 if group g is assigned to macro-group t; otherwise 0;
x_{tvp}	1 if path p is used for macro-group t to reach node v; otherwise 0;
y_{lk}^{t}	The number of fibers on link l with wavelength k used by macro-group t;
z_{tk}	1 if wavelength k is selected for macro-group t; otherwise 0.

Table 2-4 Notation of decision variables for Formulation-II

An equivalent formulation of Problem (IP1) is given by

Optimization problem (IP2)

$$Z_{IP2} = \max \sum_{g \in G} \sum_{t \in MG} A_g m_{gt} - \sum_{l \in L} B_l \sum_{t \in MG} \sum_{k \in W} y_{lk}^t \qquad \textbf{(IP2)}$$

Subject to:

$$m_{gt} = 0 \text{ or } 1 \qquad \forall g \in G, t \in MG \qquad (2.19)$$

$$m_{gt} = 0 \qquad \forall g \in G, t \in MG, o_g \neq o_t \qquad (2.20)$$

$$m_{gt} = 0 \qquad \forall g \in G, t \in MG, t > g \qquad (2.21)$$

$$\sum_{t \in MG} m_{gt} \leq 1 \qquad \forall g \in G \qquad (2.22)$$

$$\sum_{g \in G} ts_g m_{gt} \leq TS \qquad \forall t \in MG \qquad (2.23)$$

$$x_{tvp} = 0 \text{ or } 1 \qquad \forall t \in MG, v \in V, p \in P_{tv} \qquad (2.24)$$

$$\sum_{p \in P_{tv}} x_{tvp} \leq 1 \qquad \forall t \in MG, v \in V \qquad (2.25)$$

$$\sum_{g \in G} m_{gt} s_{vg} \leq \left| \{G_u \mid u = o_t\} \right| \times \sum_{p \in P_{tv}} x_{tvp} \qquad \forall t \in MG, v \in V \qquad (2.26)$$

$$y_{lk}^t \in \{0,1,2,\ldots,C_l\} \qquad \forall t \in MG, k \in W, l \in L \qquad (2.27)$$

$$\sum_{v \in V} \sum_{p \in P_{tv}} x_{tvp} d_{pl} \leq \left| D_t \right| \times \sum_{k \in W} y_{lk}^t \qquad \forall t \in MG, l \in L \qquad (2.28)$$

$$\sum_{t \in MG} y_{lk}^t \leq C_l \qquad \forall l \in L, k \in W \qquad (2.29)$$

$$\sum_{l \in L_v^+} y_{lk}^t \leq SP_v \times \sum_{l \in L_v^-} y_{lk}^t \qquad \forall v \in V, t \in MG, t \notin T_v, k \in W \qquad (2.30)$$

$$\sum_{k \in W} y_{lk}^t \leq \min\left\{C_l, \left\lceil \frac{1}{SP_v} \times |D_t| \right\rceil\right\} \qquad \forall t \in MG, l \in L, v = Dest(l) \qquad (2.31)$$

$$z_{tk} = 0 \text{ or } 1 \qquad \forall t \in MG, k \in W \qquad (2.32)$$

$$\sum_{k \in W} z_{tk} \leq 1 \qquad \forall t \in MG \qquad (2.33)$$

$$\sum_{g \in G} m_{gt} \leq \left| \{G_u \mid u = o_t\} \right| \times \sum_{k \in W} z_{tk} \qquad \forall t \in MG \qquad (2.34)$$

$$\sum_{k \in W} z_{tk} \leq \sum_{g \in G} m_{gt} \qquad \forall t \in MG \qquad (2.35)$$

$$\sum_{l \in L_v^+} y_{lk}^t \quad \leq \quad SP_v \times z_{tk} \quad \forall v \in V, t \in T_v, k \in W \qquad (2.36)$$

$$\sum_{k \in W} \sum_{l \in L_v^-} y_{lk}^t \quad = \quad 0 \quad \forall t \in MG \qquad (2.37)$$

$$\sum_{t \in T_v} \sum_{k \in W} z_{tk} \quad \leq \quad Tx_v \quad \forall v \in V \qquad (2.38)$$

$$\sum_{t \in MG} \sum_{p \in P_{tv}} x_{tvp} \quad \leq \quad Rx_v \quad \forall v \in V \,. \qquad (2.39)$$

The set of constraints is explained bellow.

1) Group aggregation constraints:

Constraints (2.19), (2.20), (2.21), (2.22), and (2.23). Each multicast group *g* should be merged to at most one macro-group, and the aggregated demands can not exceed the number of time slogs supported in a TDM frame. Constraint (2.20) requires that two multicast groups can be merged together only if they originating from the same source node. Equation (2.21) is a redundant constraint which is added to reduce computation time. A more detailed explanation is presented in section 3.1.1.

2) Routing and wavelength channel allocation constraints

Constraints (2.24), (2.25), (2.26), (2.27), (2.28), (2.30), (2.31), and (2.37). If a multicast group *g* is merged to macro-group *t*, all destination nodes of *g* should become *t*' s destination. Constraint (2.26) requires that if node *v* is a destination node of macro-group *t*, there must be a simple path starting from *t*'s source to it. Constraint (2.27) is a capacity constraint restricting the link usage on *l* by macro-group *t* assigned to wavelength *k*.

Constraint (2.28) requires that if link *l* is on the path used for macro-group *t* to reach any node *v*, the link usage of *t* with wavelength *k* on that link should be greater than zero. Constraint (2.30) is a splitting constraint which requires that the total outgoing link usage of macro-group t with wavelength k on node v should be less than or equal to the product of splitting capability of v and total incoming link usage of t on node v with the same wavelength. Equation (2.31) and Equation (2.37) are redundant constraints added to restrict solution range in the relaxed problem.

3) Wavelength assignment constraints

Constraints (2.32), (2.33), (2.34), (2.35), and (2.36). For each macro-group t, it can be assigned to at most one wavelength, which implies wavelength continuity. If any multicast group g is merged to it, Equation (2.34) ensures that it will be assigned a wavelength. On the other hand, Constraint (2.35) requires that macro-group t should not be assigned a wavelength if no group is merged into it. Equation (2.36) is also a splitting constraint special for the source node of macro-group t.

4) Optical transceiver and capacity constraints

Constraints (2.29), (2.38), and(2.39). Equation (2.29) requires that, for each wavelength, the total wavelength channels used on link l should not exceed the number of fibers on that link. For each OXC in the network, the total number of super-lighttree rooted at it should not exceed the amount of optical transmitter equipped on it. The number of being destination of any macro-group should not exceed the number of optical receiver equipped on the OXC.

The number of variables and constraints used in our formulation are both $O(|G|^2 + |G| \times |V| \times |P| + |G| \times |L| \times |W|)$, where P is the candidate paths set between all pairs of nodes in the network. The size of P is $O(2^{|L|})$ for that each link l in the network may or may not be in a given path.

However, with a slight modification, the space complexity of our formulation will not grow exponentially with network size in terms of links. The variable x_{tvp}, which represents whether source node of macro-group t reaching node v by path p, can be replaced by two 0-1 variables: x'_{tv} and x'_{tvl}. The former represents whether macro-group t use any path to reach node v while the later decides whether link l is on the path used by t from it's source to v.

The reason the modification can be made in such a way is that the variable x_{tvp} in all equations of the proposed formulation is almost represented in an aggregated form ($\sum_{p \in P_{tv}} x_{tvp}$) except for Constraint (2.24) and Constraint (2.28). For each pair of macro-group t and node v, Constraint (2.25) requires that there is at most one path connecting t's source to v. As a result, the path p used by t to reach v can always be recovered from the information recorded in x'_{tv} and x'_{tvl}. Therefore, the space complexity of our formulation is reduced to $O(|G|^2 + |G| \times |L| \times (|V| + |W|))$ where the exponential term $O(|P|) = O(2^{|L|})$ is replaced by $O(|L|)$.

Chapter 3 Solution Approach

3.1 Lagrangean Relaxation

As a convention, we can transform the maximization problem to minimization without loss of correctness. By using the Lagrangean relaxation method, the primal problem (IP2) can be transformed into the following Lagrangean relaxation problem (LR) where Constraints (2.23), (2.26), (2.28), (2.29), (2.30), (2.34), (2.35), (2.36) are relaxed. For a vector of non-negative Lagrangean multipliers, the Lagrangean relaxation problem is given by optimization problem (LR):

$$
\begin{aligned}
&Z_d(u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8) = \\
&\min -\sum_{g\in G}\sum_{t\in MG} A_g m_{gt} + \sum_{l\in L} B_l \sum_{t\in MG}\sum_{k\in W} y_{lk}^t \\
&+\sum_{t\in MG}\sum_{v\in V} u_{1tv}\left[\sum_{g\in G} m_{gt} s_{vg} - \left|\{G_u \mid u = o_t\}\right| \times \sum_{p\in P_{tv}} x_{tvp}\right] \\
&+\sum_{t\in MG}\sum_{l\in L} u_{2tl}\left[\sum_{v\in V}\sum_{p\in P_{tv}} x_{tvp} d_{pl} - |V| \times \sum_{k\in W} y_{lk}^t\right] \\
&+\sum_{t\in MG} u_{3t}\left[\sum_{g\in G} m_{gt} - \left|\{G_u \mid u = o_t\}\right| \times \sum_{k\in W} z_{tk}\right] + \sum_{v\in V}\sum_{t\in T_v}\sum_{k\in W} u_{4vtk}\left[\sum_{l\in L_v^+} y_{lk}^t - SP_v \times z_{tk}\right] \\
&+\sum_{l\in L}\sum_{k\in W} u_{5lk}\left[\sum_{t\in MG} y_{lk}^t - C_l\right] + \sum_{v\in V}\sum_{t\in\{MG-T_v\}}\sum_{k\in W} u_{6vtk}\left[\sum_{l\in L_v^+} y_{lk}^t - SP_v \times \sum_{l\in L_v^-} y_{lk}^t\right] \\
&+\sum_{t\in MG} u_{7t}\left[\sum_{g\in G} ts_g m_{gt} - TS\right] + \sum_{t\in MG} u_{8t}\left[\sum_{k\in W} z_{tk} - \sum_{g\in G} m_{gt}\right] \qquad \text{(LR)}
\end{aligned}
$$

subject to:

$$m_{gt} = 0 \text{ or } 1 \quad \forall g \in G, t \in MG \tag{3.1}$$

$$m_{gt} = 0 \quad \forall g \in G, t \in MG, o_g \neq o_t \tag{3.2}$$

$$m_{gt} = 0 \quad \forall g \in G, t \in MG, t > g \tag{3.3}$$

$$\sum_{t \in MG} m_{gt} \leq 1 \quad \forall g \in G \tag{3.4}$$

$$x_{tvp} = 0 \text{ or } 1 \quad \forall t \in MG, v \in V, p \in P_{tv} \tag{3.5}$$

$$\sum_{p \in P_{tv}} x_{tvp} \leq 1 \quad \forall t \in MG, v \in V \tag{3.6}$$

$$y_{lk}^{t} \in \{0,1,2,\ldots,C_l\} \quad \forall t \in MG, k \in W, l \in L \tag{3.7}$$

$$\sum_{k \in W} y_{lk}^{t} \leq \min\{C_l, \left\lceil \frac{1}{SP_v} \times |D_t| \right\rceil\} \quad \forall t \in MG, l \in L, v = Dest(l) \tag{3.8}$$

$$z_{tk} = 0 \text{ or } 1 \quad \forall t \in MG, k \in W \tag{3.9}$$

$$\sum_{k \in W} z_{tk} \leq 1 \quad \forall t \in MG \tag{3.10}$$

$$\sum_{k \in W} \sum_{l \in L_v^-} y_{lk}^{t} = 0 \quad \forall t \in MG \tag{3.11}$$

$$\sum_{t \in T_v} \sum_{k \in W} z_{tk} \leq Tx_v \quad \forall v \in V \tag{3.12}$$

$$\sum_{t \in MG} \sum_{p \in P_{tv}} x_{tvp} \leq Rx_v \quad \forall v \in V \tag{3.13}$$

where u_1, u_2, u_3, u_4, u_5, u_6, u_7, and u_8 are the vectors of non-negative Lagrangean multipliers $\{u_{1tv}\}$, $\{u_{2tl}\}$, $\{u_{3t}\}$, $\{u_{4vtk}\}$, $\{u_{5lk}\}$, $\{u_{6vtk}\}$, $\{u_{7t}\}$, and $\{u_8\}$. To solve (LR), we decompose the problem into the following four independent and easily solvable optimization subproblems.

3.1.1 **Subproblem 1** (related to decision variable m_{gt})

$$Z_{sub3.1}(u_1, u_3, u_7, u_8)$$
$$= \min -\sum_{g \in G}\sum_{t \in MG} A_g m_{gt} + \sum_{t \in MG}\sum_{v \in V}\sum_{g \in G} u_{1tv} m_{gt} s_{vg} + \sum_{t \in MG}\sum_{g \in G} u_{3t} m_{gt} + \sum_{t \in MG} u_{7t} \sum_{g \in G} ts_g m_{gt}$$
$$- \sum_{t \in MG} u_{8t} \sum_{g \in G} m_{gt}$$
$$= \min \sum_{t \in MG}\sum_{g \in G}\left(\sum_{v \in V} u_{1tv} s_{vg} + u_{3t} + u_{7t} ts_g - u_{8t} - A_g\right) m_{gt} \qquad \text{(SUB3.1)}$$

subject to:

m_{gt}	$=$	0 or 1	$\forall g \in G, t \in MG$	(3.1)
m_{gt}	$=$	0	$\forall g \in G, t \in MG, o_g \neq o_t$	(3.2)
m_{gt}	$=$	0	$\forall g \in G, t \in MG, t > g$	(3.3)
$\sum_{t \in MG} m_{gt}$	$\leq$	1	$\forall g \in G$.	(3.4)

Subproblem 1 can be further decomposed into $|V|$ problem because the groups will be aggregated together only if they root at the same nodes. A redundant constraint (3.3) is added to the problem in order to avoid oscillation of decision variable iteration by iteration. A formal proof is given below.

Lemma 1

Constraint (3.3) is a redundant constraint.

Proof

The lemma is proved by construction. A simple permutation and re-labeling technique can be applied to all possible assignments between groups and macro-groups to satisfy Constraint (3.3). Given an aggregated macro-group *t*, it can be relabeled to the smallest ID among all groups assigned to it. Because each group can be aggregated to at most one macro-group, no macro-group will come into collision with others in terms of ID. As a result, the assignment between groups and relabeled macro-groups satisfies the constraint (3.3).

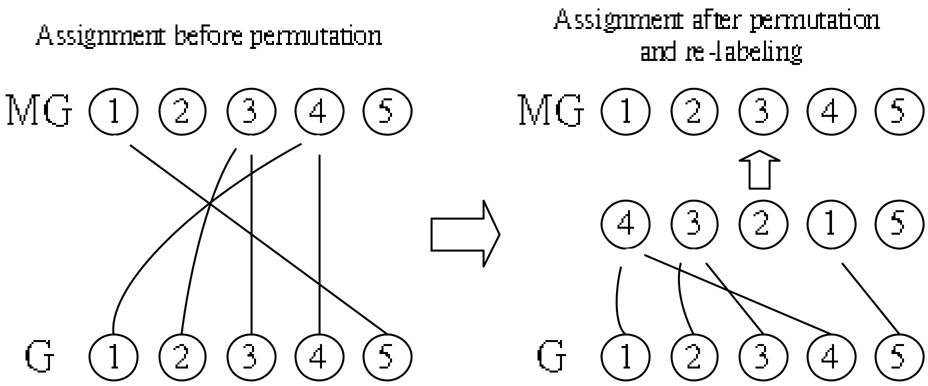


Figure 3-1 Permutation and re-labeling on macro-groups

An example is illustrated in Figure 3-1. The macro-groups are permutated according to the lowest ID among groups being assigned to them and relabeled according to the new order. For example, macro-group 4 is relabeled to 1 because multicast group 1 is assigned to it. The macro-group 3 is relabeled in this way as well. According to the computational experiments, the running time of the algorithm will be shortened and the lower bound will be slightly higher with this redundant constraint.

For each group g, it will be aggregated to the macro-group t with lowest coefficient $\sum_{v\in V} u_{1tv} s_{vg} + u_{3t} + u_{7t} t s_g - u_{8t} - A_g$. If the lowest coefficient is greater than 0, group g is dropped; otherwise g is aggregated to macro-group t and the corresponding variable m_{gt} is set to be 1.

3.1.2 Subproblem 2 (related to decision variable x_{tvp})

$$Z_{sub3.2}(u_1, u_2)$$
$$= \min - \sum_{t \in MG} \sum_{v \in V} \sum_{p \in P_{tv}} \left|\{G_u \mid u = o_t\}\right| \times u_{1tv} x_{tvp} + \sum_{t \in MG} \sum_{l \in L} \sum_{v \in V} \sum_{p \in P_{tv}} u_{2tl} x_{tvp} d_{pl}$$

$$= \min \sum_{t \in MG} \sum_{v \in V} \sum_{p \in P_{tv}} \left(\sum_{l \in L} u_{2tl} d_{pl} - \left|\{G_u \mid u = o_t\}\right| \times u_{1tv} \right) x_{tvp} \quad \text{(SUB3.2)}$$

subject to:

$$x_{tvp} = 0 \text{ or } 1 \qquad \forall t \in MG, v \in V, p \in P_{tv} \qquad (3.5)$$

$$\sum_{p \in P_{tv}} x_{tvp} \leq 1 \qquad \forall t \in MG, v \in V \qquad (3.6)$$

$$\sum_{t \in MG} \sum_{p \in P_{tv}} x_{tvp} \leq Rx_v \qquad \forall v \in V. \qquad (3.13)$$

Subproblem 2 is composed of $|MG|$ shortest path tree problems for each macro-group t, where u_{2tl} is the arc weight of link l. For each pair t and v, if the cost of the result shortest path p is less than the threshold value $\left|\{G_u \mid u = o_t\}\right| \times u_{1tv}$, set x_{tvp} to be 1, otherwise set it to be 0. If Constraint (3.13) is violated by some node v, sort values of shortest path cost minus threshold value in ascending order, and followed by setting the first Rx_v corresponding x_{tvp} to be 1.

3.1.3 Subproblem 3 (related to decision variable y_{lk}^t)

$$
\begin{aligned}
& Z_{sub3.3}(u_2, u_4, u_5, u_6) \\
& = \min \sum_{l\in L} B_l \sum_{t\in MG} \sum_{k\in W} y_{lk}^t - \sum_{t\in MG} \sum_{l\in L} |D_t| \times u_{2tl} \sum_{k\in W} y_{lk}^t + \sum_{v\in V} \sum_{t\in T_v} \sum_{k\in W} u_{4vtk} \sum_{l\in L_v^+} y_{lk}^t \\
& + \sum_{l\in L} \sum_{k\in W} u_{5lk} \sum_{t\in MG} y_{lk}^t + \sum_{v\in V} \sum_{t\in\{MG-T_v\}} \sum_{k\in W} u_{6vtk} \sum_{l\in L_v^+} y_{lk}^t - \sum_{v\in V} \sum_{t\in\{MG-T_v\}} \sum_{k\in W} u_{6vtk} SP_v \times \sum_{l\in L_v^-} y_{lk}^t \\
& = \min \sum_{k\in W} \sum_{t\in MG} \sum_{l\in L} B_l y_{lk}^t - \sum_{k\in W} \sum_{t\in MG} \sum_{l\in L} |D_t| \times u_{2tl} y_{lk}^t + \sum_{k\in W} \sum_{t\in MG} \sum_{l\in L} u_{5lk} y_{lk}^t \\
& + \sum_{k\in W} \sum_{v\in V} \sum_{t\in T_v} \sum_{l\in L_v^+} u_{4vtk} y_{lk}^t + \sum_{k\in W} \sum_{v\in V} \sum_{t\in\{MG-T_v\}} \sum_{l\in L_v^+} u_{6vtk} y_{lk}^t - \sum_{k\in W} \sum_{v\in V} \sum_{t\in\{MG-T_v\}} \sum_{l\in L_v^-} u_{6vtk} SP_v y_{lk}^t \\
& = \min \sum_{k\in W} \sum_{t\in MG} \sum_{l\in L} [B_l + u_{5lk} - |D_t| \times u_{2tl}] y_{lk}^t + \sum_{k\in W} \sum_{v\in V} \sum_{t\in T_v} \sum_{l\in L_v^+} u_{4vtk} y_{lk}^t \\
& + \sum_{k\in W} \sum_{v\in V} \sum_{t\in\{MG-T_v\}} \sum_{l\in L_v^+} u_{6vtk} y_{lk}^t - \sum_{k\in W} \sum_{v\in V} \sum_{t\in\{MG-T_v\}} \sum_{l\in L_v^-} u_{6vtk} SP_v y_{lk}^t
\end{aligned}
\qquad \text{(SUB3.3)}
$$

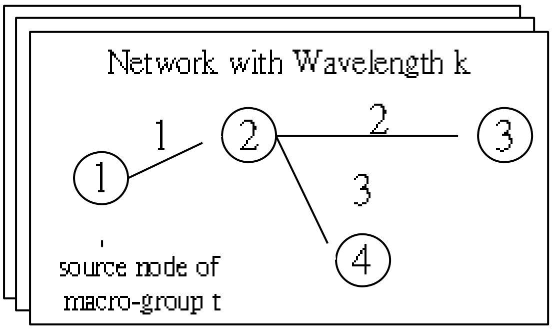
subject to:

$$y_{lk}^t \in \{0,1,2,...,C_l\} \qquad \forall t \in MG, k \in W, l \in L \qquad (3.7)$$

$$\sum_{k\in W} \sum_{l\in L_v^-} y_{lk}^t = 0 \qquad \forall v \in V, t \in T_v \qquad (3.11)$$

$$\sum_{k\in W} y_{lk}^t \le \min\{C_l, \left\lceil \frac{1}{SP_v} \times |D_t| \right\rceil\} \qquad \forall t \in MG, l \in L, v = Dest(l). \qquad (3.8)$$

Subproblem 3 can be further decomposed into |*MG*| problems. The link usage and wavelength for each macro-group $t \in MG$ should be decided. In order to minimize the objective function, the wavelength k with smallest coefficient is select for each link. The coefficient is composed of two parts: $[B_l + u_{5lk} - |D_t| \times u_{2tl}]$ and multipliers $\{u_{4vtk}\}$, $\{u_{6vtk}\}$, which is illustrated in Figure 3-2.

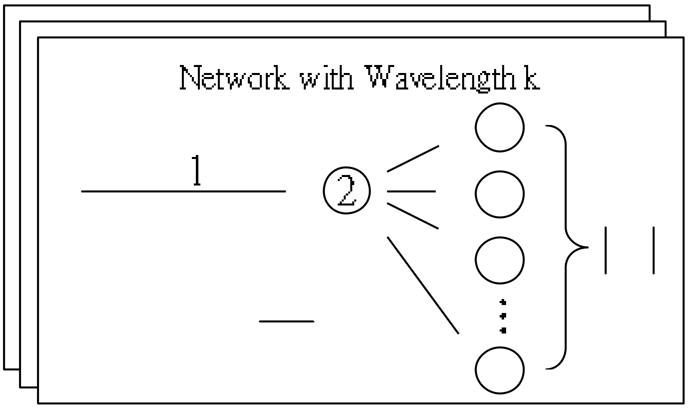


Second part coefficient

$L1:\ u_{4v_1tk} - SP_{v_2} \times u_{6v_2tk}$

$L2:\ u_{6v_2tk} - SP_{v_3} \times u_{6v_3tk}$

$L3:\ u_{6v_2tk} - SP_{v_4} \times u_{6v_4tk}$



3.1.4 Subproblem 4 (related to decision variable z_{tk})

$$
\begin{aligned}
& Z_{sub3.4}(u_3, u_4) \\
&= \min -\sum_{t\in MG}\sum_{k\in W}\left|\{G_u \mid u = o_t\}\right| u_{3t} z_{tk} - \sum_{v\in V}\sum_{t\in T_v}\sum_{k\in W} u_{4vtk} z_{tk} SP_v + \sum_{t\in MG} u_{8t} \sum_{k\in W} z_{tk} \\
&= \min -\sum_{v\in V}\sum_{t\in T_v}\sum_{k\in W} |T_v| \times u_{3t} z_{tk} - \sum_{v\in V}\sum_{t\in T_v}\sum_{k\in W} u_{4vtk} z_{tk} SP_v + \sum_{t\in MG}\sum_{k\in W} u_{8t} z_{tk} \\
&= \min -\sum_{v\in V}\sum_{t\in T_v}\sum_{k\in W} \left(|T_v| \times u_{3t} + u_{4vtk} SP_v - u_{8t}\right) z_{tk}
\end{aligned}
\qquad \text{(SUB3.4)}
$$

Subject to:

z_{tk}	=	0 or 1	$\forall t \in MG, k \in W$	(3.9)
$\sum_{k\in W} z_{tk}$	$\leq$	1	$\forall t \in MG$	(3.10)
$\sum_{t\in T_v}\sum_{k\in W} z_{tk}$	$\leq$	Tx_v	$\forall v \in V$	(3.12)

Subproblem 4 can be further decomposed into |V| problems. For each node v, every macro-group rooted at v has to be assigned a wavelength. According to Lagrangean multipliers $\{u_{4vtk}\}$, the wavelength k with largest value of u_{4vtk} is chosen for macro-group t if the coefficient $|T_v| \times u_{3t} + u_{4vtk} SP_v - u_{8t}$ is larger than 0. Otherwise, macro-group t is skipped. For each node v, if the number of macro-groups rooted at it is larger than the number of transmitters on it, macro-groups are sorted according to their coefficients in ascending order and then the first Tx_v corresponding z_{tk} are set to be 1.

3.2 The Dual Problem and the Subgradient Method

According to the weak Lagrangean duality theorem [6], for any u_1, u_2, u_3, u_4, u_5, u_6, u_7, and $u_8 \geq 0$, $Z_d(u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8)$ is a lower bound on Z_{IP2}. We construct the following dual problem (D) to calculate the tightest lower bound.

Dual Problem (D):

$$Z_D = \max Z_d(u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8) \qquad \text{(D)}$$

subject to:

$$u_1,\ u_2,\ u_3,\ u_4,\ u_5,\ u_6,\ u_7,\ u_8 \geq 0 \qquad (3.14)$$

The most popular method to solve the dual problem is the subgradient method [9]. Let the vector S be a subgradient of $Z_d(u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8)$. Then, in iteration k of the subgradient optimization procedure, the multiplier vector $m=(u_1,\ u_2,\ u_3,\ u_4,\ u_5,\ u_6,\ u_7,\ u_8)$ is updated by $m^{k+1} = m^k + t^k S^k$. The step size t^k is determined by $t^k = d\,\dfrac{Z_{IP2}^h - Z_d\left(m^k\right)}{\left\|S^k\right\|^2}$. Z_{IP2}^h is the primal objective function value for a heuristic solution and d is constant between 0 and 2.

Chapter 4 Getting Primal Feasible Solutions

By using Lagrangean relaxation and the subgradient method, we can get a theoretical lower bound of the primal problem. Besides, solutions to the Lagrangean relaxation problem (LR) and Lagrangean multipliers resulted from iterations can provide some hints to help us get primal feasible solution [13]. If the calculated decision variables happen to satisfy all constraints in the primal problem, a primal feasible solution is found. Otherwise, the modification on the infeasible solution can be made to obtain primal feasible solutions.

Owing to the complexity of the primal problem, we divide overall problem into two subproblems: the group aggregation subproblem and the constrained multicast routing and wavelength assignment (RWA) subproblem. The first one determines which group can be admitted in, the memberships between admitted-in multicast groups and the destinations of all macro-groups. After the aggregation of multicast groups and memberships of macro-group are determined, we solve constrained multicast RWA subproblem for each macro-group.

4.1 Heuristic for Group Aggregation Subproblem

In this subproblem, we have to decide which groups to be admitted in the network and the membership of the aggregated macro-groups. To solve this problem, solutions to problem

$Z_{sub3.1}$, $\{m_{gt}\}$, are considered. It would be a good starting point to get the feasible solution because, by definition, whether a group g is admitted in and to which macro-group g should be aggregated are answered. However, the capacity constraint in terms of time-slots (2.23) in the primal problem is relaxed and the solution to $Z_{sub3.1}$ may violate it. As a result, it is required to check whether the capacity constraint is violated, and, determine which group to be drop temporarily if needed. The multipliers considered in $Z_{sub3.1}$, $\{u_{1tv}\}$, $\{u_{3t}\}$, and $\{u_{8t}\}$ are good reference to drop group.

The algorithm is described as follows:

1. Based on $\{m_{gt}\}$, identify the set of un-admitted-in groups, denoted by U_g.
2. Based on $\{m_{gt}\}$, calculate the aggregated demands of time-slots of all macro-groups and identify the set of multicast group which are assigned to it, denoted by G_t.
3. Identify the set of macro-groups whose aggregated demand exceeds the number of available time-slots in a TDM frame (*TS*), denoted by T_m.
4. Remove one $t \in T_m$, calculate contribution ratio for each group g in G_t.
5. Drop $g \in G_t$ with lowest contribution ratio and insert g into U_g.
6. Repeat step 5 until the aggregated demands in terms of time-slots of t is less than or equal to *TS*.
7. Repeat step 4, 5, and 6 until T_m becomes empty.
8. For each nonempty macro-group, identify the destinations, calculate the revenue and insert it to the set T_r, which is the set of macro-group to be routed.
9. Based on revenue, sort $t \in T_r$ in descending order.

Table 4-1 Heuristic for group aggregation subproblem

After applying the algorithm described above, we get the set of macro-groups for which we solve multicast RWA in next heuristic.

4.2 Heuristic for Constrained Multicast RWA Subproblem

The constraints to be considered in this subproblem are capacity constraint and degree constraint, since the residual capacity of each link would decrease iteration by iteration and splitting capability of each node is given as input parameter. We first propose an algorithm called SPH-J to solve constrained multicast routing problem. The idea comes from SPH-Relax and SPH-Naïve algorithms proposed in [4]. In addition to splitting capability considered in SPH-Relax and SPH-Naïve, residual capacity in terms of fiber-optics of each link should also be taken into consideration.

The algorithm SPH-J is described as follows:

Algorithm SPH-J
1. Insert source node to set *TreeNodeSet*, and insert all destinations to set *DestSet*.
2. Calculate the link cost: SPHCost(l) = $B_l + \sum_{t\in MG}\sum_{k\in W} u_{6vtk}$, where v=*Dest(l)*.
3. Choose $d \in DestSet$ with feasible path and lowest cost to any node $tn \in TreeNodeSet$, degree constraint of each node and capacity constraint of each link on the acyclic path should be checked. If no such d exists, terminate this algorithm and return fail.
4. Remove d from *DestSet* and insert it with other nodes all the way in the path into *TreeNodeSet*.
5. Repeat step 3 until *DestSet* is empty.
6. Return success.

Table 4-2 Algorithm SPH-J

The link costs used in SPH-J are the sum of original link costs B_l and multipliers $\{u_{6vtk}\}$ where v is the destination node of link l. The reason is that if the destination node v of link l is more powerful in terms of splitting capability, the link cost on l would be lower than others. When solving dual problem Z_D, the higher degree of splitting capability on node v the lower the corresponding u_{6vtk} will be. As a result, $\{u_{6vtk}\}$ are added to the original link cost. With this modification, we can find out more feasible solutions with acceptable quality. The time complexity of Dijkstra shortest path algorithm using heap implementation is $O(|L|\ log|V|)$. Therefore, the time complexity of SPH-J is $O(|Dt|^2 \times |L| \times \log|V|)$.

Once the routing problem is solved, the lambda k with lowest routing cost is chosen for the macro-group. The overall algorithm solving constrained multicast RWA problem is given below.

1. Select a macro-group $t \in T_r$ with highest revenue it can earn, and check whether residual transmitter on source node and receiver on destination nodes of t are enough or not.
2. If any residual resources needed by t are not enough, drop all groups in G_t and insert into U_g. Repeat step1.
3. Else, run SPH-J algorithm |W| times for each wavelength. Select wavelength k with lowest routing cost.
4. If no such wavelength k exists, drop all groups in G_t and insert them into U_g.
5. Else, decrease the residual transmitter on source node of t, the residual receiver on destinations of t, and the residual link capacity with wavelength k of all used link according to the link usage calculated in step 3. Remove t from T_r and repeat step 1 until T_r becomes empty.

Table 4-3 Heuristic Constrained Multicast RWA Subproblem

Chapter 5 Computational Experiments

Because of the complexity of the multicast group aggregation and constrained multicast routing and wavelength assignment problems, it is not easy to get a tighter lower bound by solving the Lagrangean relaxation problem iteration by iteration. But this powerful methodology provides a lot of hints to help us get a primal feasible solution. In order to demonstrate the difference of solution quality between the algorithms proposed in this thesis and other primal heuristics, a simple algorithm is implemented to compare with the Lagrangean relaxation based algorithms.

5.1 Simple Algorithm (SA)

In chapter 4, the problem is decomposed into two subproblems: the group aggregation subproblem and the constrained multicast routing and wavelength assignment (RWA) subproblem. Without implications of the Lagrangean multipliers, memberships and demands of groups are the only information we can rely on to solve the group aggregation problem. Two groups can be merged into one macro-group if they have sufficient overlap in terms of destination nodes and the aggregated demands do not exceed the number of available timeslots in a TDM frame.

After determining the membership of macro-groups, the same heuristic described in section 4.2 is applied to solve the constrained multicast RWA problem. Note that, again, by no means can we properly adjust the link cost without implications of the Lagrangean multipliers. As a result, the cost of each link is not modified. The simple algorithm is presented below.

Algorithm SA

Step 1 (Initialization):

Read configuration file to construct WDM network and generate multicast traffic demands.

Step 2 (Group Aggregation):

For each group g rooted at node v, merge g to a macro-group t ($t < g$) with highest extent of overlap (at least 66%) in terms of destination nodes without violating capacity constraint. If no such t exists, assign g to the macro-group with ID g.

Step 3 (Constrained Multicast RWA):

Applying the same algorithms described in section 4.2 to determine the routing and wavelength assignment problem for each macro-group.

Step 4 (Termination):

Calculate the result value from step 3 and terminate this algorithm.

Table 5-1 Simple algorithm

5.2 Lagrangean Relaxation Based Algorithm (LR)

This algorithm is based on the mathematical formulation described in Chapter 2. The relaxed problem is then optimally solved as described in Chapter 3 to get a lower bound to the primal problem. We adopt a heuristic algorithm to solve group aggregation problem as described in section 4.1 and a shortest path heuristic based algorithm to solve capacity and degree constrained multicast routing and wavelength assignment problem as described in section 4.2. And we use a subgradient method to update the Lagrangean multipliers. To sum up, the Lagrangean relaxation based algorithm (LR) is presented as follows:

Algorithm LR

Step 1 (Initialization):

1) Read configuration file to construct WDM network and generate multicast traffic demands.
2) Initialize all multipliers and iteration counter *i* to be zero and step size to be 2.
3) Initialize upper bound value (UB) to be 0 and lower bound value (LB) to be the negative summation of all groups' revenue.

Step 2 (Termination Criterion):

If upper bound equals lower bound or iteration counter *i* reaches desired iterations, terminate this algorithm. Otherwise, go to next step.

Step 3 (Calculating Lower Bound):

With the given Lagrangean multipliers, optimally solve these subproblems as described in chapter 3.1 to get the value Z_d .

Step 4 (Getting Primal Feasible Solution):

Applying the algorithms described in chapter 4 to calculate the value Z_{IP2}.

Step 5 (Updating Lower Bound, Upper Bound, and Lagrangean Multipliers):

1) If Z_d > LB, set LB = Z_d .
2) If Z_{IP2} < UB, set UB = Z_{IP2}.
3) Calculate step size and update Lagrangean multipliers by using the subgradient method as described in section 3.2.
4) Increase the iteration counter *i* and go to Step 2.

Table 5-2 Lagrangean relaxation based algorithm

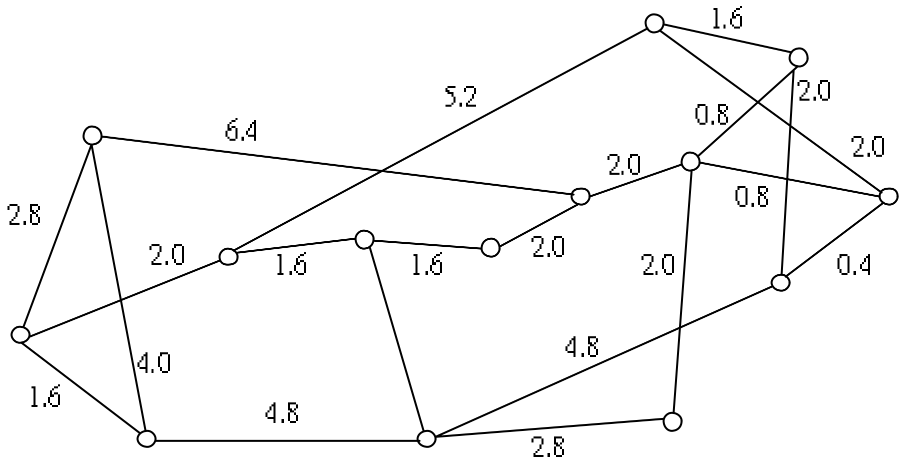
The time complexity of iterations of the proposed Lagrangean relaxation based algorithm is composed of three parts: solving Lagrangean relaxation subproblems, solving the Lagrangean dual problem, and getting primal feasible solutions. The third part dominates others because the high complexity of solving constrained multicast RWA problem. The worst case time complexity is $O(|G|\times|W|\times|Dt|^2\times|L|\times\log|V|)$.

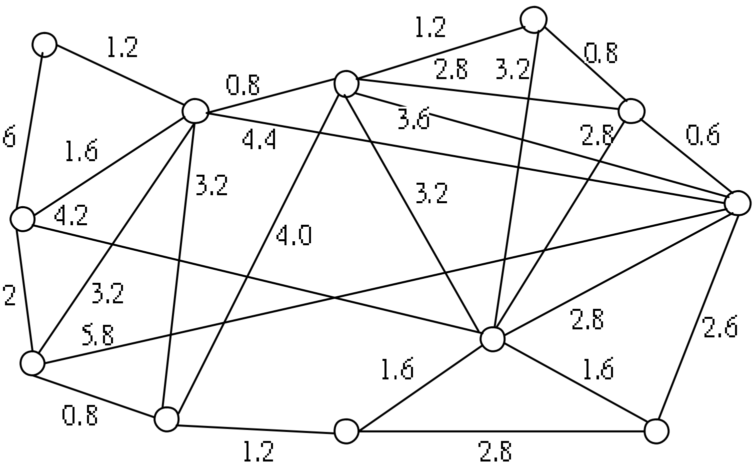
5.3 Parameters and Cases of the Experiment

Number of Iterations	10000
Improvement Counter	600
Begin to Tune Iteration	800
Initial Upper Bound	0
Initial Scalar of Step Size	2
Number of Time-Slots available in a TDM frame	10
Revenue Factor	0.7~1.0

Table 5-1 Parameters for all cases

The parameters used for all cases are listed in Table 5-1. The revenue of each group is decided by multiplying the total link cost in the network and the revenue factor. The revenue factor is chosen uniformly at random between 0.7 and 1.0. The network topology used for our numerical experiments are 14-node 42-link NSFNET network (Figure 5-1) and 12-node 50-link GTE network (Figure 5-2). The number beside each link indicates the cost of the link.


1.6
5.2
2.0
0.8
6.4
2.0
2.0
2.8
0.8
2.0
1.6
1.6
2.0
2.0
0.4
4.8
4.0
1.6
4.8
2.8


1.2
1.2
0.8
0.8
2.8
3.2
1.6
3.6
2.8
0.6
4.4
1.6
3.2
3.2
4.2
4.0
1.2
3.2
2.8
2.6
5.8
1.6
1.6
0.8
1.2
2.8

Case	1	2	3
Number of Groups	50, 70, 90, 110	50, 70, 90, 110	50, 70, 90, 110
Additional Splitting Capabilities	8, 12	8, 12	8, 12
Group Size	0.2|*N*|~0.6|*N*|	0.2|*N*|~0.6|*N*|	0.2|*N*|~0.6|*N*|
Demand	2~6	4~8	9~10
Number of Tx	3	3	3
Number of Rx	17	17	17
Number of Fibers on Each Link	2	2	2
Number of Wavelength	3	3	3

Table 5-2 Parameters in Experiment-I

Second, the relationship between the reduced cost and increased splitting capability are examined in Experiment-II. In this case, we try to compare the costs between different splitting capabilities conditions. To achieve fair comparison, the number of groups being admitted in should be the same or we can compare the costs between different numbers of groups being routed in the network. As a result, the offered loads in terms of the number of groups are set to be small to make all of them being admitted in. The parameters used in Case 4 are listed in Table 5-3.

Case	4
Number of Groups	10
Additional Splitting Capabilities	0, 4, 8, 12, 16, 20, 24
Group Size	0.3|*N*|~0.7|*N*|
Demand	2~7
Number of Tx	4

Number of Rx	20
Number of Fibers on Each Link	4
Number of Wavelength	4

Table 5-3 Parameters in Experiment-II

5.4 Experiment Results

To make the comparison easier, solutions to the minimization problem are transformed to solutions to the original maximization problem. The LR value means the optimal value calculated from the LR algorithm and the Upper Bound value represents the optimal solution to the dual problem (D). The SP value is the additional splitting capabilities added to nodes in the network uniformly at random. The results are average over two samples with random seeds 100 and 800. The improvement ratio is calculated by (LR-SA)/SA.

5.4.1 Results of Experiment-I

Case 1-NSFNET (Demand: 2 ~ 6)

SP	Seed	Group	SA	LR	UB	Gap	Imp.ratio
8	10	50	2553	3227.8	4628.9	43.41%	26.43%
		70	2872.4	3748	6524.45	74.08%	30.48%
		90	3233	4001.3	8415	110.31%	23.76%
		110	3569.4	4172.4	10266	146.05%	16.89%
	70	50	2573.4	3133.2	4691.91	49.75%	21.75%
		70	2982.4	3691.5	6582.68	78.33%	23.78%
		90	3744	3936	8532	116.77%	5.13%
		110	3821.8	4132.6	10416.9	152.07%	8.13%
12	10	50	2691.8	3353.2	4624.25	37.91%	24.57%
		70	2991	3740.4	6521.74	74.36%	25.06%
		90	3301.6	4128.2	8412.6	103.78%	25.04%
		110	3643.2	4469	10254.6	129.46%	22.67%

	70	50	2843	3377.8	4693.39	38.95%	18.81%
		70	2995.8	3818	6573.95	72.18%	27.45%
		90	3787	4226.4	8515.59	101.49%	11.60%
		110	3947.6	4564	10413.7	128.17%	15.61%

Table 5-4 Results of Case-1 in NSFNET network

Case 2-NSFNET (Demand: 4 ~ 8)

SP	Seed	Group	SA	LR	UB	Gap	Imp.ratio
8	10	50	2223.4	2662.8	4631.82	73.95%	19.76%
		70	2436	2844.6	6528.8	129.52%	16.77%
		90	2886.8	3215.8	8436.82	162.36%	11.40%
		110	2843.2	3326	10266	208.66%	16.98%
	70	50	2391	2406.2	4691.67	94.98%	0.64%
		70	2540	2949.8	6592.42	123.49%	16.13%
		90	2742.2	3131.8	8532	172.43%	14.21%
		110	2939.2	3333.4	10418.6	212.55%	13.41%
12	10	50	2300.6	2676.2	4625.77	72.85%	16.33%
		70	2680.2	2938.4	6521.1	121.93%	9.63%
		90	2697	3188.8	8414.22	163.87%	18.24%
		110	2859	3491.2	10254	193.71%	22.11%
	70	50	2552	2625	4692.58	78.77%	2.86%
		70	2625.2	3070.6	6578.11	114.23%	16.97%
		90	3012.6	3353.8	8520.84	154.07%	11.33%
		110	3288	3585.4	10412.4	190.41%	9.05%

Table 5-5 Results of Case-2 in NSFNET network

Case 3-NSFNET (Demand: 9 ~ 10)

SP	Seed	Group	SA	LR	UB	Gap	Imp.ratio
8	10	50	2023	2303.8	4641.45	101.47%	13.88%
		70	2309.8	2371.2	6524.35	175.15%	2.66%
		90	2365.2	2606.6	8437.46	223.70%	10.21%
		110	2553.2	2718.5	10266	277.63%	6.47%
	70	50	2147.6	2241.2	4691.95	109.35%	4.36%
		70	2287.6	2524.6	6599	161.39%	10.36%
		90	2266.2	2560	8532	233.28%	12.96%

		110	2726.6	2752.6	10421.9	278.62%	0.95%
12	10	50	2133.6	2425.6	4626.35	90.73%	13.69%
		70	2228.4	2559.8	6521.24	154.77%	14.87%
		90	2509.2	2723	8415.31	209.05%	8.52%
		110	2721.4	2837.6	10254.1	261.37%	4.27%
	70	50	2272.8	2301.2	4693.29	103.95%	1.25%
		70	2396.6	2587	6579.13	154.32%	7.94%
		90	2640	2782.6	8528.69	206.50%	5.40%
		110	2748.8	2915.4	10413.2	257.18%	6.06%

Table 5-6 Results of Case-3 in NSFNET network

In the following tables, the values are average from two results of rand number seed 10 and 70 to save space.

Case 1-GTE (Demand: 2 ~ 6)

SP	Group	SA	LR	UB	Gap	Imp.ratio
8	50	3696.2	4463.3	5071.49	13.63%	20.75%
	70	4288.8	5173.9	7122.5	37.66%	20.64%
	90	4884	5836.55	9148.725	56.75%	19.50%
	110	5278.5	6416.567	11235.52	75.10%	21.56%
12	50	3726.3	4360.3	5067.445	16.22%	17.01%
	70	4421	5307.3	7106.695	33.90%	20.05%
	90	4858.2	5891.1	9161.425	55.51%	21.26%
	110	5374.5	6236.8	11182.75	79.30%	16.04%

Table 5-7 Results of Case-1 in GTE network

Case 2-GTE (Demand: 4 ~ 8)

SP	Group	SA	LR	UB	Gap	Imp.ratio
8	50	3216.3	3540.3	5068.8	43.17%	10.07%
	70	3699.4	4177.6	7122.5	70.49%	12.93%
	90	3844.6	4556.7	9178.2	101.42%	18.52%
	110	4033.3	4689	11197.5	138.80%	16.26%
12	50	3179.5	3682.6	5067.34	37.60%	15.82%

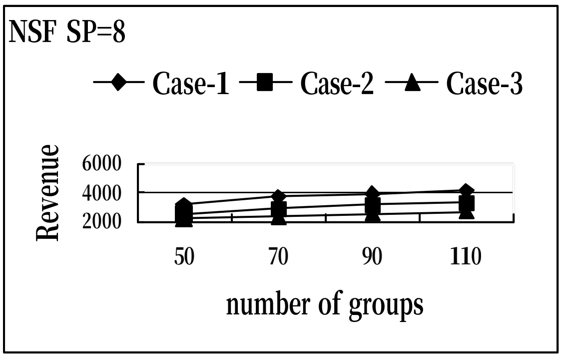
	70	3743.7	4279.9	7107.22	66.06%	14.32%
	90	3861.5	4590.8	9161.085	99.55%	18.89%
	110	4076.7	4791.2	11189.75	133.55%	17.53%

Table 5-8 Results of Case-2 in GTE network

Case 3-GTE (Demand: 9 ~ 10)

SP	Group	SA	LR	UB	Gap	Imp.ratio
8	50	3040.3	3212.1	5071.905	57.90%	5.65%
	70	3337.2	3461.8	7122.5	105.75%	3.73%
	90	3438.3	3519.2	9172.84	160.65%	2.35%
	110	3512.2	3610.3	11197.5	210.15%	2.79%
12	50	3001.3	3323.1	5067.9	52.51%	10.72%
	70	3395	3554.9	7107.71	99.94%	4.71%
	90	3457.6	3610.8	9170.065	153.96%	4.43%
	110	3556.1	3665.5	11197.5	205.48%	3.08%

Table 5-9 Results of Case-3 in GTE network



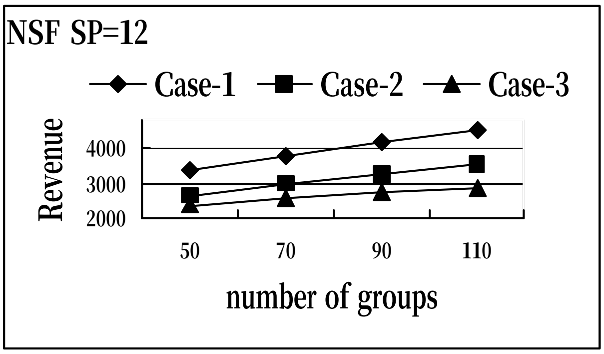


Figure 5-3 Comparisons of different cases in Experiment-I on NSFNET network

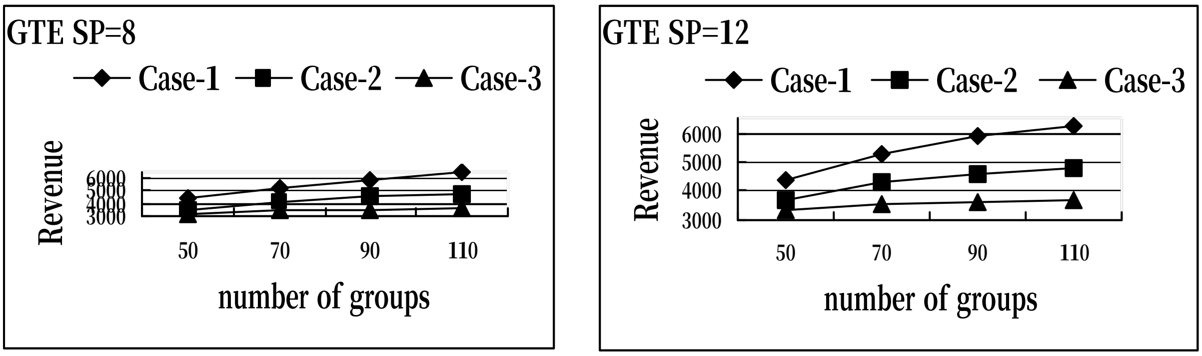


Figure 5-4 Comparisons of different cases in Experiment-I on GTE network

5.4.2 Results of Experiment-II

In the following result tables for Case-4, the values in SA Cost represent the cost calculated from simple algorithm while LR Cost means cost value calculated from Lagrangean relaxation based algorithm. The values in Ratio column represent cost down ratio which is calculated by (LR Cost – SA Cost) / SA Cost. The 'X' indicates not all groups are admitted for the given parameters, so the value is ignored.

Case 4 (G = 10)

Seed	SP	SA Cost	LR Cost	Cost Imp. Ratio
10	0	X	247.2	X
	4	243.6	208.8	-14.29%
	8	218	170.8	-21.65%
	12	214	165.2	-22.80%
	16	203.2	160	-21.26%
	20	197.2	150.4	-23.73%
	24	194	148.4	-23.51%
70	0	225.6	224.4	-0.53%
	4	196.8	186	-5.49%
	8	175.6	164	-6.61%
	12	170.8	152.4	-10.77%
	16	170.8	152.8	-10.54%

	20	168.8	150.4	-10.90%
	24	166.4	143.2	-13.94%

Table 5-10 Results of Case-4 with |G| = 10

Case 4 (G = 20)

Seed	SP	SA Cost	LR Cost	Cost Imp. Ratio
10	0	X	X	X
	4	431.6	404.4	-6.30%
	8	364.8	350.8	-3.84%
	12	365.6	334	-8.64%
	16	352.4	318.8	-9.53%
	20	344.8	304	-11.83%
	24	343.2	299.6	-12.70%
70	0	X	X	X
	4	X	377.2	X
	8	355.6	282.4	-20.58%
	12	342.4	282.4	-17.52%
	16	339.2	283.6	-16.39%
	20	329.2	278.8	-15.31%
	24	324.4	261.2	-19.48%

Table 5-11 Results of Case-4 with |G| = 20

5.5 Computation Time

Number of groups	Computation Time (s)		
	Case1	Case2	Case3
50	221.86	187.0225	152.355
70	265.56	220.6275	170.395
90	295.385	239.61	182.1075
110	316.625	259.9725	196.96

Table 5-12 Computation time of different cases running 10000 iterations

According to table 5-12, the computation time increase when the number of groups to be admitted grows. If the aggregation level is higher, the computation time grows as well. The running time per iteration of the Lagrangean relaxation based algorithm is slightly higher than simple algorithm.

5.5 Result Discussion

The results of LR are all better than SA. There are three main reasons that LR works better than SA. First, the SA makes groups aggregation decision only based on the destination nodes set and residual capacity in terms of time-slots, whereas LR makes use of the related Lagrangean multipliers. The Lagrangean multipliers include the potential cost for routing and wavelength assignment on each link in the formed tree topology.

Second, LR solves the constrained multicast routing and wavelength assignment problem based on the modified link cost, which takes the splitting capability of destination node of link into consideration. As a result, LR can find a feasible solution with higher possibility comparing to SA. The solution quality is better too.

Last, LR is iteration-based and is guaranteed to improve the solution quality iteration by iteration. Therefore, in a more complicated testing environment such as Case-1 which the extent of mergence between groups is higher, the improvement ratio is higher than in Case-3.

Chapter 6 Summary and Future Work

6.1 Summary

As WDM networks have emerged as a promising candidate for future networks with large bandwidth, efficient utilization of the limited and expensive components in networks becomes an import research issue. For better utilization, many algorithms and solutions had been proposed in the literature. However, the wavelength converters, and light splitters are assumed to be full range which may not be true. To the best of our knowledge, no algorithm or mathematical formulation has been proposed to address the problem of non-full range splitting capability in WDM networks.

To solve this problem, we present a mathematical formulation for the first time. It is assumed the OXCs are equipped with no wavelength converter but with splitters that have limited splitting capability. The problem of multicast routing and wavelength assignment in WDM networks is solved in such a constrained environment. Therefore, the limited resources are utilized in a more efficient way and higher revenues are earned by aggregating the multicast traffic demands based on OTDM technologies.

The achievement of this thesis can be expressed in terms of mathematical formulation and experiment performance. In terms of formulation, we propose a precise mathematical expression to well model the problems of multicast tree/group aggregation, constrained multicast routing and wavelength assignment on wavelength-routed WDM networks. The overall problem is modeled as an integer linear programming problem. In terms of performance, the proposed Lagrangean relaxation and subgradient based algorithms outperform the primal heuristics with acceptable computation time.

Different network topologies are tested in experiments, including NSFNET network and GTE network. And different parameters setting, including different number of wavelength available in fiber-optics, different size of multicast groups, and different demands in terms of time-slots have been tested to make this thesis more generic. As a result, we suggest that network operators apply the proposed Lagrangean relaxation based algorithms when dealing with network design problems related to supporting multicast communications in resource constrained WDM networks.

6.2 Future Work

In this paper, Quality of Service (QoS) measurements are not taken into consideration. In the future, the QoS requirements can be added to the proposed flexible formulation. For example, delay bound, jitter, and the hop count constraint can be easily added to the formulation. Due to the variety of services carried on the networks, different group aggregation admission policies can also be added to the mathematical model to fulfill different service requirements.

Besides, the feasibility of the lighttree approach depends obviously on the relative cost of optical OXCs, transmitters and receivers at different capability. Today, these costs are today rapidly changing due to the rapid evolution of electrical and optical technologies. The resources placement in WDM networks is an important issue as well. If the related technologies get maturer, different tree sharing schemes can be taken into consideration such as aggregating groups originating from different source nodes. Concatenation of two shared tree is also a possible way.

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