論文摘要

隨著網際網路不斷得加速成長，嶄新的應用服務對於頻寬的需求也隨之愈來愈大。波分多工網路技術能提供光纖網路頻更大的頻寬，已被視為下一代高速網路的技術主軸。在這樣高速高頻的環境下，一旦有任何網路斷線，都可能造成大量的資料流失以及商業上的損害。因此，提供可靠性網路的重要性不言而喻。

為因應可靠度的需求，本篇論文提出了三個問題的解決方案。首先，提出兩個方法來處理波分多工網路之初始規劃建置問題，分別採取快速保護及路徑恢復兩種機制，希望在滿足這兩種保護機制以及其他限制條件的前提下，能根據給定的頻寬需求，建立成本最低的網路。由於先前的網路考慮固定的頻寬需求，在網路建置完成之後，需考量未來資源需求擴充的彈性。因此，第三個方案試圖利用建立網路連結之極大值使用率極小化的網路，來達到負載平衡的網路，讓每一條網路連結，都能保有最多剩餘的空間，應付未來需求的變化。

我們分別將三個問題仔細地分析轉換成最佳化數學模型，在滿足資源需求、可靠度需求、頻寬限制、光波長通訊限制等條件下，求得每一個問題的最佳解。每個數學問題在本質上都是一個非線性混和整數規劃問題，問題的本身具有高度的複雜性和困難度。我們採用以拉格蘭日鬆弛法為基礎的方法來處理這三個複雜問題，分別將其分解成數個較容易解決的子問題，並根據所得到的結果各別發展了一組簡易的演算法。

關鍵詞：<u>波分多工網路、網路規劃、存活性、保護、恢復、最佳化、拉格蘭日鬆弛法、數學規劃</u>

THESIS ABSTRACT

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LIGHTPATH ROUTING AND WAVELENGTH ASSIGNMENT IN SURVIVABLE WDM NETWORKS

The Internet is growing faster than ever, and creating more and more demand for bandwidth. Wavelength division multiplexing (WDM) is widely considered as a promising technology for next-generation optical communication networks providing large transmission bandwidth. In such a high speed environment, a single link failure may cause the simultaneous failure of several fibers and the channels on them, and potentially bring enormous loss of business and critical data. Therefore, survivability of network service is extremely important.

To accommodate the demand of survivability, we propose three algorithms in this thesis. For topology design problems, we provide two algorithms to design survivable WDM network for fast protection and path restoration schemes respectively. The objectives of both problems are to minimize the total implementation cost of the network. And the third algorithm is to design and solve the load-balancing routing and wavelength assignment problem, which provides more flexibility for future usage.

All three problems are formulated as combinatorial optimization problem models, and the basic approach to the algorithm development for them is Lagrangian relaxation in conjunction with a number of optimization techniques.

Keywords: **Wavelength Division Multiplexing (WDM), Network Planning, Survivability, Protection, Restoration, Optimization, Lagrangian Relaxation, Mathematical Programming**

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Chapter 1 Introduction

1.1 Background

Wavelength division multiplexing (WDM), not surprisingly, is widely considered as a promising technology for next-generation optical communication networks providing large transmission bandwidth. WDM can be viewed as a parallel set of optical channels, divided from the tremendous bandwidth of a fiber. Each channel uses a non-overlapping light wavelength and can be operated asynchronously in parallel at any desirable speed [18], for example, 40 Gbps or more. Current development activities indicate that WDM technology will be deployed mainly in a backbone network for large regions [15]. WDM also can enhance an optical network's capacity without expensive re-cabling and then can tremendously reduce the cost of network upgrades [20].

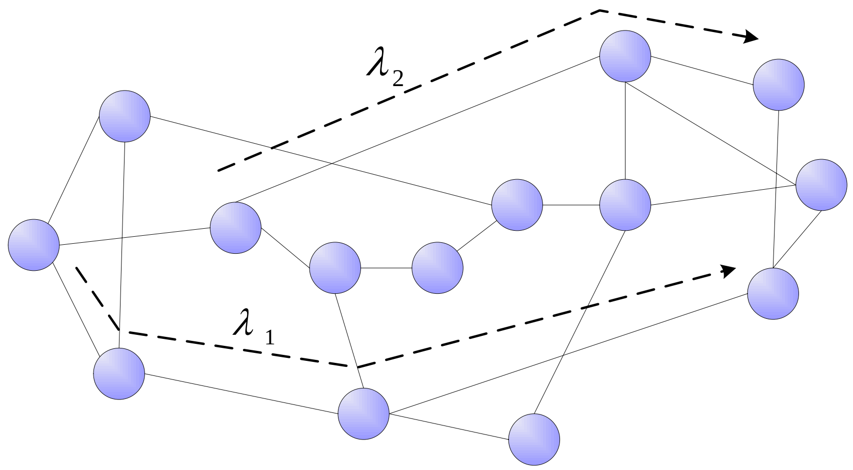

Figure 1-1 WDM Network Routing Architecture

The wavelength routing architecture of a WDM network is shown in figure 1-1, consists of OXCs (labeled from 1 to 14) interconnected by fiber links. A physical link can be further divided into three levels (illustrated in figure 1-2). A cable consists of up to *m* fibers (the value of *m* is a few of tens today), and number of wavelength supported in a fiber with current technology has exceeded one hundred and is growing up day by day. In this thesis, we concentrate on single trunk failure. The trunk component we discuss following is the cable in figure 1-2. Therefore, a single trunk failure may cause tremendous demand pair disconnected at the same time.

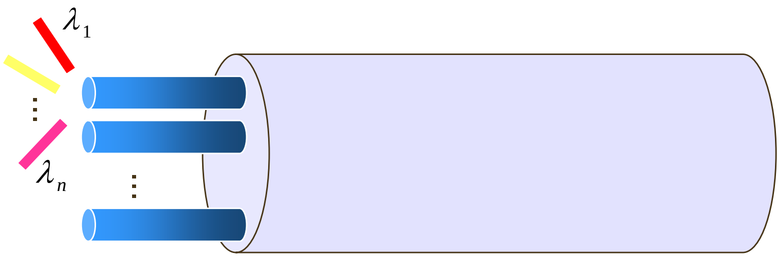

Figure 1-2 Level of a WDM Link

1.2 Motivation

The Internet is growing faster than ever, and creating more and more demand for bandwidth. WDM technology offers high speed optical networks and makes bandwidth-consumption applications such as graphics and visualization, medical image access and distribution, multimedia conferencing, broadband services to the home possible [4]. In such an environment, a single physical link failure (like cable in figure 1-2) may cause the simultaneous failure of several fibers and the channels on them, and potentially bring enormous loss of business and critical data. Therefore, survivability of network service is extremely important.

Several methods have been proposed for joint working and spare capacity planning in survivable WDM networks [3] [6] [18] [19] [22]. These methods are considered based on a static traffic demand and optimized the network cost assuming various cost models and survivability schemes. None of these methods consider the problem about demand variation and future accommodation. This is very important as it results in significant survivability and cost reductions for the network operator.

Once the network is provisioned, the critical issue is how to operate the network in such a way that the network resource is optimized used under dynamic traffic. As the traffic increases / decreases during the life time of the network, the resource allocation scheme needs to be re-arranged to provide flexibility.

In this thesis, we will discuss the design and operation of survivable WDM networks. In addition to providing algorithms for constructing minimum cost network with static traffic demand, we further provide an algorithm to perform load-balancing routing and wavelength assignment with given network topology and traffic requirement. In this way, each link in the network would have more spare capacity to accommodate dynamic future demand.

1.3 Literature Survey

In this section, we first survey two kinds of schemes for survivability – protection and restoration, and the related mathematical planning model and solution approaches for those schemes including drawbacks and improvements. Finally, the solution approach in this thesis, Lagrangian relaxation method, is also mentioned.

1.3.1 Classification of Schemes for Survivability

In optical communication network, several approaches have been proposed to achieve survivability for single link failure [24] [25]. In general, survivability schemes can be categorized into two main classes – dedicated resource reservation (protection) and dynamic restoration (illustrate in figure 1-3). Protection schemes react quickly when link failure but require pre-allocation of backup path and wavelength, which may need many spare resources. Whereas restoration approaches do not reserve any resources previously, but need to spend time for discovering and establishment of new routing path and wavelength when network failure [2] [18] [25].

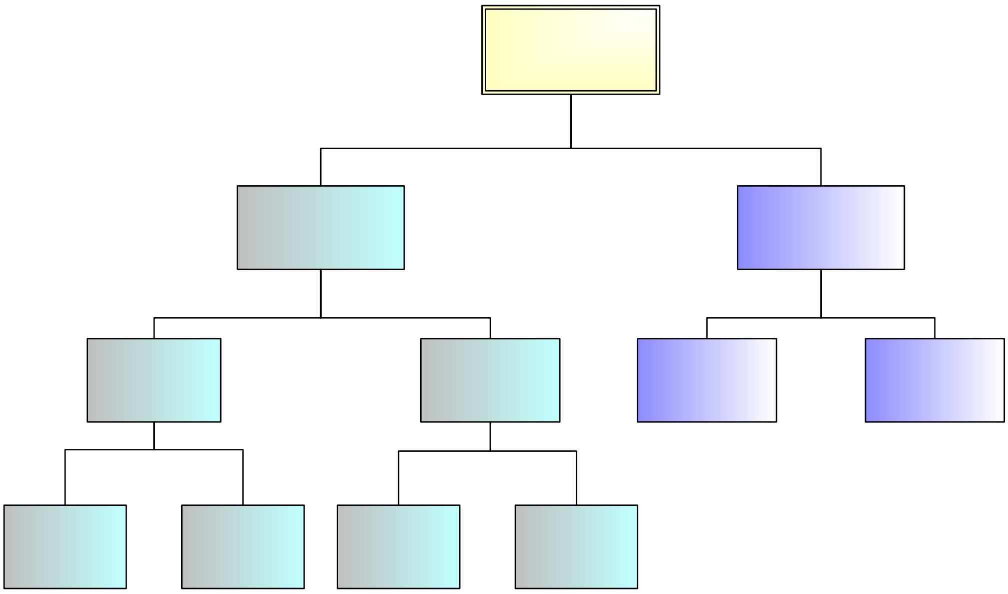

Figure 1-3 Survivability Schemes Classification

As figure 1-3 shows, survivability schemes are based on two basic paradigms: (1) path protection/restoration and (2) link protection/restoration [18]. The phases taken for network failure are discussion in [20].

(1) Path protection / restoration:

When a link fails in path protection/restoration paradigm, the backup paths for those origin-destination (O-D) pairs on that link are reserved on an end-to-end basis (illustrated in figure 1-4).

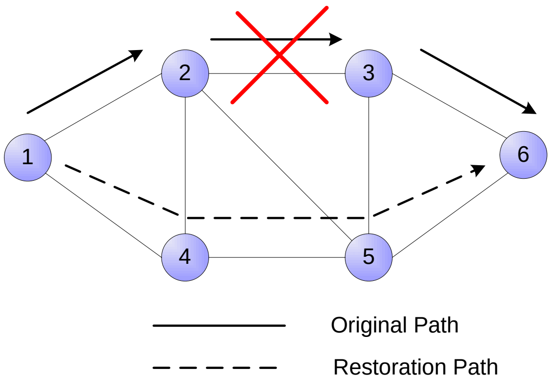

Figure 1-4 Path Protection/Restoration

- **Dedicated-path protection:** At the time of call setup, for each working path, a link-disjoint backup path and wavelength is reserved dedicated to the O-D pair, and are not shared with other O-D pair.
- **Shared-path protection:** The path and wavelength are reserved as dedicated path protection, but the wavelength reserved on a backup path can be shared with other backup paths in different failure scenarios.
- **Path restoration:** When a link fails, the O-D pairs pass through the link need to discovery a new path and wavelength on the end-to-end basis. Therefore, if there is no new path and wavelength for an O-D pair on that link, the connection is blocked.

(2) Link protection / restoration:

In link protection/restoration paradigm, backup paths reservation is based on the end nodes of the failure link (illustrated in figure 1-5 (a)). In some situation, protection / restoration based on end node of failure link will take a long route as illustrated in figure 1-5 (b), for O-D pair (4, 6); path 4-5-6 would be a better choice than the path 4-2-5-6

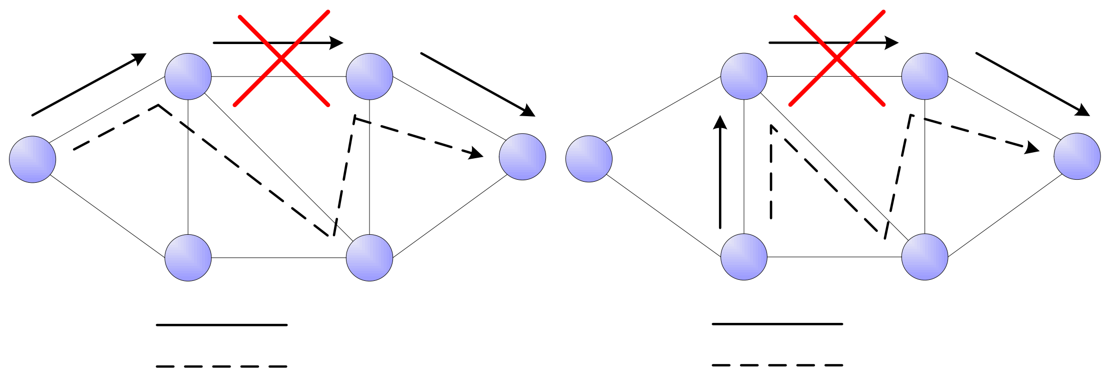

Figure 1-5 Link Protection/Restoration

- **Dedicated-link protection:** At the time of call setup, for each link on the working path for an O-D pair, a backup path and wavelength are reserved around that link, and are dedicated to the O-D pair.
- **Shared-link protection:** The path and wavelength are reserved as dedicated-link protection, but the backup wavelengths may be shared with other O-D pairs in different failure scenarios.
- **Link restoration:** In link restoration, the end of the failure link discovery a new path for each wavelength traverse on the link. If no new path found for a broken connection, the connection is blocked.

In [19] and [25], the influence of cross-connect configuration time for protection-switching time is examined for link and path protection. And the restoration time and efficiency for link and path restoration are also studied. Restoration efficiency defined here is the ratio of the number of connection restored to the total number of connections that traverse the failed link.

In protection schemes, when the cross-connect configuration time is high, dedicated-path protection scheme has a better protection-switching time than other schemes. On the other hand, when the cross-connect configuration time is low, shared-link protection benefits from less hops.

In restoration schemes, path restoration has better efficiency, higher probability in finding wavelength-continuous backup path, and fewer hops than link restoration. Whereas link restoration has a better restoration time and simpler routing complexity compared to path restoration [19] [25].

As described above, dedicated-path protection has better protection switching time when the cross-connect configuration time is high, and path restoration has better efficiency than other restoration schemes. In this thesis, we will implement these two schemes and examine them by mathematical model for static traffic demand.

1.3.2 Dimensioning of Network Models

There are several dimensions for designing a survivable WDM network For the cost model, the link cost for a WDM network includes (a) implementation cost: digging, wiring, and maintenance cost, (b) line cost: fibers, optical amplifiers, multiplexers, demultiplexers, etc. (c) channel cost [3]. In some studies, the purpose is to minimize the number of wavelength used (channel cost) in a given network topology [17] [18]. In [3], fiber topology layer and optical path layer are considered together, and the objective is to minimize the total network design cost. In really world, because implementation cost is much larger than the other two costs, we assume that the fiber and wavelength cost is too inexpensive to be neglected. Only the physical implementation cost and line cost are considered in this thesis.

For failure scenario, most of studies focus on single link / fiber failure assumption [3] [18] [19] [22]. That is, there is only one link that would fail at any one instance. Our model is also based on this assumption. In [16], the author considers the case that the primary and backup paths traversing the same physical link would fail simultaneously. In our thesis, primary and backup paths are reserved with link-disjoint constraint, so this situation could not happen. Surviving from disaster is discussed in [9]. Survivable routing on node failure scenarios are studied in [14]. A node component can be treated as a link with nodes on the both ends. Therefore, a node failure can be regarded as a link failure with some appropriate transformation, and then can be solved by our algorithm.

1.3.3 Protection Models

The design of protection model has been studied in [6] [18] [22]. In [6], the authors provide an algorithm called disjoint alternate path (DAP) algorithm to design a set of routing path between O-D pair with maximum protection from given topology. Instead of minimizing the design cost or capacity used, the goal of the algorithm is to minimize the number of disconnected pairs when a single link fails. More clearly, the algorithm desires to minimize the number of lightpath which may be broken (no alternate route) when some critical link fails. A link is critical when the one or more lightpath across it and has no disjoint alternate path in the case of link failure. The DAP algorithm starts from an arbitrary solution of routing decision, and randomly modifies the route of each O-D pair in order to avoid critical links. At the end of every iteration, the optimal value so far is kept and new list of critical links base on the new routing decision is computed. After all possible combinations ($O(N^4)$), the optimal value is found. For a given network topology with limited resource, the routing decision can be determined by this algorithm to find an optimal solution set with minimal critical path after all possible choices is tried. In our thesis, topology design and routing decision are considered together to get the optimal balance.

In [18], link and path protection are formulated as integer linear programming problems. With network topology given, the objectives of the two models are to minimize total number of wavelength used, including working and backup path. These two models can be view as traditional routing and wavelength assignment problems with survivability demand additionally. Both problems utilize the optimization tool CPLEX to solve. Link protection is not considered in our model since it is more complex and less practical. To get the optimal topology and best routing decision, topology design and survivable routing are considered together in our thesis.

Shared path protection problem is studied in [22]. The authors solve the problem by splitting an integer linear programming formulation into two parts. The first part sets up primary lightpaths, while the second part sets up the backup paths. Shared path protection, as discussed in 1.3.1, benefits from resource sharing and certainly has less cost than fast protection scheme. But the protection switching time is more than fast protection for call set up at the time when failure occurs. Besides, splitting problem into two parts and solving them successively may have the probability of escaping from optimal solution.

1.3.4 Restoration Models

There are several methods proposed for restoration schemes [3] [10] [19] [22]. In [3], the authors proposed two design stages for survivable networks. The first model is about topology design for routing and capacity planning with given traffic. In addition to capacity and demand constraints, the degree of each node must be minimal two for survivability reasons. The second model considers about link failure and rerouting. If spare capacity is not enough in some link failure situation, spare fibers may be required and incur additional cost. Both models provide generic notation for the network with or without wavelength converter, and node degree and type constraints is also considered. The solution approaches used in this paper is simulated annealing (SA) and optimization tool CPLEX, which solves integer linear programming problem by branch & bound method. SA is a nondeterministic search technique, which has the probability of escaping from optimal value [12]. In this thesis, we formulate the path restoration design problem in a single mathematical model with wavelength continuity constant. Besides, Lagrangian relaxation method is used to provide the upper and lower bound of optimal value.

In [10] [19], several comparisons between restoration schemes are studied, including protecting switching time, efficiency, wavelength continuity, and hop counts. As the results shown in Table 1-1, path restoration is superior to link restoration except switching time. Switching time depends on the distance between failure-response pairs; so that link restoration can be anticipated to react more quickly.

IP restoration algorithm is studied in [22]. By the assumption of using interior gateway protocol, each autonomous system can perform load sharing before any link failure and combat a link failure. For example, when the traffic between O-D pair is spitted into two lightpaths and each transmits half demand of traffic. When one lightpath fails, the traffic on that path can automatically switch to another path.

Failure recovery time is also discussed in this paper. As expected, reaction time for WDM protection is much shorter than IP restoration, which requires addition time for link-state message exchange and routing table recomputing. In this thesis, only WDM restoration is considered.

	Path Restoration	Link Restoration
Switching Time		∨
Wavelength-continuous Backup Path	∨	
Efficiency	∨	
Fewer Hops Path	∨	

Table 1-1 Comparison Between Path and Link Restoration

1.3.5 Lagrangian Relaxation Method

In the 1970s, Lagrangian relaxation methods were used in scheduling and the general integer programming problem [7]. Now it is a general solution approach for solving mathematical programs. Lagrangian method permits us to decompose problems and to exploit their special structure and provides us the proper solutions for those problems. In fact, the Lagrangian method has become one of the best tools for optimization problems such as integer programming, linear programming combinatorial optimization, and non-linear programming. Lagrangian method has several advantages: it is a very flexible approach that could decompose mathematical models in many different ways; it decomposes sub problems as stand-alone problems, which can be solved by any proper and known algorithm; it permits us to develop bounds on the value of the optimal objective function, which can be used to implement heuristic solution for solving complex problems and getting feasible solutions. [1] [7] [8].

Lagrangian relaxation permits modelers to exploit the underlying structure in any optimization problem by relaxing complicating constraints. This method permits us to “pull apart” models by removing constraints and instead place them in the objective function with associated Lagrangian multipliers. With relaxation of some constraints, the optimal value of the relaxed problem is always a lower bound (for minimization problems) on the objective function value of the problem. To obtain the best lower bound, that is to make the lower bound as close to the optimal value as possible, we need to choose the best multiplier so that the optimal value of the Lagrangian sub-problem is as large as possible. We can solve the Lagrangian multiplier problem in a variety of ways. The subgradient optimization technique is possibly the most popular technique for solving the Lagrangian multipliers problem [1] [7] [8].

1.4 Proposed Approach

To achieve network survivability, we try to design networks for two different schemes – dedicated-path protection (fast protection) and path restoration. With single link failure assumption, we model both schemes as nonlinear integer mathematical programming problems. The goal of both models is to minimize the maximum flow link of the network.

For routing and wavelength assignment with load-balancing in third model, our performance objective is to minimize the maximum link utilization, the algorithm based on this approach also call minimax utilization routing algorithm. The major advantages of using the minimum of the maximum link utilization as the performance objective include [13].

The routing and wavelength assignment (RWA) problem without survivability constraints has been shown to be an NP-complete problem in [21] and the survivable routing problem is also shown as an NP-complete problem in [17]. All three models in this thesis contain RWA or survivable RWA as part of problem. We can expect all of them are difficult problems. We will apply the Lagrangian relaxation method and the subgradient method to decompose and solve these three problems.

1.5 Thesis Organization

In this paper we propose three mathematical models for achieving the goal of survivability in the WDM network. The first one is a network topology design problem for fast protection scheme, and the second intends to solve the same problem but with path restoration scheme. The third model is based on given topology and survivability demand, and provides and load-balancing algorithm for routing and wavelength assignment. These three mathematical formulations are elaborated from chapter 2 to 4.

After decomposing each model into several smaller sub-problems, we then propose heuristics methods and algorithms for them. Chapter 5 presents some computational experiment results and comparisons. Chapter 6 summarizes the thesis and some future research extensions are suggested and discussed.

Chapter 2 Problem Formulation for Fast Protection

2.1 Problem Description

In this chapter, we will provide a network design algorithm for survivable optical networks using the survivability scheme – fast protection. This algorithm is used to help the decision making for constructing a WDM network with sufficient backup path for each O-D pair to survive any single link failure. The objective of this algorithm is to minimize total physical link cost spent. The cost we consider here is only the implementation cost discussed in chapter 1.

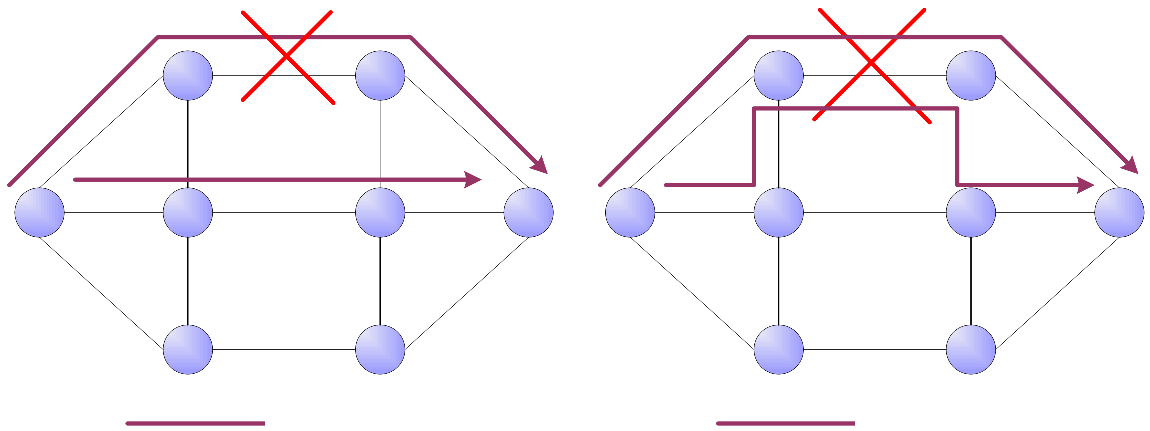

Figure 2-1 Number of Working Path Required for O-D pair

For each O-D pair with multiple lightpaths demand, we also consider the situation that more than one working paths traverse through the same physical link. Because of single link failure assumption and survivability requirement, the number of backup path needed will depend on the maximum number of lightpath among the links used by the O-D pair (illustrated in figure 2-1). In other words, the number of backup path required for each O-D pair is unknown before network topology and routing decision is made. This would increase the difficulty of our problem. And routing and wavelength assignment with tentative topology also raise the complexity. To solve these uncertainties, we introduce some extra variables to make them clearly.

Problem assumptions：
● The basic architecture used is a WDM network.
● The OXCs used in the optical network lack the capability of wavelength conversion1, which incurs the extra delay for O-E-O conversion.
● Link cost includes (a) implementation cost: digging, wiring, and maintenance cost, (b) line cost: cost of fibers, which is proportioned to the distance between two-end nodes, (c) channel cost. We assume implementation cost is much higher than other two costs, and only consider implementation and line cost in our model.
● A WDM lightpath can serve only a single traffic demand; there is no traffic grooming within wavelength.
● Single trunk failure assumption.

Table 2-1 Problem Assumptions for Fast Protection Scheme

Given：
● A set of candidate trunks and links.
● The limit number of fibers that can be placed in a single cable.
● Candidate wavelength set and the maximum number of wavelengths per fiber.
● Implementation cost and line cost per for each link.
● A set of origin-destination (O-D) pairs with traffic demand.

- Candidate paths with their routes and wavelengths used for each O-D pair.

Objective：

To minimize the total network cost.

Subject to：

- Capacity limit constraint – the number of lightpath used in a fiber should not exceed its wavelength count limit.
- Wavelength limit constraint – a wavelength can only be used once in a single fiber. If a cable are placed with *n* fibers, a wavelength can then be used *n* times.
- Wavelength continuity constraint – there is no wavelength converter in OXCs. That is, for each lightpath can only use one wavelength.
- Traffic demand constraint – the total number of lightpath assigned to an O-D pair should meet its requirement.
- Implementation limit constraint – the number of fibers put in a cable should not exceed its limit.
- Survivability constraint – for each O-D pair satisfies (a) the number of backup path reserved should be sufficient for any single link failure situation; (b) backup paths should be link-disjoint with working path.

To determine：

1. The topologies of the optical network.
2. Number of fibers to be placed in each link.
3. Working and backup path and wavelength used for each O-D pair.
4. Number of backup path required for each O-D pair.

Table 2-2 Problem Description for Fast Protection Scheme

2.2 Notation

Given Parameters	
Notation	**Descriptions**
T	The set of candidate WDM trunks.
L	The set of candidate WDM links.
W	The set of original origin-destination O-D pairs.
P_w	Candidate path set for O-D pair *w*, where the distance of each path meets the hop count requirement.
n	Limit number of fibers on a link.
r_w	The number of lightpath required for O-D pair *w*.
J	The set of candidate wavelengths in the WDM network.
a_t	The implementation cost of trunk *t*.
b_l	The fiber cost of each link *l*.
δ_{pl}	1 if lightpath *p* is on WDM link *l*; otherwise 0.
σ_{lt}	1 if directed link *l* is on WDM trunk *t*; otherwise 0.

Table 2-3 Notation of Given Parameters for Fast Protection Model

Decision Variables	
Notation	**Descriptions**
x_{pj}	Number of lightpath used for working path *p* using wavelength *j*.
y_t	1 if trunk *t* is setup; otherwise 0.
z_l	Number of fibers to be put on directed link *l*.

v_{pj}	Number of lightpath used for backup path *p* using wavelength *j*.
B_w	The number of backup lightpath required for O-D pair *w*.
u_{wt}^{o}	1 if trunk *t* is used by a working path of OD pair *w*; otherwise 0.
u_{wt}^{b}	1 if trunk *t* is used by a backup path of O-D pair *w* ; otherwise 0.

Table 2-4 Notation of Decision Variables for Fast Protection Model

2.3 Problem Formulation

Optimization Problem:

Objective function:

$$Z_{IP1} = \min \sum_{t\in T} a_t y_t + \sum_{l\in L} b_l z_l \qquad \text{(IP1)}$$

subject to:

$$\sum_{w\in W}\sum_{p\in P_w}\left(x_{pj}+v_{pj}\right)\delta_{pl} \le z_l \qquad \forall j\in J, l\in L \qquad (1)$$

$$x_{pj} = \{0, 1, 2, \dots, n\} \qquad \forall j\in J, p\in P_w, w\in W \qquad (2)$$

$$z_l \in \{0, 1, 2, \dots, n\} \qquad \forall l\in L \qquad (3)$$

$$v_{pj} = \{0, 1, 2, \dots, n\} \qquad \forall j\in J, p\in P_w, w\in W \qquad (4)$$

$$\sum_{j\in J}\sum_{p\in P_w} x_{pj} = r_w \qquad \forall w\in W \qquad (5)$$

$$z_l\sigma_{lt} \le ny_t \qquad \forall l\in L, t\in T \qquad (6)$$

$$y_t = 1 \text{ } or \text{ } 0 \qquad \forall t\in T \qquad (7)$$

$$\sum_{j\in J}\sum_{p\in P_w} x_{pj}\delta_{pl}\sigma_{lt} \le B_w \qquad \forall w\in W, t\in T \qquad (8)$$

$$\sum_{j\in J}\sum_{p\in P_w} v_{pj} = B_w \qquad \forall w\in W \qquad (9)$$

$$B_w \leq r_w \qquad \forall w \in W \qquad (10)$$

$$\sum_{j \in J} \sum_{p \in P_w} x_{pj} \delta_{pl} \sigma_{lt} \leq r_w u_{wt}^o \qquad \forall w \in W, t \in T \qquad (11)$$

$$\sum_{j \in J} \sum_{p \in P_w} v_{pj} \delta_{pl} \sigma_{lt} \leq r_w u_{wt}^b \qquad \forall w \in W, t \in T \qquad (12)$$

$$u_{wt}^o + u_{wt}^b \leq 1 \qquad \forall t \in T, w \in W \qquad (13)$$

$$u_{wt}^o = 1 \textit{ or } 0 \qquad \forall t \in T, w \in W \qquad (14)$$

$$u_{wt}^b = 1 \textit{ or } 0 \qquad \forall t \in T, w \in W \qquad (15)$$

The objective function represents to minimize the network design cost in the network, where $\sum_{t \in T} a_t y_t$ represents network implementation cost and $\sum_{l \in L} b_l z_l$ stands for total line cost. Constraint (1), (2), (3) and (4) require that the frequency of a single wavelength used on a link does not exceed the number of fibers on that link. Capacity constraint is also implied on constraint (1). Constraint (5) requires the number of path selected for each O-D must meets its requirement. Constraints (6) and (7) state the number of fibers to be put on a link does not exceed the limit. Constraint (7) enforces that WDM link can be setup only if corresponding trunk is implemented. To achieve survivability, constraint (8), (9), and (10) state the sufficient of backup path for each O-D pair. Constraint (8) indicates that the number of backup path is enough to make up the failure of any link, and constraint (9) requires that the number of backup path to be selected meets the requirement. Constraints (11) to (15) desire to satisfy the link-disjoint requirement between original and backup paths which is implicated imposed on Constraint (13).

2.4 Solution Approach

By using the Lagrangian relaxation method, we can transform the primal problem (IP1) into the following Lagrangian relaxation problem (LR) where constraints (1), (8), (11) and (12) are relaxed.

For a vector of non-negative Lagrangian multipliers, a Lagrangian relaxation problem of IP1 is given by optimization problem (LR1):

$$Z_{d1}(\alpha,\beta,\theta,\varepsilon)=$$

$$\min \sum_{t\in T} a_t y_t + \sum_{l\in L} b_l z_l + \sum_{l\in L}\sum_{j\in J}\alpha_{lj}\left[\sum_{w\in W}\sum_{p\in P_w}\left(x_{pj}+v_{pj}\right)\delta_{pl} - z_l\right] + \sum_{w\in W}\sum_{t\in T}\beta_{wt}\left[\sum_{j\in J}\sum_{p\in P_w} x_{pj}\delta_{pl}\sigma_{lt} - B_w\right] +$$

$$\sum_{w\in W}\sum_{t\in T}\theta_{wt}\left[\sum_{j\in J}\sum_{p\in P_w} x_{pj}\delta_{pl}\sigma_{lt} - r_w u^o_{wt}\right] + \sum_{w\in W}\sum_{t\in T}\varepsilon_{wt}\left[\sum_{j\in J}\sum_{p\in P_w} v_{pj}\delta_{pl}\sigma_{lt} - r_w u^b_{wt}\right]$$

subject to:

x_{pj}	=	$\{0, 1, 2, \ldots, n\}$	$\forall j \in J, p \in P_w, w \in W$	(2)
z_l	$\in$	$\{0, 1, 2, \ldots, n\}$	$\forall l \in L$	(3)
v_{pj}	=	$\{0, 1, 2, \ldots, n\}$	$\forall j \in J, p \in P_w, w \in W$	(4)
$\sum_{j\in J}\sum_{p\in P_w} x_{pj}$	=	r_w	$\forall w \in W$	(5)
$z_l\sigma_{lt}$	$\leq$	ny_t	$\forall l \in L, t \in T$	(6)
y_t	=	1 *or* 0	$\forall t \in T$	(7)
$\sum_{j\in J}\sum_{p\in P_w} v_{pj}$	=	B_w	$\forall w \in W$	(9)
B_w	$\leq$	r_w	$\forall w \in W$	(10)
$u^o_{wt} + u^b_{wt}$	$\leq$	1	$\forall t \in T, w \in W$	(13)
u^o_{wt}	=	1 *or* 0	$\forall t \in T, w \in W$	(14)

$$u_{wt}^{b} \quad = \quad 1 \text{ or } 0 \qquad \forall t \in T, w \in W \qquad (15)$$

where α, β, θ, ε are the vectors of $\{\alpha_{lj}\}$, $\{\beta_{wt}\}$, $\{\theta_{wt}\}$, $\{\varepsilon_{wt}\}$, and α, β, θ, ε are the Lagrangian multipliers and α, β, θ, $\varepsilon \geq 0$. To solve (LR1), we can decompose (LR1) into the following four independent and easily solvable optimization sub-problems.

Subproblem 2-1 (related to decision variable x_{pj})

$$Z_{sub2-1}(\alpha,\beta,\varepsilon) = \min \sum_{w\in W}\sum_{p\in P_w}\left[\sum_{l\in L}\sum_{j\in J}\alpha_{lj}x_{pj}\delta_{pl} + \sum_{t\in T}\sum_{j\in J}\left(\beta_{wt}+\theta_{wt}\right)x_{pj}\delta_{pl}\sigma_{lt}\right]$$

subject to:

$$x_{pj} \quad = \quad \{0, 1, 2, \ldots, n\} \qquad \forall j \in J, p \in P_w, w \in W \qquad (2)$$

$$\sum_{j\in J}\sum_{p\in P_w} x_{pj} \quad = \quad r_w \qquad \forall w \in W \qquad (5)$$

Subproblem 2-2 is composed of $|W|$ problems for each O-D pair w. For each problem, we want to find out r_w shortest paths from source to destination, where α_{lj} is the cost for using wavelength j on link l, and the arc cost is for O-D pair w of trunk t is $\beta_{wt}+\theta_{wt}$ (illustrated in figure 2-2).

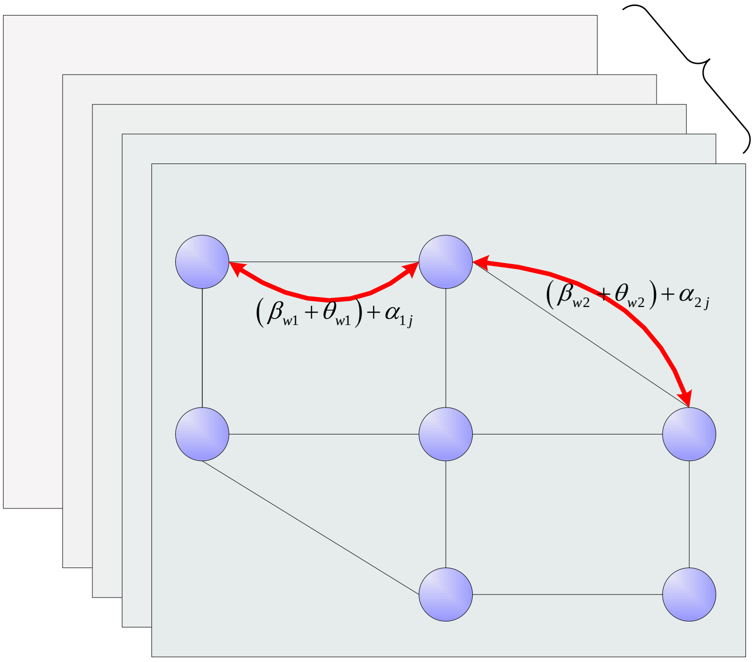

Figure 2-2 Cost for a WDM Link

We construct number of $|J|$ WDM wavelength sub-networks, each corresponding to a different wavelength j and applying WDM network topology. As illustrated in figure 2-3, we split origin and destination node into $|J|$ nodes in each sub-networks respectively.

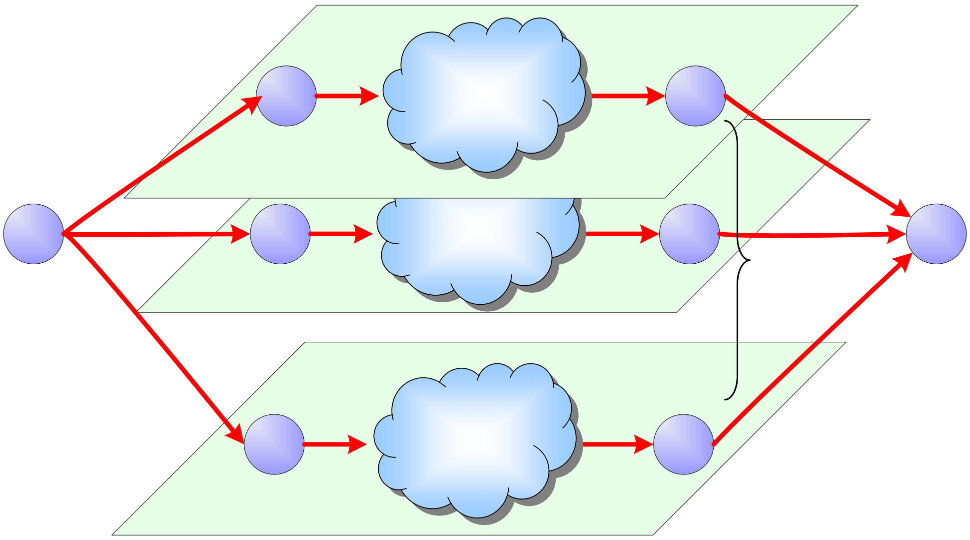

Figure 2-3 Decompose Network into Single Wavelength Sub-network

For each O-D pair w with $|J|$ sub-networks, we can regard it as a minimum cost flow problem [1] with traffic flow r_w from origin node to destination node, and each link can carry up to n traffic. By the property of minimum cost flow with capacity larger than 1, we can expect that the traffic flow would aggregate to one or some paths with minimum link cost. To avoid such a situation as possible, we can modify the minimum cost flow algorithm to spread out traffic flow into path with the same cost as possible.

Subproblem 2-2 (related to decision variables v_{pj} and B_w)

$$Z_{sub2-1}(\alpha,\beta,\varepsilon)=\min\sum_{w\in W}\left[\sum_{p\in P_w}\sum_{l\in L}\sum_{j\in J}\alpha_{lj}v_{pj}\delta_{pl}+\sum_{p\in P_w}\sum_{t\in T}\sum_{j\in J}\varepsilon_{wl}v_{pj}\delta_{pl}\sigma_{lt}-\sum_{t\in T}\beta_{wt}B_w\right]$$

subject to:

$$v_{pj} = \{0, 1, 2, \ldots, n\} \qquad \forall j\in J, p\in P_w, w\in W \qquad (4)$$

$$\sum_{j\in J}\sum_{p\in P_w} v_{pj} = B_w \qquad \forall w\in W \qquad (9)$$

$$B_w \le r_w \qquad \forall w\in W \qquad (10)$$

Subproblem 2-2 is similar to subproblem 2-1, expect the traffic demand B_w is not known in advance. We can decompose this problem into $|W|$ subproblem. For each $w\in W$, the objective of the problem is

$$\min\sum_{w\in W}\sum_{p\in P_w}\left[\sum_{l\in L}\sum_{j\in J}\alpha_{lj}v_{pj}\delta_{pl}+\sum_{t\in T}\sum_{j\in J}\varepsilon_{wl}v_{pj}\delta_{pl}\sigma_{lt}\right]-\sum_{t\in T}B_w\beta_{wt}$$

The objective function above can be decomposed into two parts, first part is $\left[-\sum_{t\in T}B_w\beta_{wt}\right]$, which is a negative constraint value for each w when B_w is given, and the second part is $\sum_{w\in W}\sum_{p\in P_w}\left[\sum_{l\in L}\sum_{j\in J}\alpha_{lj}v_{pj}\delta_{pl}+\sum_{t\in T}\sum_{j\in J}\varepsilon_{wl}v_{pj}\delta_{pl}\sigma_{lt}\right]$, which is a minimum cost flow problem. According to the restriction of constraint (11), though the increasing of B_w will decreasing the value of the first part, but also increase the value of the second part. Therefore, the combination of these two parts is a convex function. For each O-D pair $w \in W$, we solve each problem by the following algorithm:

Initially $B_w = 1$.

Current_Value = INFINITY; //Current_Value = current optimal value

Step 1: Solve the minimum cost flow problem for the second part of the objective function with traffic demand B_w, and $\left[-\sum_{t\in T}B_w\beta_{wt}\right]$.

Step 2: If the objective value compute from step 1 is smaller than *Current_Value,* set this smaller value as *Current_Value*, and B_w,= B_w, +1, then go to step 1.

Else Output *Current_Value* as the optimal value of this objective function and stop algorithm.

Subproblem 2-3 (related to decision variables y_t, and z_l)

$$Z_{sub2-3}(\alpha)=\min\sum_{t\in T}a_t y_t-\sum_{l\in L}\left[(\sum_{j\in J}\alpha_{lj}-b_l)z_l\right]$$

subject to:

z_l	$\in$	$\{0, 1, 2, \ldots, n\}$	$\forall l \in L$	(3)
$z_l\sigma_{lt}$	$\le$	ny_t	$\forall l \in L, t \in T$	(6)
y_t	$=$	1 *or* 0	$\forall t \in T$	(7)

We can solve Subproblem 2-3 for pair of directed links with the same two-end nodes, and Subproblem 2-3 can be decomposed into $|T|$ subproblems:

$$Z_{sub2-3i}(\alpha) = \min\left[a_i y_i + (\sum_{j\in J}\alpha_{lj} - b_l) z_l + (\sum_{j\in J}\alpha_{l'j} - b_{l'}) z_{l'} \right] \quad i = 1,2,\ldots|T|$$

l, l' *is on i.*

subject to:

z_l	$\in$	$\{0, 1, 2, \ldots, n\}$	$\forall l \in L$	(3)
$z_l\sigma_{lt}$	$\le$	ny_t	$\forall l \in L, t \in T$	(6)
y_t	$=$	1 *or* 0	$\forall t \in T$	(7)

For each subproblem, there are only four possibilities for decision variables ($y_i, z_i, z_{i'}$):

1. (0, 0, 0), $Z_{sub2-3i}(\alpha) = 0$.
2. (1, *n*, 0), $Z_{sub2-3i}(\alpha) = a_i - (\sum_{j\in J}\alpha_{ij} - b_l)n$.
3. (1, 0, *n*), $Z_{sub2-3i}(\alpha) = a_i - (\sum_{j\in J}\alpha_{i'j} - b_{l'})n$.
4. (1, *n*, *n*), $Z_{sub2-3i}(\alpha) = a_i - (\sum_{j\in J}\alpha_{ij} - b_l)n - (\sum_{j\in J}\alpha_{i'j} - b_{l'})n$.

And then we can try each possibility to determine the minimal value of $Z_{sub2-3i}(\alpha)$ and each decision variable. The objective function $Z_{sub2-3}(\alpha)$ can be compute from solving each $Z_{sub2-3i}(\alpha)$ optimally from the lemma described above.

Subproblem 2-4 (related to decision variables u^o_{wt} and u^b_{wt})

$$Z_{sub2-4}(\theta,\varepsilon) = \min \sum_{t\in T}\sum_{w\in W}\left[-\left(\theta_{wt}u^o_{wt} + \varepsilon_{wt}u^b_{wt}\right) r_w \right]$$

subject to:

$u^o_{wt} + u^b_{wt}$	$\le$	1	$\forall t \in T, w \in W$	(13)

$$u_{wt}^{o} \quad = \quad 1 \; or \; 0 \qquad \forall t \in T, w \in W \qquad (14)$$

$$u_{wt}^{b} \quad = \quad 1 \; or \; 0 \qquad \forall t \in T, w \in W \qquad (15)$$

We can decompose this problem into |*T*|*|*W*| subproblems,

$$Z_{sub2-4}(\theta,\varepsilon) = \min\left[-\left(\theta_{wt}u_{wt}^{o} + \varepsilon_{wt}u_{wt}^{b}\right)r_{w}\right]$$

subject to:

$$u_{wt}^{o} + u_{wt}^{b} \quad \le \quad 1 \qquad \forall t \in T, w \in W \qquad (16)$$

$$u_{wt}^{o} \quad = \quad 1 \; or \; 0 \qquad \forall t \in T, w \in W \qquad (17)$$

$$u_{wt}^{b} \quad = \quad 1 \; or \; 0 \qquad \forall t \in T, w \in W \qquad (18)$$

For each subproblems, $Z_{sub2-4wt}(\theta,\varepsilon)$ depends on the value of θ_{wt} and ε_{wt}.

If $\theta_{wt} \ge \varepsilon_{wt}$, $Z_{sub2-3wt}(\theta,\varepsilon) = \theta_{wt}r_{w}u_{wt}^{o}$, and $u_{wt}^{o} = 1$, $u_{wt}^{b} = 0$.

Else $Z_{sub2-4wt}(\theta,\varepsilon) = \varepsilon_{wt}r_{w}u_{wt}^{b}$, and $u_{wt}^{o} = 0$, $u_{wt}^{b} = 1$.

From above-mentioned method, each u_{wt}^{o} and u_{wt}^{b} pair can be easily solved, and $Z_{sub2-4}(\vartheta,\varepsilon)$ can be then computed by optimally value of each $Z_{sub2-4wt}(\theta,\varepsilon)$.

2.5 The Dual Problem and the Subgradient Method

According to the weak Lagrangian duality theorem [8], for any α, β, θ, ε ≥ 0, $Z_{d1}(\alpha,\beta,\theta,\varepsilon)$ is a lower bound on Z_{IP1}. The following dual problem (D1) is then constructed to calculate the tightest lower bound.

Dual Problem (D1):

$$Z_{d1} = \max Z_{d1}(\alpha, \beta, \theta, \varepsilon) \qquad \text{(D1)}$$

subject to:

$$\alpha,\ \beta,\ \theta,\ \varepsilon \geq 0 \qquad (26)$$

The most popular method to solve the dual problem is the subgradient method [9]. Let g be a subgradient of $Z_{d1}(\alpha, \beta, \theta, \varepsilon)$. Then, in k of the subgradient optimization procedure, the multiplier vector $\pi = (\alpha, \beta, \theta, \varepsilon)$ is updated by $\pi^{k+1} = \pi^k + t^k g^k$. The step size t^k is determined by $t^k = \lambda_k \frac{Z_{IP1}^h - Z_{d1}(\pi_k)}{\left\| g^k \right\|^2}$. Z_{IP1}^h is the primal objective function value for a heuristic solution. λ_k is constant between 0 and 2.

2.6 Getting Primal Feasible Solutions

After solving these problems by Lagrangian relaxation and the subgradient method in each iteration, we will get a theoretical lower bound of primal feasible solution, and some useful information which provides us some starting points to solve our primal problem.[13] If all decision variables calculated happen to satisfy the relaxed constraints, a primal feasible solution is found. Otherwise, some modification on such infeasible solutions could be made to obtain primal feasible solutions. Owing to the complexity of the primal problem, we divide overall problem into two subproblems. The first one is survivability RWA subproblem, which modifies working and backup routing according to the result of dual problem in a complete graph. After the determination of routing paths, we then construct a network topology based on the routing decision.

2.6.1 Heuristic for Survivability RWA Subproblem

In this subproblem, we must decide the lightpath to be setup for both working and backup traffic demand, and the number of backup for each O-D pair. To solve this subproblem, the result of Subproblem2-1, { x_{pj} }, would be a good starting point to get the feasible solution for working demand. After all working RWA decisions have been made, the backup routing path and demand of each O-D pair can then be easily chosen. The overall algorithm is described below:

1. Based on { x_{pj} }, calculate aggregate traffic flow on each WDM link.
2. Find out the set of links on which traffic flow exceeds its capacity, denoted { L_e }
3. Remove one $l \in L_e$, identify the O-D pair set that has routed traffic on *l*, denoted by O_l.
4. Select $o \in O_l$ with heaviest demand, take the traffic away, and re-route it without passing through link *l*.
5. Repeat step 4 until the traffic across link *l* becomes less than its capacity.
6. Repeat step3, 4, and 5 until L_e becomes empty.
7. For each O-D pair, find out all the links that its traffic demand routing on, calculate the maximum traffic flow *f* among those links. The number of backup lightpath required, B_w, is then be set to *f*.
8. For each O-D pair, find out *f* lightpath, which is link-disjoint to working paths decided from previous steps.

Steps 1-6 determine working routing and wavelength assignment for each O-D pair, and step 7 and 8 identify the number of backup lightpath required and the routing decision.

2.6.2 Heuristic for Topology Design Subproblem

We can get a set of links *L* that is used for routing decisions in 2.6.1. Topology formed by *L* is denoted *G*. The primal solution *G* is feasible but may be loose and the cost are over-estimated from optimal solution. To refine the quality of primal feasible solution, we then apply the algorithm below to remove dispensable trunks:

1. Calculate T_Cost = total implementation cost of *G*, including trunk and line cost.
2. Based on G, identify the trunk with lightest traffic flow, denoted t_l.
3. Remote t_l from *G*, and update topology $G = G - t_l$
4. Based on *G*, re-routing those O-D pairs with traffic on t_l.
5. If there exists a feasible re-routing decision for all O-D pairs with traffic on t_l, go to step 1. Else, Stop algorithm.

After applying this algorithm, we can get a new topology with much lower implementation cost then original one.

Chapter 3 Problem Formulation for Path Restoration

3.1 Problem Description

In this chapter, we provide a network design algorithm for path restoration scheme. The objective in this algorithm is the same to fast protection – to minimize the total implementation and line cost of the network with survivability demand. As mentioned in chapter 1, although restoration may spend more time in connection recovery compared to protection scheme, but the spare resource needed is much less than protection scheme.

Unlike fast protection scheme, path restoration scheme does not have to reserve any backup resource in advance when connection is set up, but it should ensure that there is always a sufficient amount of resource for re-routing traffic demand for every single trunk failure situation. The topology design should certainly take this into consideration. Besides, for each O-D pair, the backup routing paths would depend on the spare resource available at the time when some link failure, so routing decisions of an O-D pair may be totally different for different link failure state (illustrated in figure 3-1). To fully describe every single link failure scenario, we introduce a new notation to represent the failure state, which consists of normal state and all single trunk failure situations.

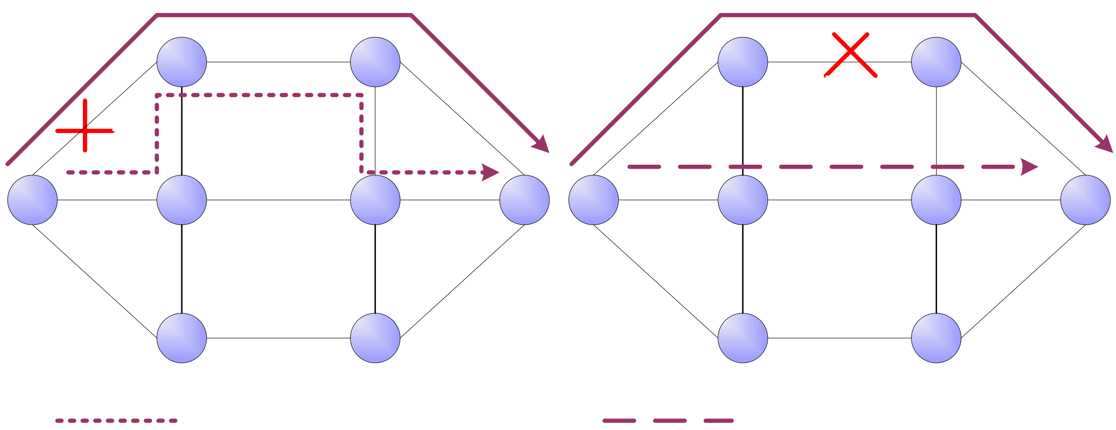

Figure 3-1 Different Routing Path for Different Link Failure Scenario

Problem assumptions：

- The basic architecture used is a WDM network.
- The OXCs used in the optical network lack the capability of wavelength conversion, which incurs the extra delay for O-E-O conversion.
- Link cost includes (a) implementation cost: digging, wiring, and maintenance cost, (b) line cost: cost of fibers, which is proportioned to the distance of link between two-end nodes, (c) channel cost. We assume implementation cost is much higher than other two costs, so we only consider implementation cost in our model.
- A WDM lightpath can serve only a single traffic demand; there is no multiplexing within wavelength.
- Single trunk failure assumption.

Table 3-1 Problem Assumptions for Path Restoration Scheme

Given：

- A set of single trunk failure state.

- A set of candidate trunks and links.
- The limit number of fibers that can be placed in a fiber cable.
- Candidate wavelength set and the maximum number of wavelengths per fiber.
- Implementation cost and line cost per for each link.
- A set of origin-destination (O-D) pairs with traffic demand.
- Candidate paths with their routes and wavelengths used for each O-D pair.

Objective：

To minimize the total network cost.

Subject to：

- Capacity limit constraint – the number of lightpath used in a fiber should not exceed its wavelength count limit.
- Wavelength limit constraint – a wavelength can only be used once in a single fiber. If a cable are placed with *n* fibers, a wavelength can then be used *n* times in that cable.
- Wavelength continuity constraint – there is no wavelength converter in OXCs. That is, each lightpath can only use one wavelength
- Traffic demand constraint – the number of lightpath assigned to an O-D pair should meet its requirement.
- Implementation limit constraint – the number of fiber put in a fiber cable should not exceed its limit.
- Survivability constraint – for any failure scenario, there is always enough resource to meet the demand of each O-D pair.

To determine： 1. The topologies of the optical network. 2. Number of fibers to be placed in each link. 3. Routing path and wavelength used for each O-D pair under each network state.

Table 2-2 Problem Description for Path Restoration Scheme

3.2 Notation

Given Parameters	
Notation	**Descriptions**
E	The set of network states (In this thesis, we only consider the failed scenario of each single trunk.)
T	The set of candidate WDM trunks.
L	The set of candidate WDM links.
W	The set of origin-destination (O-D) pairs.
P_w	Candidate path set for O-D pair *w*, where the distance of each path meets the hop count requirement.
n	Limit number of fibers on a link.
r_w	The number of lightpath required for O-D pair *w*.
J	The set of candidate wavelengths in the WDM network.
a_t	The implementation cost of trunk *t*.
b_l	The fiber cost per of link *l*.
δ_{pl}	1 if lightpath *p* is on WDM link *l*; otherwise 0.
σ_{lt}	1 if directed link *l* is on WDM trunk *t*; otherwise 0.

Table 3-1 Notation of Given Parameters for Restoration Model

Decision Variables	
Notation	**Descriptions**
x^e_{pj}	Number of lightpath usage for path *p* using wavelength *j* on network state *e*; otherwise 0.
y_t	1 if trunk *t* is implemented; otherwise 0.
z_l	Number of fibers to be put on link *l*.

Table 3-2 Notation of Decision Variables for Reatoration Model

3.3 Problem Formulation

Optimization Problem:

Objective function:

$$Z_{IP2} = \min \sum_{t \in T} a_t y_t + \sum_{l \in L} b_l z_l \qquad \text{(IP2)}$$

subject to:

$$\sum_{w \in W} \sum_{p \in P_w} x^e_{pj} \delta_{pl} \leq z_l \qquad \forall j \in J, l \in L, e \in E \qquad (1)$$

$$x^e_{pj} = \{0, 1, 2, \ldots, n\} \qquad \forall j \in J, p \in P_w, w \in W, e \in E \qquad (2)$$

$$z_l \in \{0, 1, 2, \ldots, n\} \qquad \forall l \in L \qquad (3)$$

$$z_l \sigma_{lt} \leq n y_t \qquad \forall l \in L, t \in T \qquad (4)$$

$$y_t = 1 \ or \ 0 \qquad \forall t \in T \qquad (5)$$

$$\sum_{j \in J} \sum_{p \in P_w} x^{\ e}_{\ pj} = r_w \qquad \forall w \in W, e \in E \qquad (6)$$

The objective function represents to minimize the design cost in the network. Constraint (1), (2), and (3) require that the aggregate traffic flow does not exceed the capacity of each WDM link. Constraints (4) and (5) state the number of fibers to be put on a link does not exceed the limit. Constraint (6) requires the number of path selected for each O-D must meet its requirement. All of these constraints from (1) to (6) require holding upon any failure state in *E*.

3.4 Solution Approach

By using the Lagrangian relaxation method, we can transform the primal problem (IP2) into the following Lagrangian relaxation problem (LR2) where constraint (1) is relaxed.

For a vector of non-negative Lagrangian multipliers, a Lagrangian relaxation problem of IP2 is given by optimization problem (LR2):

$$Z_{d2}(\alpha) = \min \sum_{t\in T} a_t y_t + \sum_{l\in L} b_l z_l + \sum_{e\in E}\sum_{l\in L}\sum_{j\in J} \alpha_{lj}^{e} \left[\sum_{w\in W}\sum_{p\in P_w} x_{pj}^{e}\delta_{pl} - z_l \right]$$

subject to:

x_{pj}^{e}	$=$	$\{0, 1, 2, \ldots, n\}$	$\forall j \in J, p \in P_w, w \in W, e \in E$	(2)
z_l	$\in$	$\{0, 1, 2, \ldots, n\}$	$\forall l \in L$	(3)
$z_l \sigma_{lt}$	$\leq$	$n y_t$	$\forall l \in L, t \in T$	(4)
y_t	$=$	1 *or* 0	$\forall t \in T$	(5)
$\sum_{j\in J}\sum_{p\in P_w} x_{pj}^{e}$	$=$	r_w	$\forall w \in W, e \in E$	(6)

where α is the vector of $\{\alpha_{lj}^{e}\}$, and α is the Lagrange multiplier and α ≥ 0. To solve (LR2), we can decompose (LR2) into the following two independent and easily solvable optimization sub-problems.

Subproblem 3-1 (related to decision variable x_{pj}^{e})

$$Z_{sub3-1}(\alpha) = \min \sum_{e \in E} \sum_{w \in W} \sum_{p \in P_w} \sum_{l \in L} \sum_{j \in J} x_{pj}^{e} \delta_{pl} \alpha_{lj}^{e}$$

subject to:

x_{pj}^{e}	$=$	$\{0, 1, 2, \ldots, n\}$	$\forall j \in J, p \in P_w, w \in W, e \in E$	(2)
$\sum_{j \in J} \sum_{p \in P_w} x_{pj}^{e}$	$=$	r_w	$\forall w \in W, e \in E$	(6)

Subproblem 3-1, which is similar to subproblem 2-1, can be decomposed into |*E*| * |*W*| minimum cost flow problems, with traffic demand r_w for each O-D pair and link capacity *n*. The positive Lagrangian multiplier α_{lj}^{e} can be viewed as the link cost for each link *l* using wavelength *j* in error state *e*. To represent failure state of each link, we can set the capacity of the failure link to zero for each error state $e \in E$, and then this problem can apply the same decomposing and solving approaches proposed for subproblem 2-1.

Subproblem 3-2 (related to decision variables y_t and z_l)

$$Z_{sub3-2}(\alpha) = \min \sum_{t \in T} a_t y_t - \sum_{l \in L} \left[(\sum_{e \in E} \sum_{j \in J} \alpha_{lj}^{e} - b_l) z_l \right]$$

subject to:

z_l	$\in$	$\{0, 1, 2, \ldots, n\}$	$\forall l \in L$	(3)
z_l	$\leq$	$n y_l$	$\forall l \in L$	(4)
y_l	$=$	$1 \text{ or } 0$	$\forall l \in L$	(5)

This problem is identical to subproblem 2-3, we solve it with pair of links whose two-end nodes is the same, and Subproblem 3-2 can be decomposed into |T|/2 subproblems:

$$Z_{sub2-3i}(\alpha) = \min\left[a_i y_i + (\sum_{j\in J}\alpha_{lj} - b_l)z_l + (\sum_{j\in J}\alpha_{l'j} - b_{l'})z_{l'}\right] \quad i = 1,2,\ldots|T| \quad l, l' \text{ is on } i.$$

subject to:

z_l	$\in$	$\{0, 1, 2, \ldots, n\}$	$\forall l \in L$	(3)
$z_l\sigma_{lt}$	$\le$	ny_t	$\forall l \in L, t \in T$	(6)
y_t	$=$	1 *or* 0	$\forall t \in T$	(7)

For each subproblem, there are only four possibilities for decision variables (y_i, z_i, $z_{i'}$):

1. (0, 0, 0), $Z_{sub2-3i}(\alpha) = 0$.
2. (1, n, 0), $Z_{sub2-3i}(\alpha) = a_i - (\sum_{j\in J}\alpha_{ij} - b_l)n$.
3. (1, 0, n), $Z_{sub2-3i}(\alpha) = a_i - (\sum_{j\in J}\alpha_{i'j} - b_{l'})n$.
4. (1, n, n), $Z_{sub2-3i}(\alpha) = a_i - (\sum_{j\in J}\alpha_{ij} - b_l)n - (\sum_{j\in J}\alpha_{i'j} - b_{l'})n$.

And then we can try each possibility to determine the minimal value of $Z_{sub2-3i}(\alpha)$ and each decision variable. The objective function $Z_{sub2-3}(\alpha)$ can be compute from solving each $Z_{sub2-3i}(\alpha)$ optimally from the lemma described above.

3.5 The Dual Problem and the Subgradient Method

According to the weak Lagrangian duality theorem [8], for any $\alpha \geq 0$, $Z_{d2}(\alpha)$ is a lower bound on Z_{IP2}. The following dual problem (D2) is then constructed to calculate the tightest lower bound.

Dual Problem (D2):

$$Z_{d2} = \max Z_{d2}(\alpha) \tag{D2}$$

subject to:

$$\alpha \geq 0 \tag{7}$$

The most popular method to solve the dual problem is the subgradient method [9]. Let g be a subgradient of $Z_{d2}(\alpha)$. Then, in k of the subgradient optimization procedure, the multiplier vector $\pi = (\alpha)$ is updated by $\pi^{k+1} = \pi^k + t^k g^k$. The step size t^k is determined by $t^k = \lambda_k \frac{Z_{IP2}^h - Z_{d2}(\pi_k)}{\left\| g^k \right\|^2}$. Z_{IP2}^h is the primal objective function value for a heuristic solution. λ_k is constant between 0 and 2.

3.6 Getting Primal Feasible Solutions

After solving these problems by Lagrangian relaxation and the subgradient method in each iteration, we will get a group of routing decisions for each single link failure state, including normal state. To achieve feasible solution topology, we use the topology formed from routing decisions of normal state as a string point, verify the feasibility of each error state, and make some adjustment to meet the survivability requirement. The algorithm is described below:

1. Form a topology G from routing decisions of normal state, $\{ x_{pj}^0 \}$. The trunk set that composes G is denoted T.
2. Select a trunk $t \in T$, removing it from T.

3. For each O-D pair *o* that has traffic on *t*, verify the survivability by re-routing the traffic in the topology $\{G - t\}$.
4. If there is no feasible routing decision for O-D pair o, adding new trunks to G according to the routing decision of o in state t.(x_{pj}^{t})
5. Repeat step 2, 3, and 4 until T becomes empty.

After applying the algorithm above, we can get a final topology that meets the requirement of survivability when any single link failure.

Chapter 4 Problem Formulation for Load-balancing RWA

4.1 Problem Description

In this chapter, we desire to provide a load-balancing routing and wavelength assignment algorithm with survivability demand. Chapter 2 and 3 provide two algorithms to design survivable network with static demand. Based on the topology designed previously, to accommodate future demand, survivability routing with load balancing is the primary concern of the mathematical model in this chapter.

Problem assumptions：
● The basic architecture used is a WDM network.
● The OXCs used in the optical network lack the capability of wavelength conversion, which incurs the extra delay for O-E-O conversion.
● A WDM lightpath can serve only a single traffic demand; there is no multiplexing within wavelength.

Table 4-1 Problem Assumptions for Load-balancing RWA

Given：
● The optical layer topology and number of fibers on each link.
● Candidate wavelength set and the maximum number of wavelengths per fiber.
● A set of origin-destination (O-D) pairs with traffic demand.
● Candidate paths with their routes and wavelengths used for each O-D pair.

Objective：

To minimize the maximum link utilization of the network.

Subject to：

- Capacity limit constraint – the number of lightpath used in a fiber should not exceed its wavelength count limit.
- Wavelength limit constraint – a wavelength can only be used once in a single fiber. If a cable are placed with *n* fibers, a wavelength can then be used *n* times in that cable.
- Wavelength continuity constraint – there is no wavelength converter in OXCs. That is, each lightpath can only use one wavelength.
- Traffic demand constraint – the number of lightpath assigned to an O-D pair should meet its requirement.
- Survivability constraint – for each O-D pair satisfies (a) the number of backup path reserved should be sufficient for any single link failure situation; (b) backup paths should be link-disjoint with working path.

To determine：

Working and backup routing path and wavelength assigned to each O-D pair.

Table 4-2 Problem Description for for Load-balancing RWA

4.2 Notation

Given Parameters	
Notation	**Descriptions**
T	The set of WDM trunks.
L	The set of directed WDM links.
W	The set of origin-destination (O-D) pairs.
C_l	The number of fibers on link *l*.
P_w	Candidate path set for O-D pair *w*, where the distance of each path meets the hop count requirement.
r_w	The number of lightpath required for O-D pair *w*.
J	The set of candidate wavelengths in the WDM network.
δ_{pl}	1 if lightpath *p* is on WDM link *l*; otherwise 0.
σ_{lt}	1 if directed link *l* is on WDM trunk *t*; otherwise 0.

Table 4-3 Notation of Given Parameters for Load-balancing RWA Model

Decision Variables	
Notation	**Descriptions**
x_{pj}	Number of lightpath used for working path *p* using wavelength *j*.
v_{pj}	Number of lightpath used for backup path *p* using wavelength *j*.
B_w	The number of backup lightpath required for O-D pair *w*.
u_{wt}^{o}	1 if trunk *t* is used by a working path of OD pair *w*; otherwise 0.
u_{wt}^{b}	1 if trunk *t* is used by a backup path of O-D pair *w* ; otherwise 0.

Table 4-4 Notation of Decision Variables for Load-balancing RWA Model

4.3Problem Formulation

Objective function:

$$Z_{IP3} = \min \max_{l \in L} \frac{\sum_{j \in J} \sum_{w \in W} \sum_{p \in P_w} (x_{pj} + v_{pj}) \delta_{pl}}{C_l |J|} \quad \text{(IP3)}$$

subject to:

$$\sum_{w \in W} \sum_{p \in P_w} \left(x_{pj} + v_{pj} \right) \delta_{pl} \le C_l \quad \forall j \in J, l \in L \quad (1)$$

$$x_{pj} = \{0, 1, 2, \dots, n\} \quad \forall j \in J, p \in P_w, w \in W \quad (2)$$

$$v_{pj} = \{0, 1, 2, \dots, n\} \quad \forall j \in J, p \in P_w, w \in W \quad (3)$$

$$\sum_{j \in J} \sum_{p \in P_w} x_{pj} = r_w \quad \forall w \in W \quad (4)$$

$$\sum_{j \in J} \sum_{p \in P_w} x_{pj} \delta_{pl} \sigma_{lt} \le B_w \quad \forall w \in W, t \in T \quad (5)$$

$$\sum_{j \in J} \sum_{p \in P_w} v_{pj} = B_w \quad \forall w \in W \quad (6)$$

$$B_w \le r_w \quad \forall w \in W \quad (7)$$

$$\sum_{j \in J} \sum_{p \in P_w} x_{pj} \delta_{pl} \sigma_{lt} \le r_w u_{wt}^o \quad \forall w \in W, t \in T \quad (8)$$

$$\sum_{j \in J} \sum_{p \in P_w} v_{pj} \delta_{pl} \sigma_{lt} \le r_w u_{wt}^b \quad \forall w \in W, t \in T \quad (9)$$

$$u_{wt}^o + u_{wt}^b \le 1 \quad \forall t \in T, w \in W \quad (10)$$

$$u_{wt}^o = 1 \text{ or } 0 \quad \forall t \in T, w \in W \quad (11)$$

$$u_{wt}^b = 1 \text{ or } 0 \quad \forall t \in T, w \in W \quad (12)$$

The objective function represents to minimize the maximum network link utilization in the network. Constraint (1), (2), and (3) require that the frequency a single wavelength used on a link does not exceed the number of fibers on that link. Capacity constraint is also implied on constraint (1). Constraint (4) requires the number of path selected for each O-D must meet its requirement. To achieve survivability, constraint (5), (6), and (7) state the sufficient of backup path for each O-D pair. Constraint (5) indicates that the number of backup path is enough to make up the failure of any link, and constraint (6) requires that the number of backup path to be selected meets the requirement. Constraints (8) to (12) desire to satisfy the link-disjoint requirement between original and backup paths which is implicated imposed on Constraint (10).

Let

$$s \quad = \quad \max_{l \in L} \frac{\sum_{j \in J} \sum_{w \in W} \sum_{p \in P_w} (x_{pj} + v_{pj}) \delta_{pl}}{C_l |J|}$$

An equivalent formulation of Problem (*IP3*) is:

$$Z_{IP4} \quad = \quad \min s \qquad \text{(IP4)}$$

subject to:

$$0 \le s \le 1 \qquad (14)$$

$$\sum_{j \in J} \sum_{w \in W} \sum_{p \in P_w} (x_{pj} + v_{pj}) \delta_{pl} \quad \le \quad sC_l |J| \qquad \forall l \in L \qquad (15)$$

$$\sum_{w \in W} \sum_{p \in P_w} (x_{pj} + v_{pj}) \delta_{pl} \quad \le \quad C_l \qquad \forall j \in J, l \in L \qquad (16)$$

$$x_{pj} \quad = \quad \{0, 1, 2, \dots, n\} \qquad \forall j \in J, p \in P_w, w \in W \qquad (17)$$

$$v_{pj} \quad = \quad \{0, 1, 2, \dots, n\} \qquad \forall j \in J, p \in P_w, w \in W \qquad (18)$$

$$\sum_{j \in J} \sum_{p \in P_w} x_{pj} \quad = \quad r_w \qquad \forall w \in W \qquad (19)$$

$$\sum_{j \in J} \sum_{p \in P_w} x_{pj} \delta_{pl} \sigma_{lt} \leq B_w \quad \forall w \in W, t \in T \tag{20}$$

$$\sum_{j \in J} \sum_{p \in P_w} v_{pj} = B_w \quad \forall w \in W \tag{21}$$

$$B_w \leq r_w \quad \forall w \in W \tag{22}$$

$$\sum_{j \in J} \sum_{p \in P_w} x_{pj} \delta_{pl} \sigma_{lt} \leq r_w u_{wt}^o \quad \forall w \in W, t \in T \tag{23}$$

$$\sum_{j \in J} \sum_{p \in P_w} v_{pj} \delta_{pl} \sigma_{lt} \leq r_w u_{wt}^b \quad \forall w \in W, t \in T \tag{24}$$

$$u_{wt}^o + u_{wt}^b \leq 1 \quad \forall t \in T, w \in W \tag{25}$$

$$u_{wt}^o = 1 \text{ or } 0 \quad \forall t \in T, w \in W \tag{26}$$

$$u_{wt}^b = 1 \text{ or } 0 \quad \forall t \in T, w \in W \tag{27}$$

Constraints (16) – (27) are the same to constraints (1) – (13). Constraint (14) and (15) require that the utilization of each link not to exceed *s*.

4.4 Solution Approach

By using the Lagrangian relaxation method, we can transform the primal problem (IP4) into the following Lagrangian relaxation problem (LR3) where Constraints (14), (15), (20), (23), (24) are relaxed:

For a vector of non-negative Lagrangian multipliers, a Lagrangian relaxation problem of IP4 is given by optimization problem (LR3):

$$Z_{d3}(\alpha,\beta,\phi,\varepsilon,\theta)=$$

$$\min s+\sum_{l\in L}\alpha_l\left[\sum_{l\in L}\sum_{w\in W}\sum_{p\in P_w}(x_{pj}+v_{pj})\delta_{pl}-sC_l\,|\,J\,|\right]+\sum_{j\in J}\sum_{l\in L}\beta_{lj}\left[\sum_{w\in W}\sum_{p\in P_w}(x_{pj}+v_{pj})\delta_{pl}-C_l\right]$$

$$+\sum_{w\in W}\sum_{t\in T}\phi_{wt}\left[\sum_{j\in J}\sum_{p\in P_w}x_{pj}\delta_{pl}\sigma_{lt}-B_w\right]+\sum_{w\in W}\sum_{t\in T}\varepsilon_{wt}\left[\sum_{j\in J}\sum_{p\in P_w}x_{pj}\delta_{pl}\sigma_{lt}-r_w u^o_{wt}\right]$$

$$+\sum_{w\in W}\sum_{t\in T}\theta_{wt}\left[\sum_{j\in J}\sum_{p\in P_w}v_{pj}\delta_{pl}\sigma_{lt}-r_w u^b_{wt}\right]$$

subject to:

$$0\le s\le 1 \qquad (14)$$

$$x_{pj} = \{0, 1, 2, \ldots, n\} \qquad \forall j\in J, p\in P_w, w\in W \qquad (17)$$

$$v_{pj} = \{0, 1, 2, \ldots, n\} \qquad \forall j\in J, p\in P_w, w\in W \qquad (18)$$

$$\sum_{j\in J}\sum_{p\in P_w}x_{pj} = r_w \qquad \forall w\in W \qquad (19)$$

$$\sum_{j\in J}\sum_{p\in P_w}v_{pj} = B_w \qquad \forall w\in W \qquad (21)$$

$$B_w \le r_w \qquad \forall w\in W \qquad (22)$$

$$u^o_{wt}+u^b_{wt} \le 1 \qquad \forall t\in T, w\in W \qquad (25)$$

$$u^o_{wt} = 1 \text{ or } 0 \qquad \forall t\in T, w\in W \qquad (26)$$

$$u^b_{wt} = 1 \text{ or } 0 \qquad \forall t\in T, w\in W \qquad (27)$$

where α, β, ϕ, ε, θ are the vectors of $\{\alpha_l\}$, $\{\beta_{lj}\}$, $\{\phi_{wt}\}$, $\{\varepsilon_{wt}\}$, $\{\theta_{wt}\}$ and α, β, ϕ, ε, θ are the Lagrangian multipliers and α, β, ϕ, ε, θ ≥ 0. To solve (LR3), we can decompose (LR3) into the following three independent and easily solvable optimization sub-problems

Subproblem 4-1 (related to decision variable *s*)

$$Z_{sub4-1}(\alpha) = \min s\left(1 - \sum_{l \in L} \alpha_l C_l \mid J \mid\right)$$

subject to:

$$0 \le s \le 1 \tag{14}$$

Because α_l is a positive multiplier, and C_l and $|J|$ are positive constants, to obtain optimal value of $Z_{sub4-1}(\alpha)$, there are only two cases:

1. $\left[1 - \sum_{l \in L} \alpha_l C_l \mid J \mid\right] \ge 0$, s = 0 and $Z_{sub4-1}(\alpha) = 0$
2. $\left[1 - \sum_{l \in L} \alpha_l C_l \mid J \mid\right] < 0$, s = 1 and $Z_{sub4-1}(\alpha) = \left[1 - \sum_{l \in L} \alpha_l C_l \mid J \mid\right]$

Subproblem 4-2 (related to decision variable x_{pj})

$$Z_{sub2-1}(\alpha, \beta, \varepsilon) = \min \sum_{w \in W} \sum_{p \in P_w} \sum_{j \in J} \left[\sum_{l \in L} \left(\alpha_l + \beta_{lj}\right) x_{pj} \delta_{pl} + \sum_{t \in T} \left(\phi_{wt} + \varepsilon_{wt}\right) x_{pj} \delta_{pl} \sigma_{lt} \right]$$

subject to:

$$x_{pj} = \{0, 1, 2, \ldots, n\} \qquad \forall j \in J, p \in P_w, w \in W \tag{17}$$

$$\sum_{j \in J} \sum_{p \in P_w} x_{pj} = r_w \qquad \forall w \in W \tag{19}$$

Subproblem 4-2, which is identical to subproblem 2-2, is composed of $|W|$ problems for each O-D pair w. Each problems is a minimum cost flow problem, where β_{lj} is the cost for using wavelength *j* on link *l*, and the arc cost is for O-D pair *w* of link *l* is $\left(\alpha_l + \phi_{wt} + \varepsilon_{wt}\right)$. We can apply the same approaches to solve subproblem 4-2.

Subproblem 4-3 (related to decision variables v_{pj}, and B_w)

$$Z_{sub2-1}(\alpha,\beta,\varphi,\varepsilon,\theta)=\min\sum_{w\in W}\left[\sum_{p\in P_w}\sum_{j\in J}\sum_{l\in L}\left(\alpha_l+\beta_{lj}\right)v_{pj}\delta_{pl}+\sum_{p\in P_w}\sum_{j\in J}\sum_{t\in T}\left(\varepsilon_{wt}+\theta_{wt}\right)v_{pj}\delta_{pl}-\sum_{t\in T}\phi_{wt}B_w\right]$$

subject to:

$$v_{pj} = \{0, 1, 2, \ldots, n\} \qquad \forall j\in J, p\in P_w, w\in W \qquad (18)$$

$$\sum_{j\in J}\sum_{p\in P_w} v_{pj} = B_w \qquad \forall w\in W \qquad (21)$$

$$B_w \le r_w \qquad \forall w\in W \qquad (22)$$

Subproblem 4-3 is identical to subproblem 2-2. We can apply the same algorithm proposed in subproblem 2-2 to solve this subproblem.

Subproblem 4-4 (related to decision variables u_{wt}^o and u_{wt}^b)

$$Z_{sub4-3}(\varepsilon,\theta)=\min\sum_{t\in T}\sum_{w\in W}\left[-\left(\varepsilon_{wt}u_{wt}^o+\theta_{wt}u_{wt}^b\right)r_w\right]$$

subject to:

$$u_{wt}^o+u_{wt}^b \le 1 \qquad \forall t\in T, w\in W \qquad (23)$$

$$u_{wt}^o = 1 \text{ or } 0 \qquad \forall t\in T, w\in W \qquad (24)$$

$$u_{wt}^b = 1 \text{ or } 0 \qquad \forall t\in T, w\in W \qquad (25)$$

This problem is identical to subproblem 2-4, which can be decomposed into |*T*|*|*W*| subproblems, and each can be solved by the comparison of ε_{wt} and θ_{wt}.

4.5 The Dual Problem and the Subgradient Method

According to the weak Lagrangian duality theorem [8], for any α, β, ϕ, ε, θ ≥ 0, $Z_{d3}(\alpha,\beta,\varphi,\varepsilon,\theta)$ is a lower bound on Z_{IP4}. The following dual problem (D3) is then constructed to calculate the tightest lower bound.

Dual Problem (D3):

$$Z_{d3} = \max Z_{d3}(\alpha,\beta,\varphi,\varepsilon,\theta) \tag{D3}$$

subject to:

$$\alpha,\ \beta,\ \phi,\ \varepsilon,\ \theta \geq 0 \tag{28}$$

The most popular method to solve the dual problem is the subgradient method [9]. Let g be a subgradient of $Z_{d3}(\alpha,\beta,\varphi,\varepsilon,\theta)$. Then, in k of the subgradient optimization procedure, the multiplier vector $\pi = (\alpha,\beta,\varphi,\varepsilon,\theta)$ is updated by $\pi^{k+1} = \pi^k + t^k g^k$. The step size t^k is determined by $t^k = \lambda_k \dfrac{Z_{IP4}^h - Z_{d3}(\pi_k)}{\left\| g^k \right\|^2}$. Z_{IP3}^h is the primal objective function value for a heuristic solution. λ_k is constant between 0 and 2.

4.6 Getting Primal Feasible Solutions

Owing to the complexity of the primal problem, we divide overall problem into two subproblems. The first one is survivability RWA subproblem, which modifies working and backup routing according to the topology and capacity given in this problem. After the determination of routing paths, we then construct a network topology based on the routing decision. After all routing decisions are made, we then re-routing network traffic to achieve minmax link utilization objective.

4.6.1 Heuristic for Survivability RWM Subproblem

In this subproblem, we must decide the lightpath to be setup for both working and backup traffic demand, and the number of backup for each O-D pair. The basic idea of this algorithm is similar to the one described on section 2.6.1.

1. Based on { x_{pj} }, calculate aggregate traffic flow on each WDM link.
2. Find out the set of links on which traffic flow exceeds its capacity, denoted { L_e }
3. Remove one $l \in L_e$, identify the O-D pair set that has routed traffic on *l*, denoted by O_l.
4. Select $o \in O_l$ with heaviest demand, take the traffic away, and re-route it without passing through link *l*.
5. Repeat step 4 until the traffic across link *l* becomes less than its capacity.
6. Repeat step3, 4, and 5 until L_e becomes empty.
7. For each O-D pair, find out all the links that its traffic demand routing on, calculate the maximum traffic flow *f* among those links. The number of backup lightpath required, B_w, is then be set to *f*.
8. For each O-D pair, find out *f* lightpath, which is link-disjoint to working paths decided from previous steps.

Steps 1-6 determine working routing and wavelength assignment for each O-D pair, and step 7 and 8 identify the number of backup lightpath required and the routing decision.

4.6.2 Heuristic for MinMax Utilization Subproblem

We next propose an approach to enhance the quality of minmax routing problem. The basic idea is to adjust link arc weight h_l according to the current link flow. More precisely, if the utilization of link *l* is the maximum in the network, then we artificially increase the arc weight h_l and re-route all the traffic on *l* in an attempt to reduce the traffic flow of link *l*. It is clear that the utilization of link *l* does not increase when h_l is increased (if all the other link set metrics remain unchanged). The overall algorithm is given below:

1. Set the iteration counter *k* to be 1. Initialize the link arc weights h_l according to the link cost of subproblem 4-2 and subproblem 4-3 (Lagrangian multipliers).
2. If *k* is greater than a pre-specified counter limit, stop.
3. Select link with maximum link utilization, denoted *l*. Add the arc weight of *l* by a positive value t_l^k.
4. Apply min-cost-flow algorithm to re-routing all traffic demand on *l*.
5. Calculate the aggregate flow for each link.
6. Increase *k* by 1 and go to Step 2

t_l^k can be chosen by different ways. However, the following two properties of $\{t_l^k\}$ are suggested: (i) $\sum_{k=1}^{\infty} t_l^k$ approaches infinity and (ii) t_l^k approaches 0 as *k* approaches infinity. The first property is meant to prevent the algorithm from being stalled, and the second property decreases the possibility of oscillation. If a sequence of t_l^k satisfies the first property, then every h_l will be unbounded when *k* approaches infinity. In our algorithm, we initially set t_l^k to the inverse of capacity, e.g. Link 1, capacity = 1 unit, t_1^0 = 1; Link 2, capacity = 2 unit, and t_2^0 = 1/2. t_l^k is increased by $\frac{1}{capacity + s}$ if link is selected by s times.

Chapter 5 Computational Experiments

5.1 Topology Design Problems

5.1.1 Simple Algorithms (SA)

To evaluate the performance of our algorithms, we develop two simple algorithms for protection and restoration topology design problems respectively. To achieve fairness, the tuning philosophies of simple algorithms are identical to the algorithms proposed in our model.

5.1.1.1 Simple Algorithm for protection

1. For each O-D pair *w*, apply min cost flow algorithm to calculate the working path in complete graph. Without loss of generality, we use the trunk cost as arc weights in min cost flow algorithm.
2. Apply the same algorithm described in section 2.6 to find out the backup paths and feasible topology.

5.1.1.2 Simple Algorithm for restoration

1. For each O-D pair *w* in all error state *e*, apply min cost flow algorithm to calculate the working path for in complete graph. Without loss of generality, we use the trunk cost as arc weights in min cost flow algorithm.
2. Apply the same algorithm described in section 3.6 to find out feasible topology.

5.1.2 Experiment Parameters for Topology Problems

Number of Nodes	8 ~ 12
Trunk Cost	10 ~ 100

Line(Fiber) Cost	0.01 * Trunk Cost
Random Number Seed of Trunk Cost	100
Number of Iteration	2000
Maximum Unimprovement Counter	50
Begin to Tune	200
Initial Upper Bound	Cost of Complete Graph
Initial Scalar of Step Size	2
Test Platform	Windows 2000, 2G Hz CPU, 1G RAM

Table 5-1 Command Testing Parameters for Topology Design

Case	Max Fiber	Max Lambda	Demand	seed	Test Model*
1	8	8	1~3	100/200	P/R
2	8	8	1~6	100/200	P/R
3	8	16	1~6	100/200	P/R
4	8	16	1~3	100/200	P

*P = Protection Model, R = Restoration Model

Table 5-2 Test Cases of Topology Design Problems

5.1.3 Experiment Results

5.1.3.1 Case 1:

Case	Max Fiber	Max Lambda	Demand	seed	Test Model*
1	8	8	1~3	100/200	P/R

Seed	Node	SA	UB	LB	Gap(%)	SA Trunk	Trunk

100	8	523.4	243.6	69.36387	251.1915	15	9
	9	308.8	274.6	78.03053	251.9135	11	11
	10	383.8	276.2	92.70311	197.9404	15	13
	11	595.2	322.3	111.946	187.9067	23	15
	12	368.7	313.1	128.1728	144.2796	18	16
200	8	301	265	72.55373	265.2465	10	9
	9	540.3	266.3	82.06298	224.5069	17	10
	10	366.4	277.7	90.52801	206.7559	15	13
	11	391.8	311.8	113.0614	175.7794	16	15
	12	368.5	300.8	125.6207	139.4511	18	16

Table 5-3 Computational Result of Protection Model in Case 1

Seed	Node	SA	UB	LB	Gap(%)	SA Trunk	Trunk
100	8	261.9	216.9	45.897552	372.5742	9	8
	9	265	230.1	50.618927	354.573	10	9
	10	230.7	209.9	61.294827	242.4433	11	11
	11	271.6	265	69.734642	280.012	12	12
	12	306.5	272.4	77.679108	250.6734	15	13
200	8	264.1	219.7	51.17627	265.2465	9	8
	9	227.2	220.5	54.025028	224.5069	10	9
	10	229.9	202.9	56.861328	206.7559	11	10
	11	261.8	236	68.659698	175.7794	12	11
	12	313.4	271.6	73.552513	139.4511	15	13

Table 5-4 Computational Result of Restoration Model in Case 1

Comparison between 2 models:

Seed = 100 Seed = 200

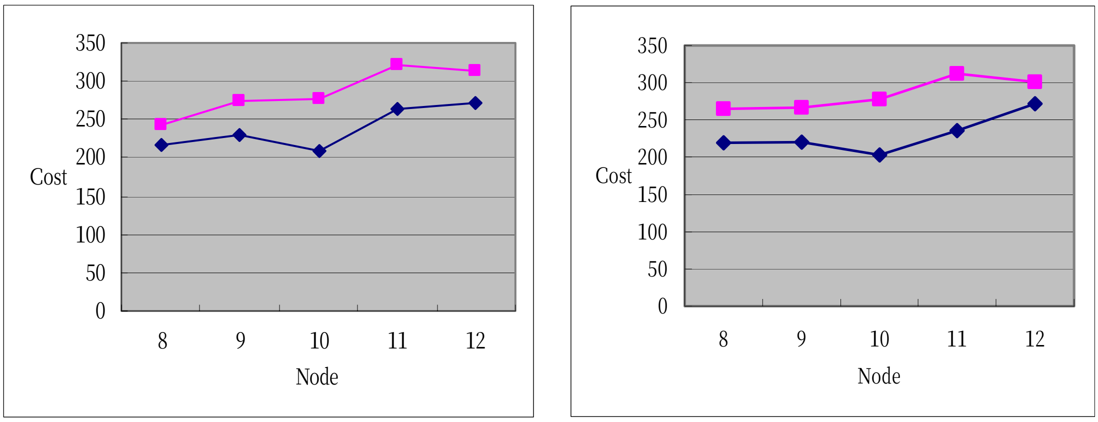

Figure 5-1 Comparison of Implementation Cost in Case 1

Seed = 100 Seed = 200

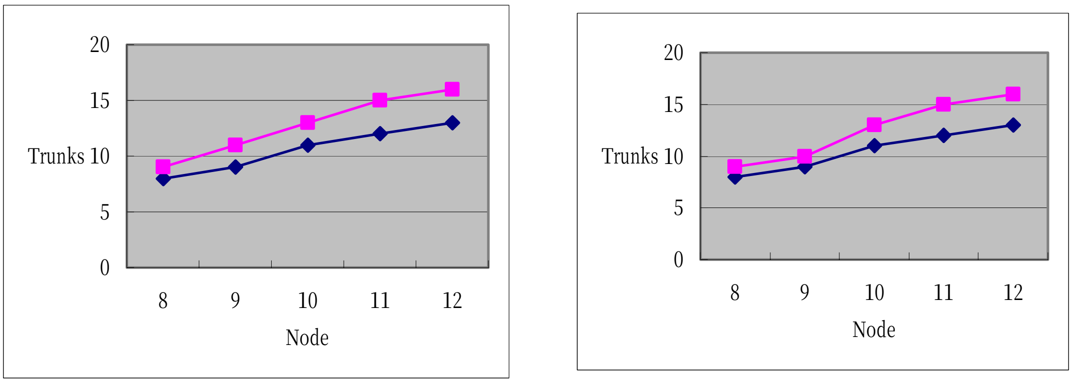

Figure 5-2 Comparison of Number of Trunks in Case 1

5.3.1.2 Case 2:

Case	Max Fiber	Max Lambda	Demand	seed	Test Model*
2	8	8	1~6	100/200	P/R

Seed	Node	SA	UB	LB	Gap(%)	SA Trunk	Trunk
100	8	324.6	290	101.3955	251.1915	11	10
	9	368.4	301.1	126.1427	251.9135	13	12
	10	405.7	346.6	146.7532	197.9404	16	15
	11	498.3	428.8	175.3887	187.9067	20	18
	12	562.2	452.4	203.0779	144.2796	23	21
200	8	334.8	323	110.813	265.2465	11	11
	9	474.2	324.8	135.8017	224.5069	16	12
	10	582.3	393.8	158.1878	206.7559	19	16
	11	487	440	184.8539	175.7794	20	19
	12	565.5	486.9	206.621	139.4511	23	21

Table 5-5 Computational Result of Protection Model in Case 2

Seed	Node	SA	UB	LB	Gap(%)	SA Trunk	Trunk
100	8	278.7	223.9	76.04682	194.4239	9	8
	9	341.9	262.7	90.91807	188.9415	10	10
	10	303.2	258.1	105.6153	144.3776	11	11
	11	340.3	275.5	116.0377	137.4228	12	13
	12	331.4	294.3	135.4356	117.2989	15	14
200	8	279.3	219.7	51.17627	169.7387	9	8
	9	337.1	220.5	54.025028	149.5144	11	9
	10	251	202.9	56.861328	109.7477	11	11
	11	313.5	236	68.659698	117.0699	12	11
	12	338.8	271.6	73.552513	114.1071	14	13

Table 5-6 Computational Result of Restortion Model in Case 2

Comparison between 2 models:

Seed = 100 Seed = 200

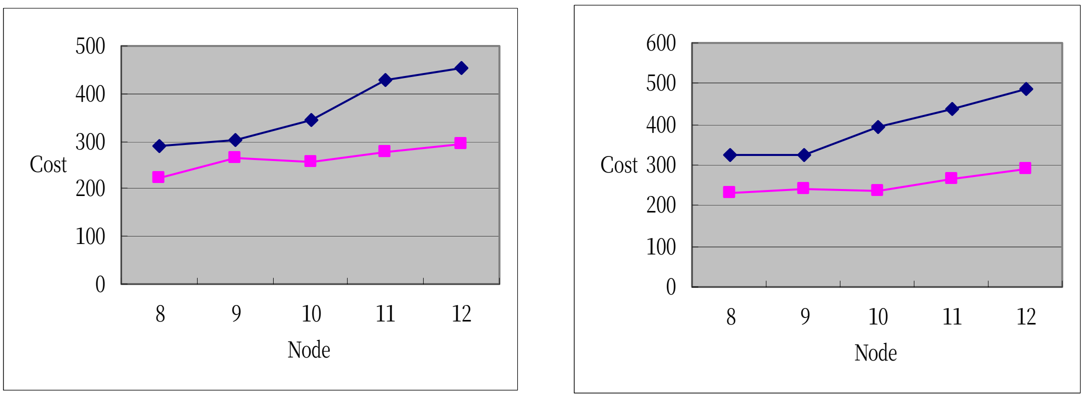

Figure 5-3 Comparison of Implementation Cost in Case 2

Seed = 100 Seed = 200

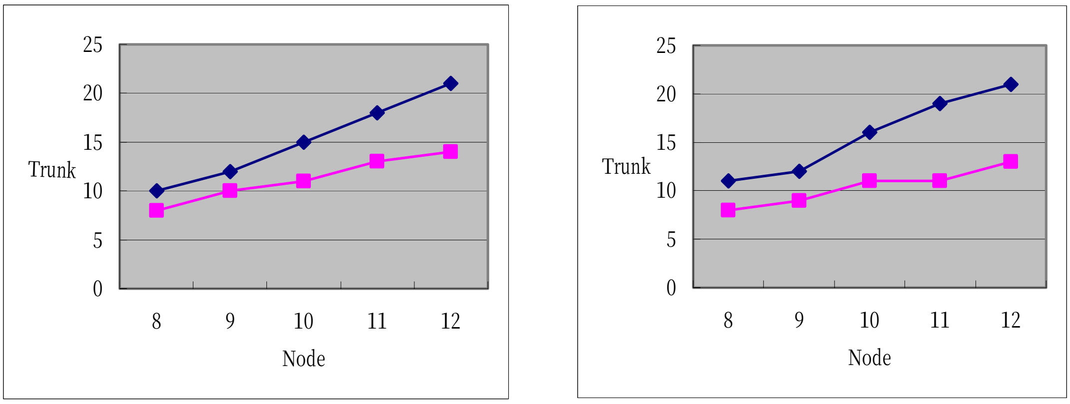

Figure 5-4 Comparison of Number of Trunks in Case 2

5.1.3.3 Case 3:

Case	Max Fiber	Max Lambda	Demand	seed	Test Model*
3	8	16	1~6	100/200	P/R

Seed	Node	SA	UB	LB	Gap(%)	SA Trunk	Trunk

100	8	416.1	255.2	48.92498	421.6149	10	8
	9	312.4	255	60.14375	323.9842	11	9
	10	278.2	254.3	69.69959	264.8515	12	12
	11	498.3	312	82.94238	276.1648	20	13
	12	448.1	278.4	97.75674	184.7885	19	15
200	8	301.4	255.2	54.15602	371.2311	10	8
	9	474.2	255	66.59787	282.8951	16	10
	10	496.1	255.2	75.45925	238.1958	19	12
	11	394.4	312.5	88.10313	254.698	16	14
	12	582.5	278.4	96.4437	188.6658	24	15

Table 5-7 Computational Result of Protection Model in Case 3

Seed	Node	SA	UB	LB	Gap(%)	SA Trunk	Trunk
100	8	278.7	217.5	35.76006	508.2205	9	8
	9	341.9	225.6	43.96958	413.082	10	10
	10	303.2	205.4	49.82267	312.2621	11	10
	11	340.3	224.9	55.4738	305.4166	12	11
	12	331.4	240	61.82351	288.2018	15	12
200	8	279.3	232.9	41.24984	464.6083	9	8
	9	337.1	232.5	47.20492	392.5334	11	9
	10	251	232.1	54.51144	325.7822	10	11
	11	313.5	224.3	57.37252	290.9537	12	11
	12	338.8	234.2	62.38481	275.4119	14	12

Table 5-8 Computational Result of Restoration Model in Case 3

Comparison between 2 models:

Seed = 100 Seed = 200

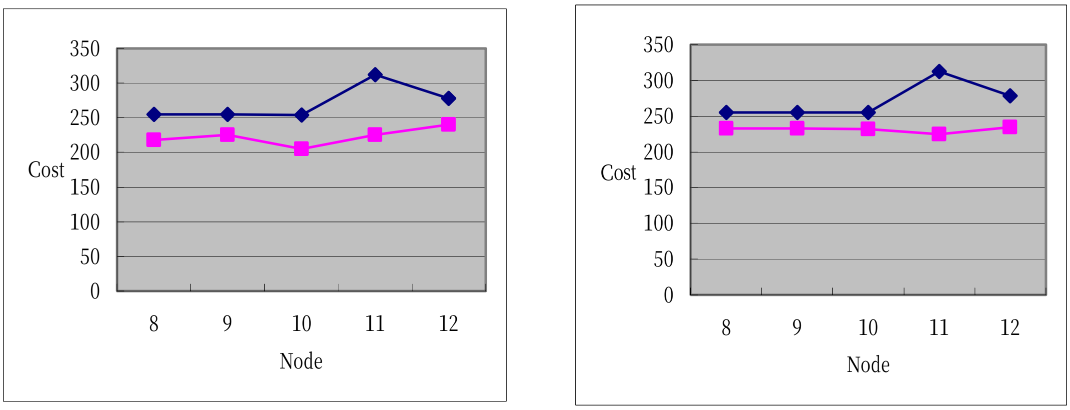

Figure 5-5 Comparison of Implementation Cost in Case 3

Seed = 100 Seed = 200

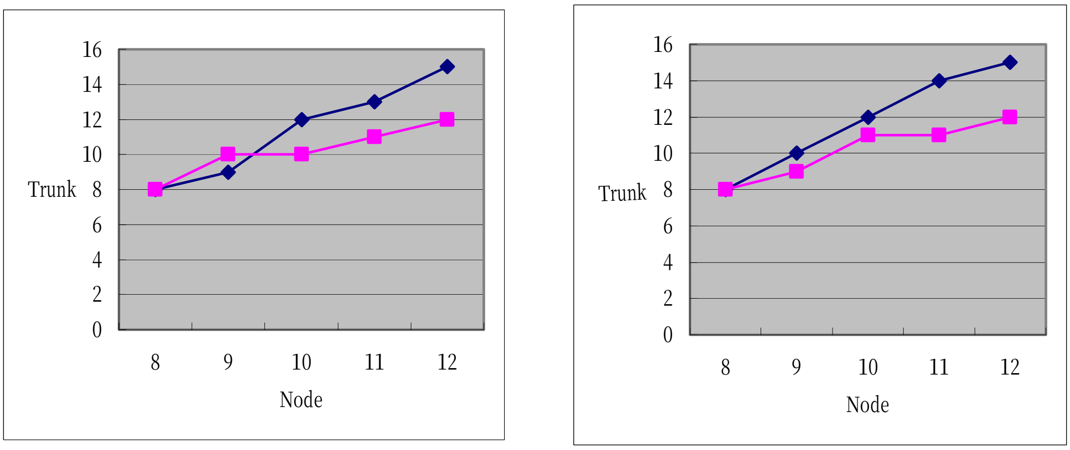

Figure 5-6 Comparison of Number of Trunks in Case 3

5.1.3.4 Case 4:

Case	Max Fiber	Max Lambda	Demand	seed	Test Model*
4	8	16	1~3	100/200	P

Model	Seed	Node	UB	LB	Gap(%)	Trunk	Time

Protection	100	8	243	33.67276	621.6516	8	5361
		9	253.6	37.71374	572.434	10	13716
		10	231.4	42.71944	441.6738	11	32121
		11	255.2	54.53704	367.9389	12	89696
		12	243.6	62.6502	288.8256	13	105383
	200	8	243.1	35.2422	589.798	8	7333
		9	243.1	40.44542	501.0569	10	19073
		10	231.4	43.43478	432.7528	12	44388
		11	254.9	55.0574	362.9713	14	105232
		12	243.6	61.39943	296.7464	15	172977

Table 5-9 Computational Result of Protcection Model in Case 4

5.1.4 Influence from Integer Property

5.1.4.1 Topology Design

With linear relaxation, resource can be divided infinitely. As shown in figure 5-7, if there is one unit of traffic demand between two nodes, the resource required in our model would be one trunk. But with linear relaxation, total resource required would be only $\frac{1}{\max\ lambda} \times \frac{1}{\max\ fiber}$. For this reason, the lower bound of our model would be proportioned to $\frac{1}{\max\ lambda} \times \frac{1}{\max\ fiber}$, which causes larger duality gap.

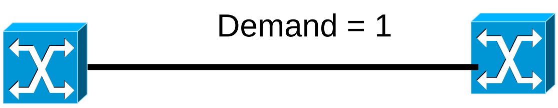

Figure 5-7 Influence on Topology Design from Integer Property

5.14.2 Routing and Wavelength Assignment

Figure 5-8 (a) depict the normal routing and backup decision in our model. If routing With linear relaxation, the routing traffic demand can be divided infinitely, so that the backup path required becomes much fewer than original requirement.

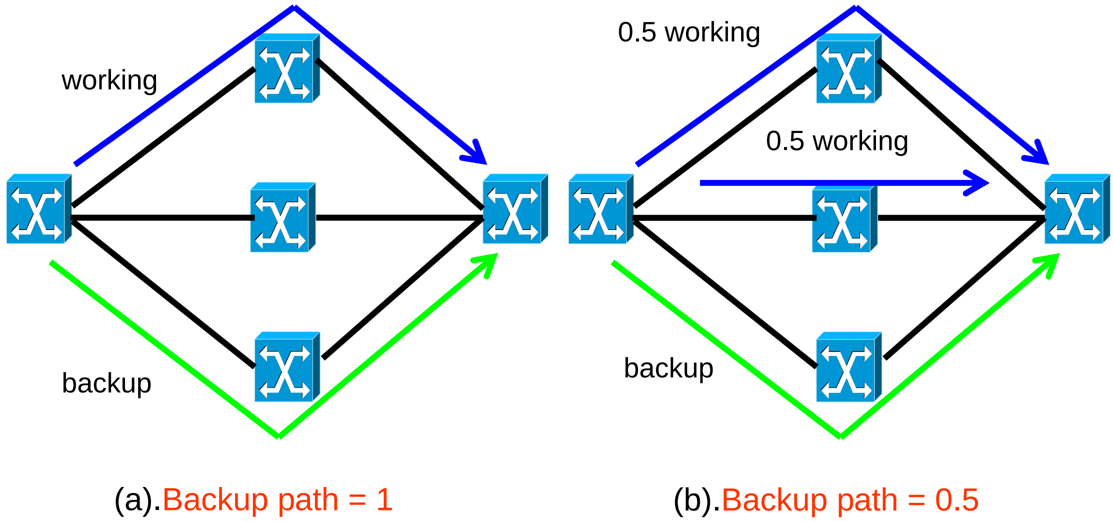

Figure 5-8 Influence on RWA from Integer Property

5.1.4.3 Disjoint-ness

The disjoint requirement is forced by constraint:

$$u^{o}_{wt} + u^{b}_{wt} \le 1 \qquad \forall t \in T, w \in W \qquad (16)$$

With linear relaxation, the disjoint-ness between working and backup path would be much weaker. As shown in Figure 5-9.

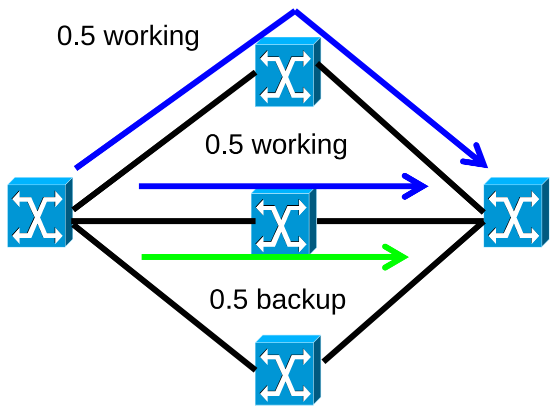

Figure 5-9 Influence on Disjoint-ness from Integer Property

Because of the complexity of network design problem and integer property of our models, we cannot get a tighter lower bound by solving the Lagrangian relaxation problem iteration by iteration under some conditions. Although we cannot get a tighter lower bound, this powerful methodology provides a lot of hints to help us get a primal feasible solution.

5.1.5 Computational Time

5.1.5.1 Case 1:

Model	Seed	Node	LR	Avg. Time	SA
Protection	100	8	1622	0.811	1
		9	4830	2.415	5
		10	7490	3.745	6
		11	14869	7.4345	9
		12	23568	11.784	14
	200	8	1798	0.899	1
		9	3370	1.685	3
		10	12852	6.426	6
		11	19202	9.601	13
		12	23009	11.5045	14
Restoration	100	8	1195	0.5975	3
		9	4153	2.0765	13
		10	7875	3.9375	24
		11	16740	8.37	44

		12	24266	12.133	88
	200	8	596	0.298	4
		9	2942	1.471	13
		10	5396	2.698	23
		11	9744	4.872	44
		12	20529	10.2645	88

Table 5-10 Computational Time of Case 1

Model	Seed	Node	LR	Avg. Time	SA
Protection	100	8	1861	0.9305	2
		9	3710	1.855	4
		10	5615	2.8075	5
		11	9398	4.699	8
		12	16330	8.165	11
	200	8	1100	0.55	1
		9	2559	1.2795	3
		10	4759	2.3795	5
		11	9211	4.6055	8
		12	15552	7.776	11
Restoration	100	8	603	0.3015	4
		9	3022	1.511	14
		10	6349	3.1745	24
		11	10760	5.38	45
		12	19308	9.654	90
	200	8	1379	0.6895	5

		9	4213	2.1065	16
		10	7519	3.7595	27
		11	14979	7.4895	44
		12	27251	13.6255	91

Table 5-11 Computational Time of Case 2

Model	Seed	Node	LR	Avg. Time	SA
Protection	100	8	5333	2.666333	2
		9	13733	6.866667	4
		10	27093	13.54633	5
		11	58412	29.206	8
		12	152117	76.05833	11
	200	8	3996	1.998	1
		9	12037	6.018333	3
		10	23807	11.90333	5
		11	49679	24.83933	8
		12	118783	59.39167	11
Restoration	100	8	3649	1.8245	4
		9	10788	5.3941	14
		10	20535	10.2675	24
		11	66146	33.0730	45
		12	69388	34.6938	90
	200	8	2644	1.3222	5
		9	15923	7.9614	16
		10	20001	10.0004	27

		11	60975	30.4873	44
		12	93442	46.7208	91

Table 5-12 Computational Time of Case 3

According to Table 5-10, 5-11, and 5-12, the computational time of SA and LR is not significant different, which is not usual for solve complex problem like this problem. The reason of no significant computation time different between SA and LR is during solving dual problem of LR. That is because we already generate a set of decision variables of this problem, which are good reference to our primal problem, especially for decision variables x_{pj} in protection model and x_{pj}^0 in restoration model. And in SA, before tuning solution to feasible, we also have to generate x_{pj} and x_{pj}^0, which takes almost the same computational time as LR.

5.1.6 Result Discussion

According to Table 5-3, 5-4, 5-5, 5-6, 5-7, and 5-8, the results of LR are all better than SA whether on the basis of total cost or number of trunks. There are two major reasons that LR works better than SA. First, SA makes routing decision only based on implementation cost of each trunk, whereas LR make use of multipliers including the influence of O-D pair, link capacity, wavelength used, and disjoint-ness effect and path selected. The comprehensive consideration of all factors makes good performance than simplex. Second, LR is iteration-based and guaranteed to improve the result iteration by iteration. Besides, the result of each iteration can also be used as a good hint to improve the lower bound of the problem, which leads to good result of feasible solution.

5.2 Load-balancing RWA problem

To prove the correctness of our models, we examine them by the topologies of NSF network (14-node 21-link) and GTE network (12-node 25-link).

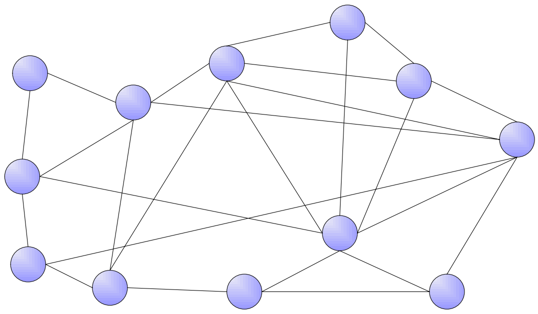

Figure 5-10 Experimental Topologies for Load-balancing RWA

5.2.1 Simple Algorithm for MinMax Problem (SA)

1. For each O-D pair *w*, apply min cost flow algorithm to calculate the working path in complete graph. We use identical cost for each trunk in the algorithm.
2. Apply the same algorithm described in section 3.6 to tune the routing solution feasible.

5.2.2 Experiment Parameters

Testing Topologies	NSF, GTE networks
Link Capacity	16 fibers
Max Lambda per Fiber	16
Number of Iteration	1000

Initial Upper Bound	1
Initial Scalar of Step Size	4
Maximum Unimprovement Counter	50
Begin to Tune	300
MinMax Tune Counter	20
Demand seed	100
Test Platform	Windows 2000 with Pentium 2G CPU

Table 5-13 Command Testing Parameters for Load-balancing RWA

5.2.3 Experiment Results

Demand	SA	UB	LB	Gap(%)
1~2	0.128906	0.105469	0.07635	38.14
1~4	0.234375	0.167969	0.12684	32.43
1~6	0.28125	0.234375	0.175137	33.82
1~8	0.348471	0.285156	0.219849	29.71
1~10	0.4375	0.378906	0.30253	25.25
1~12	0.5	0.4375	0.355408	23.10
3~12	0.640625	0.578125	0.45536	26.96
4~12	0.738281	0.632813	0.511518	23.71
5~12	0.839844	0.710938	0.565007	25.83

Table 5-14 Computational Result in GTE Networks

5.2.4 Computational Time

Demand	LR	Ave. Time	SA
1~2	172526	172.526	676
1~4	176876	176.876	939
1~6	100235	100.235	1191
1~8	151294	151.294	760
1~10	119347	119.347	1893
1~12	119858	119.858	770
3~12	188460	188.46	687
4~12	162902	162.902	774
5~12	129893	129.893	737

Table 5-15 Computational Time in GTE Networks

The computational time of SA is much larger than LR according to the result of Table 5-15. In LR, the routing decision is based on the multiplexer of each iteration, and iteration by iteration, all multipliers are auto-adjust to better and better values, which lead the routing decision close to optimal value. The value of dual problem approaches closer and closer to optimal value, so that effort required tuning solution feasible then becomes less and less. Whereas in SA, minimum hop routing decision may disperse all traffic flow and require much more effort to tune solution feasible.

5.2.5 Result Discussion

According to Table 5-14 the results of LR are all better than SA. Similar to the result of topology design, there are two major reasons that LR works better than SA. First, SA makes routing decision only based on the hop count, whereas LR make use of multipliers, which including the influence of O-D pair, link capacity, wavelength used, and disjoint-ness effect and path selected. The comprehensive consideration of all factors makes good performance than simplex. Second, LR is iteration-based and guaranteed to improve the result iteration by iteration. Besides, the result of each iteration can also be used as a good hint to improve the lower bound of the problem, which leads to good result of feasible solution.

Chapter 6 Summary and Future Work

6.1 Summary

Optical networks with WDM technology can confer enormous bandwidth on each fiber link, but may potentially cause large amount of data loss when some link fails. Therefore, design and provision of survivable WDM networks are extremely important.

Several design approaches have been proposed to solve such problems with different schemes. In this thesis, three design approaches are proposed. First, we present two approaches to design survivable optical network for fast protection scheme and path restoration scheme respectively. And the third model tries to design and solve the load-balancing routing and wavelength assignment problem, which provides more flexibility for future usage. Unlike most researches, we do not use optimization tools to solve our problems but using mathematical based solution approaches, Lagrangian relaxation method. By Lagrangian relaxation method, we decompose each problem into several easier subproblems and propose several algorithms and heuristics to optimally solve them. The result to each subproblem can provide us some hints to improve our heuristics. In terms of performance, our Lagrangian relaxation based solution has more significant result to optimal solution.

6.2 Future Work

First, there are many protection / restoration schemes fro survivable WDM networks, only two of them are proposed in this thesis. Fast protection benefits from quick reaction to failure and path restoration gains from less cost. Shared path protection, which tries to strike a balance between them, can be view as an extension of our design. Second, we only consider single link failure in our thesis. The error state introduced in chapter three can be easily extended to any combination of failure including multiple links. Third, virtual private network (VPN) is getting more and more important when considering about network design. Design survivable network would be an significant topic for VPN design problem. Fourth, we only consider about restoration and protection, respectively. For O-D pair with different survivability service level or different survivability scheme requirements, our models can be further integrated to fit the needs. Fifth, we only consider the WDM network, for the architecture of IP over WDM network, the resource required for survivability demand of IP layer would be a new issue for WDM network design.

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