國立臺灣大學資訊管理研究所碩士論文

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支援多速率訊務無線區域網路之產能最佳化 Throughput Optimization in Wireless Local Area Networks to Support Multi-Rate Traffic

研究生: 賴坤威 撰

中華民國九十三年七月

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僅將此論文獻給我最敬愛的家人與朋友



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I

論文摘要

在無線區域網路環境中,當行動裝置(Mobile station)與無線基地臺的距離相 對遙遠時,傳輸訊號將受到環境干擾而衰減,使得傳輸訊號品質愈來愈差。此時, 行動裝置會藉由調整其調變方式,也就是調降其傳輸速率,來克服此問題。當無 線區域網路處於多種傳輸速率並存之狀況時,使用傳輸速率較低的行動裝置將會 使得其它使用高速率傳輸的行動裝置的產能下降至與低速相同。之所以會有此異 常現象,在於無線區域網路中,各行動裝置擁有相同存取媒體的機率。當使用較 慢速的無線裝置取得存取媒體的使用權時,由於其傳輸速率較慢,所以必定將佔 據較多的時間來傳送資料,因此將使得使用較高速的行動裝置產能降低。

在本論文中,我們除了分析此異常問題外,同時提出兩個解決方案來解決此 問題,而這兩方案主要是籍由使用時間公平性及產能公平性的觀點來解決。我們 透過動態調整不同傳輸速率行動裝置之競爭視窗(Contention window)及傳送封包 長度來達成公平性要求。同時提出兩個非線性整數規劃模型來調整相關參數,使 得系統能夠達到產能最佳化同時滿足時間及產能公平性的要求。

Ho ?

基本上,我們所提出的方式改善了系統的效能同時也解決了效能異常的問題。我們將這兩個非線性整數規劃問題變成非線性實數規劃問題,同時使用 penalty method 搭配 gradient-based 的方法來解決此問題。在本論文中,我們做 了一些實驗來展現此解決方案的效能。另外也對一些相關參數的調整做了適當的 建議。

關鍵詞:無線區域網路,產能最佳化,公平性指標,Penalty 方法,梯度搜尋法

Π

THESIS ABSTRACT

GRADUATE INSTITUTE OF INFORMATION MANAGEMENT

NATIONAL TAIWAN UNIVERSITY

NAME: KUN-WEI LAI

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THROUGHPUT OPTIMIZATION IN WIRELESS LOCAL AREA NETWORKS TO SUPPORT MULTI-RATE TRAFFIC

In an infrastructure configuration of Wireless Local Area Networks (WLAN), a mobile station that is far away from the access point (AP) will suffer from signal fading and interference. Hence the mobile station changes it modulation scheme, i.e., it degrades its data rate, to accommodate the fading environment. In such case, a mobile station with lower bit rate will reduce the throughput of all other transmitting stations with higher bit rates since all mobile stations have the same priority to access the wireless medium. When a mobile station with lower bit seize the wireless medium, it will occupy the wireless medium a long time because of its low bit rate and hence penalize other mobile stations with higher bit rates.

In this paper, we analyze the anomaly problem and propose a solution based on two criteria: time fairness index and throughput fairness index. We dynamically assign different sizes of minimum contention window and transmitted payload lengths to stations with different data rates. Two nonlinear and integer programming problems are formulated to optimal the total system throughput in the constraints of time fairness index and throughput fairness index.

Basically, the solution improves the system performance and avoids the anomaly problem. We relax theses two problems and provide solutions based on penalty method with gradient-based approach. Some experiments are demonstrated to show the effectiveness of the proposed solutions. The guidelines for adjusting the related parameters are also presented in the paper.

Keywords: WLAN, Throughput Optimization, Fairness Index, Penalty Method, Gradient Search Algorithm



Contents

謝詞		I
論文摘要.		II
THESIS A	BSTRACT	III
Contents		V
List of Tab	les	VI
List of figu	ıres	VII
Chapter 1	Introduction	1
1.1	Background	1
1.2	Motivation	4
1.3	Proposed Approaches	5
1.4	Thesis Organization	5
Chapter 2	Throughput analysis	7
2.1	The analytical model	7
Chapter 3	Problem Formulation	15
3.1	Notation	15
3.2	Index of Fairness for Access Time and Throughput	16
3.3	Problem Formulation	18
3.4	Difficulty of Formulated Problem	19
Chapter 4	Solution Approaches	22
4.1	Penalty Method with Gradient-Based Approach	22
Chapter 5	Computational Experiments	26
5.1	Escalation factor β	26
5.2	Initial penalty multiplier	29
5.3	Initial points x ⁽⁰⁾	31
5.4	Number of mobile stations	34
Chapter 6	Discussion and Conclusion	41
References	3	42

List of Tables

Table 1-1. The throughputs of two scenarios
Table 5-1. Penalty form for $\beta = 8$
Table 5-2. Penalty form for $\beta = 2$
Table 5-3. Penalty form for $\beta = 2$. (continued)
Table 5-4. Penalty form for $\beta = 512$
Table 5-5. Summary of sequential penalty technique algorithm by using
different values of escalation factors β
Table 5-6. Summary of sequential penalty technique algorithm by using
different values of initial multiplier μ_0
Table 5-7. Summary of sequential penalty technique algorithm by using
different values of initial points x ⁽⁰⁾
Table 5-8. Initial points which obtain the most optimal at each given value
of <i>TFI</i>
Table 5-9. Number of mobile stations ($n_1=1$ $n_2=1$ $n_3=1$ $n_4=1$)
Table 5-10. Number of mobile stations (1 1 10 1, local optimal)35
Table 5-11. Number of mobile stations (1 1 10 1, second best local optimal)
Table 5-12. TFI=1 and TPFI=1
Table 5-13. TFI=1 and TPFI=0.7
Table 5-14. TFI=0.9 and TPFI=0.6

List of figures

Figure 1-1 Time diagram of channel access by two stations: one transmit	S
data with 11 Mbps and the other transmits data with 1 Mbps	3
Figure 2-1. State transition diagram of $S_k(t)$.	8
Figure 2-2. A transmission cycle	11
Figure 3-1. Non-convexity property of the problem P ₁ '	20
Figure 4-1. Sequential Penalty Technique Algorithm.	23
Figure 4-2. Gradient-Based Algorithms	24
Figure 5-1. Total throughput vs. $(L_1 L_2)$.	32
Figure 5-2. Total throughput vs. (W ₁ W ₂)	33
Figure 5-3. Time fairness index vs. Total throughput	34

2.40

Chapter 1 Introduction

1.1 Background

Recently, wireless local area networks (WLANs) are becoming increasingly popular due to the bloom of standards for WLANs. For example, the IEEE 802.11 standard [2] is the most adopted protocol recently in many hot spots, such as airports, parks, and conference rooms. The IEEE 802.11 standard specifies the physical (PHY) and medium access control (MAC) layers. Later, the IEEE 802.11b [3] and 802.11g [4] standards are emerging to further enhance the IEEE 802.11 PHY layer protocol to support the maximum data rate up to 11 Mbps and 54 Mbps, respectively. The 802.11g standard is backward compatible with 802.11b and 802.11 standards.

Higher data rates are achieved by more efficient modulation schemes. Modulation is the process of translating a data stream into a form suitable for transmitting on the physical medium. Taking digital modulation into consideration, the process involves in translating the data stream into a sequence of symbols. Each symbol is then encoded into a certain number of bits depending on its modulation scheme. The symbol sequence is then transmitted at a fixed symbol rate. Hence, when the number of encoded bits per symbol and the symbol rate is given, the data rate is determined. The performance metric of a modulation scheme is often measured by its ability to preserve the accuracy of transmitted bits. In wireless networks, the bit error rate (BER) is depending on the channel fading, path loss, and interference. Typically, this will results in low signal-to-noise ration (SNR). If a symbol is transmitted with little number of encoded bits (i.e., with high data rate) in a low quality environment, it will be hard to decode the received signal in the receiver. Therefore, there is a tradeoff between BER and data rate. That is, when the data transmitted in higher data rate, it will suffer from higher BER.

In the IEEE 802.11b standard, there are four modulation schemes whose data rates are 11 Mbps, 5.5 Mbps, 2 Mbps, and 1 Mbps, respectively, are supported to accommodate the data rate and BER in different fading environments. In an infrastructure configuration of WLAN, some mobile stations may be far away from an access point (AP). Hence, the mobile station will suffer from high BER. To react against this situation, the mobile station will select the modulation scheme with lower data rate to preserve higher BER.

Therefore, different mobile stations in the transmission range of an AP may transmit data with different data rates. A performance anomaly problem is hence emerging, as described in [5]. The performance anomaly problem is the situation that the throughput of all mobile stations transmitting at higher data rate degrades below the level of lower data rate. Such a behavior penalizes fast mobile stations and privileges the slow ones.

The reason for this anomaly is the basic CSMA/CA channel access method which

guarantees that the long term channel access probability is equal for all mobile station. When one mobile station captures the channel for a long time because its bit rate is low, it penalizes other hosts that use the higher rate. Figure 1-1 shows that the channel access of a fast mobile station and a slow one over time. The CSMA/CA protocol guarantee that the long term channel access probability of the fast stations is equal to the one of slow station. Hence, in the long run, there two stations seize the channel alternatively over time. The station with slow data rate will capture the system for a long time and cause the performance anomaly.

Table 1-1 shows the throughputs for two scenarios, which is run in *ns*-2 simulator [1]. Each scenario has two stations. Scenario 1 has two stations which transmit data at 11 Mbps. In scenario 2, one station transmits data at 11 Mbps and the other transmits at 1 Mbps. It can be seen from Table 1-1 that the performance (total throughput) of scenario 1 is more efficient than scenario 2.

In addition, IEEE 802.11b and IEEE 802.11g standard are backward compatible with IEEE 802.11 standard. Mobile stations with implementing IEEE 802.11g and IEEE 802.11b protocols transmit data with higher data rates than those ones with implementing IEEE 802.11. The performance anomaly problem also incurs and deteriorates the system performance due to the backward compatibility.



Figure 1-1 Time diagram of channel access by two stations: one transmits data with 11 Mbps and the other transmits data with 1 Mbps.

	Data ra	ate (Mb/s)	Throughput	(Mb/s)	
_	А	В	А	В	
Scenario 1	11	11	3.09	3.36	
Scenario 2	1	11	0.76	0.73	

Table 1-1. The throughputs of two scenarios

1.2 Motivation

It is desire to have a mechanism that can avoid the performance anomaly problem in the IEEE 802.11 WLANs. As seen from Figure 1-1, the anomaly problem occurs due to the channel access times of all station are different. This motivates our work for solving the anomaly problem. Therefore, the intuitive idea to conquer the problem is to try to make the access times of all stations in the system remain the same. A question will be arising: how to adjust the channel access time so that the access times of all stations will be the same? The question can be solved by tuning the MAC frame sizes of stations or the values of backoff parameters so that the access time will be the same.

In the paper, we address on the problem: *how to maximize the total system throughput with constraints that the access times of all stations are the same*. The access time is affected by two factors: MAC frame size and backoff parameters. Tuning the values of backoff parameters and MAC frame sizes for stations can determine the channel access probabilities and transmission times for different stations. In order to measure the access time for each station, we specify an index, called time fairness index. The index is used to be a quantification of access time. First, we estimate the system throughput by considering the multi-rate mobile stations. An analytical model based on renewal theory is adopted to estimate the system throughput. Second, the optimal problem formulation for maximizing the system throughput with the constraints that the access time of all stations are the same. The time fairness index is used to quantify the time access usage in the system. The time fairness index is affected by two variables, MAC frame size and backoff parameters, which are also the decision variables of the optimal problem. Finally, considering the throughput usage for multi-rate stations, we also specify an index, called throughput fairness index. Another optimal problem is derived: maximizing the total system throughput by constraining the time fairness and throughput fairness.

1.3 Proposed Approaches

The solution approaches for these two optimization problem is based on penalty method with gradient-based approach. We first relax the integer constraints of these two problems and solve the relaxed problems by developing two heuristic algorithms. Then the fractional solutions obtained from the heuristic algorithms are rounding to integers.

1.4 Thesis Organization

The rest of this paper is organized as follows. Section 2 introduces an analytical model that can estimate the total system throughput. In Section 3, the problem

formulation is presented. Section 4 gives the solution approach for the formulation. Performance evaluation is demonstrated via *ns*-2 simulator and illustrated in Section 5. We conclude the paper in Section 6.



Chapter 2 Throughput analysis

In this chapter, we introduce an analytical model to estimate the system throughput. The idea for the proposed analytical model are originated from [7] and [8].

2.1 The analytical model

The environment we consider is a single wireless cell coordinated with an AP. Each mobile station that intends to transmit a packet has to forward its packet to the access point (AP) first, even if it is destined for a mobile station located in the same cell. The communication channel is error-free and of no obstacle. Besides, there is no hidden terminal problem.

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Suppose that there are *r* traffic classes with distinct bit rate in the system, where $r \ge 1$. We use n_k to denote the number of mobile station that belongs to traffic class *k*, where $1 \le k \le r$. We refer to a packet that belongs to traffic class *k* as class-*k* packet and a mobile station that generates class-*k* packets as class-*k* station. The parameters CW_{min} and *m* (maximum backoff stage) of class-*k* station are denoted by W_k and m_k , respectively.

Suppose that each class-k packet has constant length L_k of MSDU data and the bit rate

of class-k station is R_k . Hence it takes L_k/R_k seconds for a *class-k station* to transmit a class-k packet. We consider the saturation condition [6], i.e., each mobile station always has a packet ready to transmit. The propagation delay for all packets is assumed a constant π .

A discrete and integral time scale is adopted: [t, t+1) represents a logical time unit. Each mobile station decreases its backoff counter or transmits a packet at the beginning of each logical time unit. The length of each logical time unit can be any of the following.

- the length of a time slot (δ)
- the time length required for a successful transmission
- the time length required for a colliding transmission

Suppose that a class-*k* station transmits a packet at time *t*, and let $p_k(t)$ be the collision probability. Like [6], we assume that $p_k(t)$ is constant and independent of time, i.e., $p_k(t)=p_k$ for all integers $t\geq 0$. Also let $S_k(t)$ be the backoff stage of the class-*k* station at time *t*, where $0\leq S_k(t)\leq m_{k+u}$. Since $S_k(t+1)$ depends only on $S_k(t)$, $\{S_k(t): t\geq 0\}$ is a discrete-time Markov chain and its transition diagram is depicted in Figure 2-1.



Figure 2-1. State transition diagram of $S_k(t)$.

It is easy to compute the steady-state probability distribution, denoted by S_k , as follows.

$$\Pr\{S_k = s\} = \begin{cases} \frac{1 - p_k}{1 - p_k^{m_k + u_k + 1}} & \text{if } s = 0;\\ \frac{1 - p_k}{1 - p_k^{m_k + u_k + 1}} & p_k^s & \text{if } 1 \le s \le m_k + u_k;\\ 0 & \text{if } s > m_k. \end{cases}$$
(1)

We use B_k to denote the backoff counter that the class-*k* station will be assigned, where $0 \le B_k \le 2^{m_k} W_{k,\min} - 1$. The distribution of B_k conditioning on backoff stage *s* is uniform, i.e.,

$$\Pr\{B_{k} = i \mid S_{k} = s\} = \begin{cases} \frac{1}{2^{s} W_{k}}, & \text{for } i = 0, 1, 2, \dots, 2^{s} W_{k} - 1; 0 \le s \le m_{k-1} \\ \frac{1}{2^{m_{k}} W_{k}}, & \text{for } i = 0, 1, 2, \dots, 2^{s} W_{k} - 1; m_{k} \le s \le m_{k+u} \end{cases}$$
(2)

Consequently, the average backoff counter of the class-k station in backoff stage s is computed as

$$E[B_k \mid S_k = s] = \begin{cases} \frac{2^s W_k - 1}{2}, 0 \le s \le m_{k-1} \\ \frac{2^{m_k} W_k - 1}{2}, m_k \le s \le m_{k+u} \end{cases}$$
(3)

and the average backoff counter of the class-k station, denoted by $E[B_k]$, is computed as

$$\begin{split} E[B_{k}] &= \sum_{s=0}^{m_{k}+u_{k}} E[B_{k} \mid S_{k} = s] \Pr\{S_{k} = s\} \\ &= \sum_{s=0}^{m_{k}-1} \frac{2^{s} W_{k} - 1}{2} p_{k}^{s} \frac{(1-p_{k})}{(1-p_{k}^{m_{k}+u_{k}+1})} + \sum_{s=m_{k}}^{m_{k}-1} \frac{2^{m_{k}} W_{k} - 1}{2} p_{k}^{s} \frac{(1-p_{k})}{(1-p_{k}^{m_{k}+u_{k}+1})} \\ &= \frac{1}{(1-p_{k}^{m_{k}+u_{k}+1})} \left(\frac{W_{k} - 1}{2} (1-p_{k}) + \frac{2^{W_{k}} - 1}{2} p_{k}^{1} (1-p_{k}) + \dots + \frac{2^{m_{k}-1} W_{k} - 1}{2} p_{k}^{m_{k}-1} (1-p_{k}) + \frac{2^{m_{k}} W_{k} - 1}{2} p_{k}^{m_{k}} (1-p_{k}) + \dots + \frac{2^{m_{k}} W_{k} - 1}{2} p_{k}^{m_{k}+u_{k}+1} \right) \\ &= \frac{1}{(1-p_{k}^{m_{k}+u_{k}+1})} \left(\frac{W_{k} - 1 + (2p_{k}W_{k} - p_{k}W_{k}) + \dots + (2^{m_{k}} p_{k}^{m_{k}} W_{k} - 2^{m_{k}-1} p_{k}^{m_{k}}W_{k}) - (2^{m_{k}} p_{k}^{m_{k}+u_{k}+1}W_{k} - p_{k}^{m_{k}+u_{k}+1})}{2} \right) \\ &= \frac{1}{(1-p_{k}^{m_{k}+u_{k}+1})} \left(\frac{W_{k} - 1 - (2^{m_{k}} p_{k}^{m_{k}+u_{k}+1}) + p_{k}W_{k} \sum_{l=0}^{m_{k}-1} (2p_{k})^{l}}{2} \right) \\ &= \frac{1}{(1-p_{k}^{m_{k}+u_{k}+1})} \left(\frac{(1-2p_{k})(W_{k} - 1 - 2^{m_{k}} p_{k}^{m_{k}+u_{k}+1}) + p_{k}W_{k} (1 - (2p_{k})^{m_{k}})}{2(1-2p_{k})} \right) \\ &= \frac{(1-2p_{k})(W_{k} - 1 - 2^{m_{k}} p_{k}^{m_{k}+u_{k}+1}) + p_{k}W_{k} (1 - (2p_{k})^{m_{k}})}{2(1-2p_{k})(1-p_{k}^{m_{k}+u_{k}+1})} . \end{split}$$

$$(4)$$

where the last equality holds as $p_k \neq 1/2$. If $p_k = 1/2$, $E[B_k]$ is simply given by omitting the last equality.

The class-k station has to wait $E[B_k]$ logical time units before it can transmit a packet. In other words, the probability for the class-k station to transmit a packet at any given time unit is computed as

$$q_k = \frac{1}{E[B_k] + 1}.$$
 (5)

The computation of q_k involves p_k . The latter is equal to the probability that one or more other stations transmit packets at the same logical time unit as the class-*k* station. That is,

$$p_{k} = \left(1 - (1 - q_{k})^{n_{k} - 1} \prod_{\substack{1 \le j \le r \\ j \ne k}} (1 - q_{j})^{n_{j}}\right).$$
(6)

We can solve (5) and (6) for q_k and p_k ($1 \le k \le r$) by numerical techniques.

When multiple mobile stations contend the channel at the same time, several idle

periods and several colliding transmissions will be involved before a successful transmission, as depicted in Figure 2-2. We refer to such a cycle as a *transmission cycle*. An idle period is a time interval in which the channel remains idle due to the backoff procedure. A new transmission cycle is initiated whenever a successful transmission ends.



We assume that the time lengths of all transmission cycles are independently and identically distributed. According to renewal arguments [9], at steady state, the saturation bandwidth for traffic class k, denoted by ρ_k , is given as

$$\rho_{k} = \frac{E[P_{k}]}{E[T_{I}] + E[T_{C}] + E[T_{S}]},$$
(8)

where $E[P_k]$ is the average number of bits successfully transmitted for traffic class k during a transmission cycle; $E[T_I]$, $E[T_C]$ and $E[T_S]$ are the average time lengths of all idle periods, all colliding transmission periods and the successful transmission period during a transmission cycle, respectively. The computations of $E[P_k]$, $E[T_I]$, $E[T_C]$ and $E[T_S]$ are detailed below.

 $E[P_k]$ can be computed as

$$E[P_k] = \kappa_k L_k, \tag{9}$$

where κ_k is the probability that a class-*k* packet is successfully transmitted during a transmission cycle. κ_k can be computed as

$$\kappa_{k} = \Pr\{\text{the transmitting station is a class-k station | number of transmitting stations = 1}\}$$

$$= \frac{n_{k}q_{k}(1-q_{k})^{n_{k}-1}\prod_{1 \le j \le r, j \ne k} (1-q_{j})^{n_{j}}}{\sum_{i=1}^{r} n_{i}q_{i}(1-q_{i})^{n_{i}-1}\prod_{1 \le i \le r, j \ne k} (1-q_{j})^{n_{j}}}.$$
(10)

We note that $\sum_{k=1}^{r} \kappa_k = 1$.

Let T_{PHY} , T_{MAC} , T_{ACK} , T_{RTS} and T_{CTS} be the time lengths required to transmit a physical layer header, a MAC header, an ACK, a RTS and a CTS, respectively. According to the IEEE 802.11 and 802.11e specifications, $E[T_I]$ can be computed as

$$E[T_{S}] = \sum_{i=1}^{r} \kappa_{i} (T_{PHY} + T_{RTS} + T_{SIFS} + \pi + T_{PHY} + T_{CTS} + T_{SIFS} + \pi + T_{PHY} + T_{MAC,i} + L_{i} / R_{i} + T_{SIFS} + \pi + T_{PHY} + T_{ACK} + \pi + T_{DIFS})$$
(11)

Let N_c be the number of colliding transmission periods in a transmission cycle. Since the time length of each colliding transmission period is $T_{RTS}+T_{DIFS}+\pi$, $E[T_C]$ can be computed as

$$T_{C} = E[N_{c}](T_{RTS} + T_{DIFS} + \pi),$$
(12)

where $E[N_c]$ is the average number of colliding transmission periods in a transmission cycle. Assume that p_c is the collision probability when a mobile station (regardless of any kind traffic class-*i* station, where $1 \le i \le r$) is transmitting a packet. p_c can be computed as

$$p_{c} = \Pr\{\text{number of transmitting stations} \ge 2 \mid \text{number of transmitting stations} \ge 1\}$$

$$= \frac{1 - \Pr\{\text{number of transmitting stations} = 0\} - \Pr\{\text{number of transmitting stations} = 1\}}{\Pr\{\text{number of transmitting stations} \ge 1\}}$$

$$= \frac{1 - \prod_{1 \le j \le r} (1 - q_{j})^{n_{j}} - \sum_{i=1}^{r} n_{i}q_{i}(1 - q_{i})^{n_{i}-1} \prod_{1 \le j \le r, j \ne i} (1 - q_{j})^{n_{j}}}{1 - \prod_{1 \le j \le r} (1 - q_{j})^{n_{j}}}.$$
(13)

Clearly, the distribution of N_c is given as

$$\Pr\{N_c = i\} = (1 - p_c)p_c^i, \text{ for } i = 0, 1, 2, \cdots,$$
(14)

and hence

$$E[N_c] = \frac{p_c}{1 - p_c}.$$
(15)

We assume that the time lengths of idle periods are independently and identically distributed. Let N_s be the number of time slots contained in an idle period. As shown in Figure 2-2, $E[T_I]$ can be computed as

$$E[T_{I}] = (E[N_{c}] + 1)(\delta E[N_{s}]).$$
(16)

The distribution of N_s is

$$\Pr\{N_s = i\} = \left(1 - \prod_{1 \le j \le r} (1 - q_j)^{n_j}\right) \left(\prod_{1 \le j \le r} (1 - q_j)^{n_j}\right)^i, \text{ for } i = 0, 1, 2, \cdots,$$
(17)

and hence

$$E[N_{s}] = \sum_{i=0}^{\infty} i \left(1 - \prod_{1 \le j \le r} (1 - q_{j})^{n_{j}} \right) \left(\prod_{1 \le j \le r} (1 - q_{j})^{n_{j}} \right)^{i}$$

$$= \left(\frac{\prod_{1 \le j \le r} (1 - q_{j})^{n_{j}}}{1 - \prod_{1 \le j \le r} (1 - q_{j})^{n_{j}}} \right).$$
(18)



Chapter 3 Problem Formulation

In this section, we first describe the notation used in the formulation. Second, we give a formal description about time fairness index and throughput fairness index. Third, the problem formulations are presented. Finally, we show the non-convex property of the formulated problems.

3.1 Notation

Notation	Descriptions					
p_k	Collision probability of a class-k station.					
q_k	Packet transmission Probability of a class-k station.					
f_k	The average fraction of time occupied by per class k , $0 \leq f_k \leq 1$					
$ ho_k$	The saturation bandwidth for traffic class k					
κ_k	The probability that a class-k packet is successfully transmitted					
	during a transmission cycle.					
$E[P_k]$	The average number of bits successfully transmitted for a class k					
	during a transmission cycle.					
$E[T_I]$	The average time lengths of all idle periods.					
$E[T_C]$	The average time lengths of all colliding periods.					
$E[T_S]$	The average time lengths of successful transmission during a					
	transmission cycle.					
$E[T_k]$	The average time lengths of a transmission cycle for a class k.					
$E[T_{S,k}]$	The average time lengths of successful transmission a packet during a					
	transmission cycle for a class k.					
N _c	The number of colliding transmission periods in a transmission cycle					
p_c	The collision probability when a mobile station is transmitting a					

	packet
N_s	The number of time slots contained in an idle period
TPFI	Throughput Fairness index, $0 < TPFI \le 1$

Given Parameters								
Notation	Descriptions							
r	The number of classes with distinct bit rate in the system, where $r \ge 1$.							
n_k	The number of mobile station that belongs to class <i>k</i> , where $0 \le k \le r-1$.							
m_k	maximum backoff stage of class-k station.							
u_k	The number of maximum retransmission for class-k station.							
R_k	The bit rate of class-k station.							
π	Propagation delay for all packets.							
δ	The slot time.							
TFI	Time Fairness index, $0 < TFI \le 1$							
T_{RTS} ,	The time lengths required to transmit a RTS							
T_{CTS}	The time lengths required to transmit a CTS							
T_{PHY}	The time lengths required to transmit a physical layer header							
$T_{MAC,k}$	The time lengths required to transmit a MAC header of class-k station							

Decision Variables								
Notation	Descriptions							
W_k	CW_{min} value of class-k station, where $W_{min} \leq W_k \leq W_{max}$.							
L_k	The packet length (MSDU) of class-k packet, $L_{min} \leq L_k \leq L_{max}$.							

3.2 Index of Fairness for Access Time and

Throughput

In order to quantify the access time usages for mobile stations with different data rates

in the system, we specify the time fairness index (TFI), which is defined as follows.

$$TFI = \frac{\left(\sum_{k=1}^{r} n_k f_k\right)^2}{\sum_{k=1}^{r} n_k \left(\sum_{k=1}^{r} n_k f_k^2\right)}$$

where f_k is the amount of time used for class-*k* stations (1<=k<=r). f_k is defined as follows.

$$E[T_k] = \kappa_k (E[T_I] + E[T_C] + E[T_{S,k}])$$

$$f_k = \frac{E[T_k]}{n_k \sum_{i=1}^r E[T_i]}$$

Similarly, by quantifying the throughput usage in the system, a throughput fairness

index is defined as follows.

der.	
TPFI	$= \underbrace{\left(\sum_{k=1}^{r} n_k t p_k\right)}_{(k+1)},$
5	$\sum_{k=1}^{r} n_k \left(\sum_{k=1}^{r} n_k t p_k^2 \right)$

where
$$tp_K = \frac{\rho_k}{n_k}$$

The definition of fairness index carries four characteristics, which is listing as follows.

- Population size independence: The index is applicable to any number of users (i.e., mobile stations), finite or infinite. In particular, it should be applicable to two mobile stations sharing the wireless channel.
- Scale independence: The index is independent of scale, i.e., the unit of measurement is not matter.
- 3. Bound: The index is between bounded between 0 and 1, i.e., a totally fair system has a fairness of 1 and a totally unfair system has a fairness of 0. That is, fairness can be expressed as a percentage. For example, a scheme with a fairness of 0.1

could be shown to be fair to 10% of the mobile stations and unfair to 90%.

4. Continuity: The index is continuous. Any slight change in allocation is shown up in the fairness index.

The fairness index defined here has a very intuitive interpretation. Many examples that are illustrated to explain the characteristics of the fairness index can refer to [11].

3.3 Problem Formulation

The performance anomaly problem is mainly caused by the access time of mobile stations with lower data rates penalizes the one of mobile stations with lower data rates. The first problem is formulated as

Objective function: P₁

$$Z = \max \sum_{k=1}^{r} \frac{E[P_k]}{E[T_I] + E[T_C] + E[T_S]}$$
 (total throughput)

subject to:

$$TFI = a \qquad a \in [0,1]$$

$$L_{\min} \le L_k \le L_{\max} \qquad k = 1,...,r$$

$$W_{\min} \le W_k \le W_{\max} \qquad k = 1,...,r$$

$$L_k, W_k \text{ are integers} \qquad k = 1,...,r$$

In the first formulated model P_1 , we only consider the time fairness among multiple classes. Since the objective of P_1 is to maximize the total system throughput, the model P_1 might designate higher priority to mobile stations with higher data rate than the ones with lower data rates. Hence, the throughput of mobile stations with lower data rate will be significantly less than the one of mobile stations with higher data rate. In the following formulated model, we take both the time fairness and throughput fairness in to consideration, which is formulated as

Objective function P₂

$$Z = \max \sum_{k=1}^{r} \frac{E[P_k]}{E[T_I] + E[T_C] + E[T_S]} \quad \text{(total throughput)}$$

subject to:

$$\begin{aligned} TFI &= a & a \in [0,1] \\ TPFI &= b & b \in [0,1] \\ L_{\min} &\leq L_k \leq L_{\max} & k = 1,...,r \\ W_{\min} &\leq W_k \leq W_{\max} & k = 1,...,r \\ L_k, W_k \text{ are integers } & k = 1,...,r \end{aligned}$$

3.4 Difficulty of Formulated Problem

The formulated problems P_1 is integer and nonlinear programming problem. For convenience, we first relax the integer constraint and discuss the difficulty of the relaxed problems, which are P1' problem.

Objective function: P₁'

$$Z = \max \sum_{k=1}^{r} \frac{E[P_k]}{E[T_I] + E[T_C] + E[T_S]} \quad \text{(total throughput)}$$

subject to:

$$\begin{aligned} TFI &= a & a \in [0,1] \\ L_{\min} &\leq L_k \leq L_{\max} & k = 1, ..., r \\ W_{\min} &\leq W_k \leq W_{\max} & k = 1, ..., r \end{aligned}$$

The main difficulties of the problem are the objective function and TFI=a, which are nonlinear. We show the feasible set is not a convex set. That is, the problem is intractable. A special case is indicated in Figure 3-1 to illustrate the non-convex property of problem P₁'. For simplicity and convenience, we assume that there are two classes in the system, under which the data rates for class 1 and class 2 are 11 Mbps and 1 Mbps respectively. The number of mobile stations for class 1 and class 2 are one and ten. The minimum contention window for class 1 and class 2 are fixed as 72 and 32. The TFI is fixed as 0.96.



In Figure 3-1, we tune the packet lengths for these two mobile stations and show the feasible set of the problem P1'. We find out that the feasible set has two separate regions. We choose two feasible points, which are x_1 and x_2 in Figure 3-1. Expecting x_1 and x_2 , the convex combinations of these two points are not feasible solutions. Hence the feasible set of problem P₁' is not a convex set. In other words, the problem P₁' is not a convex programming problem and is difficult. This imply that P₁ is also an intractable problem.

Similarly, relax the integer constraint of problem P_2 , which is denoted by P_2 '.

Objective function P₂'

$$Z = \max \sum_{k=1}^{r} \frac{E[P_k]}{E[T_I] + E[T_C] + E[T_S]} \quad \text{(total throughput)}$$

subject to:

$$TFI = a \qquad a \in [0,1]$$
$$TPFI = b \qquad b \in [0,1]$$
$$L_{\min} \le L_k \le L_{\max} \qquad k = 1,...,r$$
$$W_{\min} \le W_k \le W_{\max} \qquad k = 1,...,r$$

The non-convex property is also possessed in the problem P_2 ' since the feasible region is nonlinear due to *TFI*.



Chapter 4 Solution Approaches

4.1 Penalty Method with Gradient-Based Approach

In this section, we first solve the problems P_1 ' and P_2 ' by using penalty method with gradient-based approaches [10]. Then the solved solutions are then rounding to integers.

Considering the problem \mathbf{P}_1 , the penalty function is used to solve the transformed problem \mathbf{P}_1 , which is of the form:

Objective function P₁"

$$Z = \max \sum_{k=1}^{r} \frac{E[P_k]}{E[T_I] + E[T_C] + E[T_S]} - \mu (a - TFI)^2$$

subject to:

$$\begin{split} L_{\min} &\leq L_k \leq L_{\max} \qquad \quad k = 1, ..., r \\ W_{\min} &\leq W_k \leq W_{\max} \qquad \quad k = 1, ..., r \end{split}$$

where μ is the penalty multiplier. The problem \mathbf{P}_1 " can be solved by using an iterative algorithm, as shown in Figure 4-1. The penalty multiplier is iteratively increasing to

obtain the feasible solution as possible as it can. In each iteration, penalty multiplier μ is given and the optimum solution of \mathbf{P}_1 " is solved by a gradient-based method, as shown in Figure 4-2.

Formalization of principle strategy for slowly increasing the penalty multiplier μ produces the sequential penalty technique of algorithm, as shown in Figure 4-1. Multiplier μ begins relatively small and grows with each search. For each value of μ , unconstrained penalty problem is solved beginning with the optimum of the preceding search. If the result is ever feasible in the original model, we stop with an optimum. Otherwise, we continue until the unconstrained optimum is sufficiently close to feasible. The escalation factor β is used to increase the penalty multiplier.

Step0: Initialization. Form penalty model, and choose initial penalty multiplier μ₀ > 0 relatively small and starting solution x⁽⁰⁾. Also, initialize solution index t ← 0, and pick an escalation factor β > 1. (We choose β = 4 in my program)
Step1: Linear Constrained Optimization. Beginning from x^(t), solve penalty optimization problem with μ = μ_t to produce optimum x^(t+1). (Using Gradient Search Algorithm described below to solve it).
Step2: Stopping. If x^(t+1) is feasible or sufficiently close to feasible in the constrained model given, stop and output x^(t+1).
Step3: Increase. Enlarge the penalty multiplier as μ_{t+1} ← βμ_t. Then advance

 $t \leftarrow t+1$, and return to Step1.

Figure 4-1. Sequential Penalty Technique Algorithm.

Step0: Initialization. Choose any starting solution $x^{(0)}$, pick stopping tolerance $\varepsilon > 0$, and set solution index $t \leftarrow 0$. Step 1. Step Size. Choose a big number λ_{i} , which is sufficiently large, as step size satisfied all constraints as well as improving objective function value. Also, let $\Re = \{\emptyset\}$. Step 2: Gradient. Calculate objective function gradient $\nabla f(x^{(t)}) = \begin{bmatrix} \frac{\partial f(x)}{\partial x_1} & \frac{\partial f(x)}{\partial x_2} & \dots & \frac{\partial f(x)}{\partial x_{2r}} \end{bmatrix} \text{ at current point } x^{(t)}, \text{ where } (x_1, x_2, \dots, x_r) =$ (W_1, W_2, \ldots, W_r) and $(x_{r+1}, x_{r+2}, \ldots, x_{2r}) = (L_1, L_2, \ldots, L_r)$. Step 3. Stationary Point. If $\max_{1 \le i \le 2r} \left\{ \lambda_i \frac{\partial f(x)}{\partial x_i} \right\} < \varepsilon$, stop. Point $x^{(t)}$ is sufficiently close to a stationary point. Step 4. Direction. Choose gradient move direction. $\Delta x^{(t+1)} \leftarrow \nabla f(x^{(t)})$ (our model is to maximize objective function value). Step 5. New Point. Update $x^{(t+1)} \leftarrow x^{(t)} + \lambda_{t+1} \Delta x^{(t+1)}$. When $\max_{1 \le i \le 2r} \left\{ \lambda_i \frac{\partial f(x)}{\partial x_i} \right\} < \varepsilon$, select the point $x^{(t+1)} = x \in \Re$, in which f(x) is the maximum in set \Re . If $f(x^{(t+1)}) \ge f(x^{(t)}), \quad \Re = \Re \cup x^{(t+1)}, \quad \lambda_t = \frac{\lambda_t}{2}$ and return Step 4. Otherwise, $\lambda_t = \frac{\lambda_t}{2}$ and return to Step 4. Step6. Advance. Increment $t \leftarrow t+1$, and return to Step1.

Figure 4-2. Gradient-Based Algorithms

Considering the problem P_2 ', the penalty function is used to solve the transformed problem P_2 ', which is of the form:

Objective function P₂"

$$Z = \max \sum_{k=1}^{r} \frac{E[P_k]}{E[T_I] + E[T_C] + E[T_S]} - \mu[(a - TFI)^2 + (b - TPFI)^2]$$

subject to:

$$\begin{split} L_{\min} &\leq L_k \leq L_{\max} \qquad \quad k = 1, ..., r \\ W_{\min} &\leq W_k \leq W_{\max} \qquad \quad k = 1, ..., r \end{split}$$

The solutions approach to P_2 " is similar with P_1 " by using penalty function. The algorithm is the same with P_1 " and is depicted in Figure 4-1 and Figure 4-2.



Chapter 5 Computational Experiments

We consider a single wireless cell coordinated by an AP, in which multiple mobile stations contend the channel access. The physical layer is according to the IEEE 802.11b standard. There are four modulation schemes specified in IEEE 802.11b, under which four data rates (i.e., 11 Mbps, 5.5 Mbps, 2 Mbps and 1 Mbps) are possible for the data transmissions.

5.1 Escalation factor β

We first demonstrate the effect of escalation factor β . We show an example as *TFI*=0.8. Table 5-1 shows the penalty form of sequential penalty technique algorithm by using β =8. With initial multiplier $\mu = \mu_0 = 0.1$, a first search produces the local optimum

$$x^{(1)} = (32, 32, 32, 32, 41, 41, 41, 41).$$

At t = 1, the value of *TFI* is 0.23261 which violates the first constraint of the problem P₁' (i.e., *TFI*=0.8). Then the multiplier is increased by factor $\beta = 8$ and a new search is initiated from $x^{(1)}$. The resulting optimum $x^{(2)}$ starts a third search by using $\mu = \beta \mu_0$. The process continues until μ is large enough that the unconstrained optimum approaches feasibility. Hence we stop the procedure and obtain local optimal solution $x^* = (386.41, 229.5, 649.48, 35.247, 912.68, 1608.4, 2100.8, 2304)$. Similarly, Table 5-2 and Table 5-4 show the penalty forms as $\beta = 2$ and $\beta = 512$. Table 5-5 summarizes

the results of sequential penalty technique algorithm by using different values of escalation factors β . It is noted that as the value of β is higher, the convergence speed is faster. However, the initial penalty multiplier μ_0 plays an important role and should be considered jointly with escalation factors. In below, we show the effect of initial penalty multiplier.

t	μ	W1	W2	W3	W4	L1	L2	L3	L4	constraint violation	TFI value	Given TFI	Total throughput
(0 0	32	32	32	32	41	41	41	41	0.17762	0.97762	0.8	0.23261
1	0.1	800.24	1013.7	976.34	32	1487.9	1797.1	2070.1	2304	-0.39938	0.40062	0.8	5.0841
2	0.8	834.12	1024	977.39	32	1470	1791.8	2070.3	2304	-0.40412	0.39588	0.8	5.1133
3	6.4	1024	984.1	784.57	32	940.86	1620.1	2077.4	2304	-0.43544	0.36456	0.8	5.3445
4	51.2	386.96	230.59	63.267	32.001	913.04	1608.6	2100.6	2304	-0.02502	0.77498	0.8	4.2998
5	409.6	386.49	229.66	64.713	34.602	912.73	1608.4	2100.8	2304	-0.0046	0.7954	0.8	4.2156
e	3276.8	386.42	229.52	64.915	35.164	912.69	1608.4	2100.8	2304	-0.00058	0.79942	0.8	4.1984
7	26214	386.41	229.5	64.944	35.236	912.68	1608.4	2100.8	2304	-7.68E-05	0.79992	0.8	4.1962
8	2.10E+05	386.41	229.5	64.948	35.246	912.68	1608.4	2100.8	2304	-9.59E-06	0.79999	0.8	4.1959
9	1.68E+06	386.41	229.5	64.948	35.247	912.68	1608.4	2100.8	2304	-1.20E-06	0.8	0.8	4.1958
10	1.34E+07	386.41	229.5	64.948	35.247	912.68	1608.4	2100.8	2304	-1.50E-07	0.8	0.8	4.1958

Table 5-1. Penalty form for $\beta = 8$.

Table 5-2. Penalty form for $\beta = 2$.

t	Multiplier	w1	w2	w3	w4	L1	L2	L3	L4	Constraint violation	TFI	Given TFI	Total throughput
0	0	32	32	32	32	41	41	41	41	0.17762	0.97762	0.8	0.23261
1	0.1	800.24	1013.7	976.34	32	1487.9	1797.1	2070.1	2304	-0.39938	0.40062	0.8	5.0841
2	0.2	833.1	1024	977.92	32	1471.9	1792.3	2070.3	2304	-0.40394	0.39606	0.8	5.1121
3	0.4	1024	1024	988.32	32	1364.1	1756.8	2071.8	2304	-0.42388	0.37612	0.8	5.239
4	0.8	1024	1024	988.32	32	1364.1	1756.8	2071.8	2304	-0.42388	0.37612	0.8	5.239
5	1.6	1024	1024	988.32	32	1364.1	1756.8	2071.8	2304	-0.42388	0.37612	0.8	5.239
6	3.2	1024	1024	988.32	32	1364.1	1756.8	2071.8	2304	-0.42388	0.37612	0.8	5.239
7	6.4	1024	1024	988.32	32	1364.1	1756.8	2071.8	2304	-0.42388	0.37612	0.8	5.239
8	12.8	881.19	675.52	747.79	75.5	1318.2	1740.3	2077	2304	-0.17372	0.62628	0.8	3.906
9	25.6	966.73	921.47	118.36	60.115	1273.2	1723	2089.7	2304	-0.10046	0.69954	0.8	4.2911

10	51.2	917.45	910.71	129.97	73.771	1266.2	1720.6	2094.2	2304	-0.05282	0.74718	0.8	4.062
11	102.4	909.7	861.78	147.6	82.264	1259.2	1718.3	2098.3	2304	-0.02304	0.77696	0.8	3.9353
12	204.8	902.24	845.86	151.38	85.072	1256.5	1717.3	2100	2304	-0.01289	0.78711	0.8	3.8931
13	409.6	892.24	825.19	151.3	86.494	1252.8	1716	2102.3	2304	-0.00571	0.79429	0.8	3.8674
14	819.2	891.86	817.94	149.71	87.051	1251.8	1715.7	2102.9	2304	-0.0038	0.7962	0.8	3.8596
15	1638.4	845.19	774.3	137.32	83.433	1248.1	1714.4	2105.3	2304	-0.00263	0.79737	0.8	3.8745
16	3276.8	842.51	769.23	144.44	82.912	1246.9	1713.9	2106	2304	-0.0007	0.7993	0.8	3.876
17	6553.6	842.97	768.75	144.29	83.045	1246.8	1713.9	2106.1	2304	-0.00042	0.79958	0.8	3.8745
18	13107	831.25	753.69	137.28	82.287	1245.1	1713.3	2107.1	2304	-0.00023	0.79977	0.8	3.8768
19	26214	830.9	753.02	137.02	82.298	1245	1713.3	2107.1	2304	-0.00015	0.79985	0.8	3.8763
20	52429	827.96	753.23	145.96	81.305	1244.6	1713.1	2107.4	2304	-6.26E-05	0.79994	0.8	3.8844
21	1.05E+05	827.91	753.54	146.39	81.317	1244.6	1713.1	2107.4	2304	-2.36E-05	0.79998	0.8	3.8842
22	2.10E+05	827.75	753.52	146.54	81.302	1244.6	1713.1	2107.5	2304	-2.55E-05	0.79997	0.8	3.8843
23	4.19E+05	827.8	753.35	146.69	81.296	1244.5	1713.1	2107.5	2304	-5.57E-06	0.79999	0.8	3.8843
24	8.39E+05	827.79	753.33	146.68	81.296	1244.5	1713.1	2107.5	2304	-2.82E-06	0.8	0.8	3.8843
25	1.68E+06	827.78	753.33	146.67	81.296	1244.5	1713.1	2107.5	2304	-8.78E-07	0.8	0.8	3.8843
26	3.36E+06	827.78	753.32	146.67	81.296	1244.5	1713.1	2107.5	2304	-7.77E-07	0.8	0.8	3.8843
27	6.71E+06	827.75	753.3	146.67	81.294	1244.5	1713.1	2107.5	2304	-3.79E-07	0.8	0.8	3.8843
28	1.34E+07	827.75	753.3	146.67	81.294	1244.5	1713.1	2107.5	2304	-2.03E-07	0.8	0.8	3.8843
29	2.68E+07	827.75	753.3	146.67	81.293	1244.5	1713.1	2107.5	2304	-8.19E-08	0.8	0.8	3.8843

Table 5-3. Penalty form for $\beta = 2$. (continued)



Table 5-4. Penalty form for $\beta = 512$.

Τ	Multiplier	w1	w2	w3	w4	L1	L2	L3	L4	constraint violation	TFI	Given TFI	Total throughput
0	0	32	32	32	32	41	41	41	41	0.17762	0.97762	0.8	0.23261
1	0.1	800.24	1013.7	976.34	32	1487.9	1797.1	2070.1	2304	-0.39938	0.40062	0.8	5.0841
2	51.2	628.32	505.03	400.93	75.228	1430	1797	2132	2304	-0.01818	0.78182	0.8	3.6003
3	26214	627.37	505.17	401.87	79.429	1429.9	1797	2132	2304	-9.20E-05	0.79991	0.8	3.5303
4	1.34E+07	627.41	505.15	401.9	79.453	1429.9	1797	2132	2304	-1.62E-07	0.8	0.8	3.5299
5	6.87E+09	627.41	505.15	401.9	79.453	1429.9	1797	2132	2304	-3.64E-10	0.8	0.8	3.5299

	Total	penalty				Converge	ence poin	t			Constraint	TFI	Total
β	rotai	pondity				eonioige	nee pom				constraint		roun
	loop	multiplier	w1	w1	w3	w4	L1	L2	L3	L4	Violation	value	Throughput
2	29	2.68E+07	827.75	753.3	146.67	81.293	1244.5	1713.1	2107.5	2304	-8.19E-08	0.8	3.8843
4	15	2.68E+07	719.43	299.95	381.65	72.836	1312.5	1742.2	2090.4	2304	-8.61E-08	0.8	3.5413
8	11	1.07E+08	386.41	229.5	64.948	35.247	912.68	1608.4	2100.8	2304	-1.88E-08	0.8	4.1958
16	9	4.30E+08	656.92	797.86	866.03	137.86	1401.6	1784	2095.8	2304	-2.82E-09	0.8	2.9468
32	7	1.07E+08	501.73	153.58	160.39	37.621	1440.7	1784.9	2073.7	2304	-1.10E-08	0.8	3.837
64	6	1.07E+08	829.42	404.98	795.5	537.94	859.84	1595.7	2085	2304	-4.12E-08	0.8	1.9453
128	6	3.44E+09	391.11	801.13	294.39	291.06	1451.9	1788.3	2075.4	2304	9.44E-10	0.8	2.1779
256	5	4.30E+08	310.09	316.02	86.178	316.38	1386.9	1762.6	2140.8	2304	-1.99E-09	0.8	2.5377
512	5	6.87E+09	627.41	505.15	401.9	79.453	1429.9	1797	2132	2304	-3.64E-10	0.8	3.5299
1024	4	1.07E+08	853.25	980.49	522.21	125.15	1475	1797.1	2077.1	2304	-1.42E-08	0.8	3.2786
2048	4	8.59E+08	393.27	435.58	521.38	284.1	1466.8	1795.9	2077.6	2304	1.06E-10	0.8	1.9968
4096	4	6.87E+09	408.06	640.59	637.01	156.06	1477.2	1797.3	2075.2	2304	-2.23E-09	0.8	2.4762

Table 5-5. Summary of sequential penalty technique algorithm by using different values of escalation factors β .

5.2 Initial penalty multiplier

In Table 5-6, we show the effect of initial penalty multiplier with $\beta = 8$. As seen from Table 5-6, as the values of initial penalty multiplier is less than 1, there are small impacts on the optimal value (i.e., total system throughput) and the convergence speed, as shown in the total throughput and total step columns in Table 5-6. As the value of initial penalty multiplier is lager enough (e.g., ≥ 10), the convergence speeds are fast, as depicted in total step column of Table 5-6. However, the values of optimal system total throughputs are small.

initial multiplier	total step	Penalty multiplier	w1	W1	w3	w4	L1	L2	L3	L4	Constraint Violation	TFI	Given TFI	Total Throughput
0.0001	15	4.40E+08	718.5	550.53	118.22	67.621	1158.1	1631.3	2051.4	2304	-4.20E-09	0.8	0.8	3.968
0.001	13	6.87E+07	381.52	182.59	334.56	50.409	1184.1	1649.8	2043.1	2304	-2.84E-08	0.8	0.8	3.5219
0.01	12	8.59E+07	408.75	250.18	78.918	254.57	1328.6	1727.6	2140.4	2304	-2.58E-09	0.8	0.8	2.7449
0.1	11	1.07E+08	386.41	229.5	64.948	35.247	912.68	1608.4	2100.8	2304	-1.88E-08	0.8	0.8	4.1958
1	10	1.34E+08	628.1	565.93	99.164	55.089	1442.1	1834.1	2055.4	2304	-1.63E-08	0.8	0.8	4.0309
10	9	1.68E+08	705.39	106.82	99.657	71.826	131.04	283.03	1017.5	1308.2	-8.06E-09	0.8	0.8	2.8179
100	7	2.62E+07	317.09	69.38	47.651	35.998	44.71	275.64	528.42	593.81	-6.03E-08	0.8	0.8	1.9756
1000	5	4.10E+06	254.61	465.5	130.72	447.81	87.647	349.38	329.52	268.61	-1.68E-08	0.8	0.8	0.8811
10000	7	2.62E+09	296.09	63.143	47.495	96.722	41.08	41.236	41.293	41.158	9.77E-08	0.8	0.8	0.22935
100000	4	5.12E+07	407.48	66.856	66.715	110.98	41.04	41.238	41.251	41.155	3.02E-08	0.8	0.8	0.22177
1000000	2	8.00E+06	218.62	292.62	74.464	152.16	41.007	41.005	41.02	41.011	-4.81E-08	0.8	0.8	0.2047
1000000	1	1.00E+07	138.98	246.52	49.24	77.419	41.001	41.001	41	41.001	6.14E-08	0.8	0.8	0.22433

Table 5-6. Summary of sequential penalty technique algorithm by using different values of initial multiplier μ_0



5.3 Initial points x⁽⁰⁾

In Table 5-7, we show the effect of initial points $x^{(0)}$. The eight decision variables $(W_1 W_2 W_3 W_4 L_1 L_2 L_3 L_4)$ are assigned with end points for initial feasible points. For eight decision variables, we have $2^8 = 256$ combinations. Due to the page limit, we select sixteen combinations for comparison. It is remarkable that the practically implementation should compare all the 256 combinations.

			Ι	nitia	l poin	t						Optimal	l point				Total	TFI
No	wl	w2	w3	w4	L1	L2	L3	L4	w1	w2	w3	w4	L1	L2	L3	L4	Throughput	111
1	32	32	32	32	41	41	41	41	386.41	229.5	64.948	35.247	912.68	1608.4	2100.8	2304	4.1958	0.8
2	32	32	32	32	41	41	2304	2304	534.87	504.13	444.52	245.05	41	41	2304	2304	3.196	0.8
3	32	32	32	32	2304	2304	41	41	655.48	368.73	167.17	35.997	2226.4	2303.8	355.02	689.94	2.0875	0.8
4	32	32	32	32	2304	2304	2304	2304	951.83	844.66	190.15	94.479	1136.1	1834	2304	2304	3.8452	0.8
5	32	32	1024	1024	41	41	41	41	435.92	266.62	166.72	35.927	1200.5	2304	245.63	644.59	1.9938	0.8
6	32	32	1024	1024	41	41	2304	2304	207.37	118.09	538.92	815.4	41	112.38	2304	2304	1.5091	0.8
7	32	32	1024	1024	2304	2304	41	41	885.42	516.57	648.88	86.206	2140.1	2304	193.8	855.08	1.8968	0.8
8	32	32	1024	1024	2304	2304	2304	2304	945.49	425.85	102.38	47.844	1916.4	2211.2	2304	2304	4.1498	0.8
9	1024	1024	32	32	41	41	41	41	253.42	725.2	319.21	199.14	41	63.839	2108.6	2304	3.2569	0.8
10	1024	1024	32	32	41	41	2304	2304	920.28	471.31	553.52	297.78	41	41	2304	2304	3.0208	0.8
11	1024	1024	32	32	2304	2304	41	41	319.13	131.84	32.711	203.8	2070.9	2264.5	1113.7	2304	2.3229	0.8
12	1024	1024	32	32	2304	2304	2304	2304	556.2	537.86	919.39	224.51	41	1443.8	2304	2304	2.8979	0.8
13	1024	1024	1024	1024	41	41	41	41	506.45	418.32	867.49	113.97	1176.3	1402	1536.2	2304	2.9588	0.8
14	1024	1024	1024	1024	41	41	2304	2304	119.42	421.72	201.78	110.67	41	41	2304	2304	3.5431	0.8
15	1024	1024	1024	1024	2304	2304	2304	2304	746.61	779.86	122.12	54.947	2173.9	2273.6	1961.8	2304	3.9595	0.8
16	1024	1024	1024	1024	2304	2304	2304	2304	971.38	868.91	122.04	64.238	2216.1	2284.9	2304	2304	3.9922	0.8

Table 5-7. Summary of sequential penalty technique algorithm by using different values of initial points $x^{(0)}$.

The reason we select end points for decision variables of packet length and contention window is that the total system throughput has the monotonic property. For simplicity, we assume there are two classes. Class 1 is for 11 Mbps and class 2 is for 1 Mbps. As observed from Figure 5-1, we show that the total system throughput versus the packet lengths, for which contention window is (32 32) and $(n_1, n_2) = (10, 10)$. In Figure 5-2, we show that the total system throughput versus the contention window, for which packet lengths is (2304, 2304) and $(n_1, n_2) = (10, 10)$. Figure 5-1 and Figure 5-2 show

that the global optimum incurs in end points.

As seen from Table 5-7, the selection of initial feasible points has great impact on the optimal value. With different initial feasible points, the resulting optimal points and optimal values has great range of difference.



Figure 5-1. Total throughput vs. $(L_1 L_2)$.



From Section 5.1 to Section 5.3, we discuss the effect of parameters used in the proposed penalty function with gradient-based method. In Figure 5-3, we show all the derived suboptimal solutions (i.e., total system throughputs) versus the values of *TFI*, where $\beta = 8$, $\mu_0 = 0.1$, $(n_1, n_2, n_3, n_4) = (1, 1, 1, 1)$. In Table 5-8, we show the initial points that obtain the most optimal at each given value of *TFI*.



Table 5-8. Initial points which obtain the most optimal at each given value of TFI

TFI value	total throughput	w1	w2	w3	w4	L1	L2	L3	L4
1	3.1705	32	32	1024	1024	2304	2304	2304	2304
0.9	3.7382	1024	1024	1024	1024	2304	2304	2304	2304
0.8	4.1958	32	32	32	32	41	41	41	41
0.7	4.4711	32	32	1024	1024	2304	2304	2304	2304
0.6	4.7643	1024	1024	1024	1024	2304	2304	2304	2304
0.5	5.2094	1024	1024	32	32	41	41	2304	2304
0.4	5.1398	1024	1024	32	32	41	41	41	41
0.3	5.7924	1024	1024	32	32	41	41	2304	2304

5.4 Number of mobile stations

In Table 5-9, Table 5-10 and Table 5-11, we study the impact on local optimum caused by number of mobile stations. We study three scenarios where the related values of

parameters are the same with Table 5-7 excepting the number of mobile stations. The initial feasible points for scenario 1 and scenario 2 are selected from the point that results in the best local optimum, i.e., as seen from Table 5-7, the point is (32, 32, 32, 32, 41, 41, 41, 41).

For scenario 3, the initial feasible point is selected as the second best local optimum, i.e., as seen from Table 5-7, the point is (32, 32, 1024, 1024, 2304, 2304, 2304, 2304). Table 5-9, Table 5-10 and Table 5-11 show the penalty forms for these three scenarios. As seen form Table 5-10 and Table 5-11, the resulting local optimum of scenario 2 is less than the one of scenario 3. Hence, as the number of mobile stations is changing, the 256 combinations should be re-calculated to obtain the "best" initial feasible point since the best initial feasible point for $(n_1, n_2, n_3, n_4) = (1, 1, 1, 1)$ is not the best initial feasible point for $(n_1, n_2, n_3, n_4) = (1, 1, 1, 1)$.

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t	mu	w1	w2	w3	w4	L1	L2	L3	L4	constraint violation	TFI_value	given_TFI	Total_throughput
0	0	32	32	32	32	41	41	41	41	0.17762	0.97762	0.8	0.23261
1	0.1	800.24	1013.7	976.34	32	1487.9	1797.1	2070.1	2304	-0.39938	0.40062	0.8	5.0841
2	0.8	834.12	1024	977.39	32	1470	1791.8	2070.3	2304	-0.40412	0.39588	0.8	5.1133
3	6.4	1024	984.1	784.57	32	940.86	1620.1	2077.4	2304	-0.43544	0.36456	0.8	5.3445
4	51.2	386.96	230.59	63.267	32.001	913.04	1608.6	2100.6	2304	-0.02502	0.77498	0.8	4.2998
5	409.6	386.49	229.66	64.713	34.602	912.73	1608.4	2100.8	2304	-0.0046	0.7954	0.8	4.2156
6	3276.8	386.42	229.52	64.915	35.164	912.69	1608.4	2100.8	2304	-0.00058	0.79942	0.8	4.1984
7	26214	386.41	229.5	64.944	35.236	912.68	1608.4	2100.8	2304	-7.68E-05	0.79992	0.8	4.1962
8	2.10E+05	386.41	229.5	64.948	35.246	912.68	1608.4	2100.8	2304	-9.59E-06	0.79999	0.8	4.1959
9	1.68E+06	386.41	229.5	64.948	35.247	912.68	1608.4	2100.8	2304	-1.20E-06	0.8	0.8	4.1958

Table 5-9. Number of mobile stations ($n_1=1 n_2=1 n_3=1 n_4=1$)

Table 5-10. Number of mobile stations (1 1 10 1, local optimal)

t	Mu	w1	w2	w3	w4	L1	L2	L3	L4	constraint violation	TFI_value	given_TFI	Total_throughput
(0	32	32	32	32	41	41	41	41	0.18847	0.98847	0.8	0.25534
1	0.1	429.46	661.11	32	920.87	77.868	152.86	2304	259.37	-0.0219	0.7781	0.8	4.0522
2	0.8	429.46	661.11	32	920.87	77.868	152.86	2304	259.37	-0.0219	0.7781	0.8	4.0522
3	6.4	429.46	661.11	32	920.87	77.868	152.86	2304	259.37	-0.0219	0.7781	0.8	4.0522
2	51.2	432.18	660.99	32	920.05	41	144.96	2304	263.01	-0.02264	0.77736	0.8	4.0537
5	409.6	130.32	574.99	50.248	872.69	41	58.432	2304	300.99	-0.00367	0.79633	0.8	3.9994
e	3276.8	127.08	574.73	54.977	872.52	41	58.106	2304	301.12	-0.00056	0.79944	0.8	3.9893
7	26214	126.62	574.71	55.748	872.51	41	58.067	2304	301.14	-6.03E-05	0.79994	0.8	3.9876

8	3 2.10E+05	126.56	574.71	55.824	872.51	41	58.062	2304	301.14	-7.67E-06	0.79999	0.8	3.9874
ç	0 1.68E+06	126.55	574.71	55.834	872.51	41	58.062	2304	301.14	-9.06E-07	0.8	0.8	3.9874
10	0 1.34E+07	126.55	574.71	55.836	872.51	41	58.062	2304	301.14	-1.20E-07	0.8	0.8	3.9874
1	1.07E+08	126.55	574.71	55.836	872.51	41	58.062	2304	301.14	-1.51E-08	0.8	0.8	3.9874

Table 5-11. Number of mobile stations (1 1 10 1, second best local optimal)

t	mu	w1	w2	w3	w4	L1	L2	L3	L4	constraint violation	TFI_value	given_TFI	Total_throughput
0	0	32	32	1024	1024	2304	2304	2304	2304	-0.64642	0.15358	0.8	1.3393
1	0.1	1024	1013.5	665.49	974.78	2229.1	2291.4	2304	2304	0.053958	0.85396	0.8	3.0493
2	0.8	1024	1024	577.59	966.27	2215.8	2287.9	2304	2304	0.087753	0.88775	0.8	3.1466
3	6.4	1024	1024	32	930.74	2115.4	2261.1	2304	2304	-0.00114	0.79886	0.8	4.0266
4	51.2	1024	1024	32.005	110.49	555.54	1740.1	2304	2304	0.006741	0.80674	0.8	4.0885
5	409.6	1024	1024	32	110.49	555.54	1740.1	2304	2304	0.006735	0.80673	0.8	4.0885
6	3276.8	1024	1024	36.618	205.62	555.33	1740.1	2304	2304	-3.44E-05	0.79997	0.8	4.0686
7	26214	1024	1024	36.644	205.48	555.31	1740.1	2304	2304	-2.32E-06	0.8	0.8	4.0686
8	2.10E+05	1024	1024	36.644	205.46	555.31	1740.1	2304	2304	-3.73E-07	0.8	0.8	4.0686
9	1.68E+06	1024	1024	36.645	205.46	555.31	1740.1	2304	2304	-3.37E-08	0.8	0.8	4.0686
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Similarly, considering the problem P_2 ', the salutation approaches are the same by comparing with solving the problem P_1 '. In addition, the convergence property is also the same with problem P_1 '. We show some penalty forms in Table 5-12, Table 5-13 and Table 5-14, in which $\beta = 8$, $\mu_0 = 1$, $(n_1, n_2, n_3, n_4) = (1, 1, 1, 1)$, and $x^{(0)} = (1024, 1024512324141412304)$.

Table 5-12. TFI=1 and TPFI=1.

t	Multiplier	w1	w2	w3	w4	L1	L2	L3	L4	constraint violation of TFI	TFI	Given TFI	constraint violation of TPFI	TFI value	Given TFI	Total throughput
0	1	1024	1024	512	32	41	41	41	2304	-0.72229	0.27771	1	-0.74998	0.25002	1	5.7971
1	1	1024	1024	1024	32	41	41	35.429	2304	-0.72864	0.27136	1	-0.74977	0.25023	1	5.8558
2	8	339.11	999.22	427.64	466.38	126.76	143.26	1637	2304	-0.13025	0.86975	1	-0.45034	0.54966	1	2.2742
3	64	522.18	1023.6	646.47	655.22	465.7	305.41	1658.7	2244.1	-0.14341	0.85659	1	-0.32844	0.67156	1	1.82
4	512	937.96	449.28	759.71	899.52	763.34	600.62	1601.5	2149.1	-0.09484	0.90516	1	-0.12293	0.87707	1	1.5951
5	4096	631.3	377.77	553.91	664.8	881.42	714.22	1539.2	2115.5	-0.11384	0.88616	1	-0.0849	0.9151	1	1.6848
6	32768	1023.9	629.55	658.27	901.8	1123.7	923.81	1422.7	2000	-0.08721	0.91279	1	-0.06899	0.93101	1	1.5683
7	2.62E+05	987.7	599.94	560.5	755.54	1179.8	973.21	1392.4	1965.7	-0.07382	0.92618	1	-0.08166	0.91834	1	1.6542
8	2.10E+06	829.38	505.36	349.43	504.43	1453.8	1183.9	1220.1	1790.4	-0.07691	0.92309	1	-0.07085	0.92915	1	1.7745
9	1.68E+07	993.62	511.09	344.29	427.24	1722.3	1353.7	1047.7	1523.5	-0.06517	0.93483	1	-0.05669	0.94331	1	1.732
10	1.34E+08	1018.5	494.11	230.95	277.82	2069	1515.9	789.89	1014.5	-0.04599	0.95401	1	-0.04093	0.95907	1	1.6688
11	1.07E+09	1016.9	518.83	189.7	170.95	2258.2	1553.9	604.33	599.45	-0.02435	0.97565	1	-0.02624	0.97376	1	1.5264
12	8.59E+09	1014.2	492.18	172.4	147.23	2304	1550	547.16	479.77	-0.01886	0.98114	1	-0.0193	0.9807	1	1.4599
13	6.87E+10	996.82	485.15	121.68	103.85	2304	1469.5	338.13	280.25	-0.00912	0.99088	1	-0.00937	0.99063	1	1.2433
14	5.50E+11	988.36	431.87	109.79	94.373	2304	1283	290.19	250.27	-0.00847	0.99153	1	-0.00743	0.99257	1	1.1833
15	4.40E+12	1016.2	368.66	109.94	97.064	2304	1037.7	278.63	241.89	-0.00671	0.99329	1	-0.00604	0.99396	1	1.1495
16	3.52E+13	1024	311.13	109.51	96.986	2304	841.18	269.03	234.61	-0.00515	0.99485	1	-0.00449	0.99551	1	1.1196
17	2.81E+14	1023.1	213.97	100.81	89.762	2304	537.1	238.64	208.9	-0.00172	0.99828	1	-0.00181	0.99819	1	1.0395
18	2.25E+15	1008.3	166.38	92.256	82.469	2304	394.71	213.26	188.53	-0.00035	0.99965	1	-0.00039	0.99961	1	0.9732
19	1.80E+16	1009	146.71	89.074	79.689	2304	332.8	198.45	176.84	-1.93E-05	0.99998	1	-1.67E-05	0.99998	1	0.93085
20	1.44E+17	1010.4	142.94	88.231	79.051	2304	320.73	195.12	174.16	-8.61E-07	1	1	-7.45E-07	1	1	0.92123
21	1.15E+18	1010.7	142.15	88.038	78.927	2304	318.22	194.41	173.58	-2.99E-08	1	1	-2.55E-08	1	1	0.91916

t	Multiplier	w1	w2	w3	w4	L1	L2	L3	L4	constraint violation of TFI	TFI	Given TFI	constraint violation of TPFI	TFI value	Given TFI	Total throughput
0	1	1024	1024	512	32	41	41	41	2304	-0.72229	0.27771	1	-0.44998	0.25002	0.7	5.7971
1	1	1024	1024	1024	32	41	41	10.093	2304	-0.72882	0.27118	1	-0.44993	0.25007	0.7	5.8559
2	8	48.024	141.93	46.14	32	41	6.478	1904.8	2304	-0.30122	0.69878	1	-0.23725	0.46275	0.7	4.2269
3	64	32	139.17	57.523	51.303	1.9936	6.8485	1905.1	2303.9	-0.19263	0.80737	1	-0.21024	0.48976	0.7	3.4104
4	512	1023.2	472.35	539.77	783.43	503.14	458.88	1929.6	2253.9	-0.05729	0.94271	1	0.004076	0.70408	0.7	1.9604
5	4096	695.65	540.09	559.5	553.12	510.71	466.75	1930.8	2250.4	-0.01125	0.98875	1	-0.00142	0.69858	0.7	2.0674
6	32768	721.52	482.58	545.26	488.35	540.68	507.99	1933.3	2233.1	-0.00375	0.99625	1	-0.00022	0.69978	0.7	2.1358
7	2.62E+05	754.95	476.27	552.83	461.54	567.59	544.22	1934.5	2217.2	-0.00196	0.99804	1	-0.00125	0.69875	0.7	2.1647
8	2.10E+06	818.35	525.03	591.4	498.65	574.79	553.94	1934.9	2212.6	-0.00162	0.99838	1	-0.00013	0.69987	0.7	2.1228
9	1.68E+07	812.37	529.04	578.36	475.14	597.12	583.93	1936.4	2197.8	-0.00095	0.99905	1	0.000157	0.70016	0.7	2.1496
10	1.34E+08	804.92	538.05	578.59	472.72	600.65	588.58	1936.7	2195.3	-0.00083	0.99917	1	-0.00041	0.69959	0.7	2.1508
11	1.07E+09	802.74	534.83	579.61	469.67	602.54	591.07	1936.9	2194	-0.0008	0.9992	1	-0.00013	0.69987	0.7	2.1518
12	8.59E+09	864.21	574.56	606.63	478.57	627.8	624.23	1939.4	2175.5	-0.00025	0.99975	1	-9.14E-06	0.69999	0.7	2.1401
13	6.87E+10	872.3	581.6	607.19	479.29	631.93	629.61	1939.9	2172.4	-0.00019	0.99981	1	-1.58E-06	0.7	0.7	2.1406
14	5.50E+11	897.75	593.5	618.94	487.63	635.01	633.62	1940.2	2170	-0.00013	0.99987	1	-2.26E-06	0.7	0.7	2.1313
15	4.40E+12	907.85	602.8	627.45	490.83	638.64	638.36	1940.7	2167.2	-8.81E-05	0.99991	1	-1.02E-06	0.7	0.7	2.1253

Table 5-13. TFI=1 and TPFI=0.7.

t	Multiplier	w1	w2	w3	w4	L1	L2	L3	L4	constraint violation of TFI	TFI	Given TFI	constraint violation of TPFI	TFI value	Given TFI	Total throughput
0	1	1024	1024	512	32	41	41	41	2304	-0.62229	0.27771	0.9	-0.34998	0.25002	0.6	5.7971
1	1	1024	1024	1024	32	41	41	1.2165	2304	-0.62888	0.27112	0.9	-0.34998	0.25002	0.6	5.8559
2	8	676.48	672.26	45.192	32	41	41	1452.4	2304	-0.38315	0.51685	0.9	-0.16759	0.43241	0.6	4.9165
3	64	526.77	609.47	460.61	647.06	25.03	39.022	1523	2304	0.003585	0.90359	0.9	-0.08444	0.51556	0.6	2.1055
4	512	174.02	276.02	194.68	290.73	160.72	148.04	1524.3	2304	0.002802	0.9028	0.9	-0.00831	0.59169	0.6	2.3758
5	4096	162.8	278.96	204.65	289.7	166.88	152.84	1524.1	2304	0.000232	0.90023	0.9	0.000274	0.60027	0.6	2.3198
6	32768	162.81	279.5	203.55	288.17	167.3	153.22	1524.1	2304	-4.84E-05	0.89995	0.9	-7.97E-05	0.59992	0.6	2.3256
7	2.62E+05	162.84	279.44	203.56	288.14	167.45	153.36	1524.2	2304	-5.26E-06	0.89999	0.9	-2.05E-06	0.6	0.6	2.3255
8	2.10E+06	162.79	279.2	203.36	288.01	167.47	153.38	1524.2	2304	3.75E-06	0.9	0.9	-2.59E-06	0.6	0.6	2.3259
9	1.68E+07	162.79	279.2	203.35	288.01	167.47	153.38	1524.2	2304	2.35E-08	0.9	0.9	1.60E-08	0.6	0.6	2.3259
10	1.34E+08	162.79	279.2	203.35	288.01	167.47	153.38	1524.2	2304	2.38E-08	0.9	0.9	-2.51E-08	0.6	0.6	2.3259
11	1.07E+09	162.79	279.2	203.35	288.01	167.47	153.38	1524.2	2304	5.37E-09	0.9	0.9	-5.95E-10	0.6	0.6	2.3259
12	8.59E+09	162.79	279.2	203.35	288.01	167.47	153.38	1524.2	2304	4.89E-10	0.9	0.9	-4.71E-10	0.6	0.6	2.3259

The solutions obtained from P_1 ' and P_2 ' are floating number which violate the integer constraints of original problems P_1 and P_2 . The rounding method we adopt is greedy to compare all the possible rounding solutions and choose the point with the largest optimum value or the one with the most feasible value.



Chapter 6 Discussion and Conclusion

In this paper, we addressed on the performance anomaly problem which is arising due to multiple rate support in IEEE 802.11. Mobile stations transmitting with different data rates are categorized into multiple traffic classes. In order to avoid the performance anomaly problem, we characterized the access time usage and throughput usage in the system. With these characteristics, we tuned the access time usage and throughput usage by adjusting the minimum contention window sizes and packet lengths among multiple classes. Two nonlinear and integer programming problems were formulated to solve the performance anomaly problem. The solution approaches were based on penalty function. We showed the convergence analysis and some examples were also demonstrated to illustrate the effects of parameters. To our best knowledge, there is no recent publication addressed on the solution of performance anomaly problem.

In the further research, there are two research directions. On the one hand, in addition to access time and throughput criteria, the delay criterion or other QoS-related criterion can be derived. These criteria can be used to bind to the constraints range and more realistic solution can be obtained. On the other hand, the proposed capacity estimation model can be used as information for system administrators to manage the radio resources. The resource management problem in WLAN is a novel topic in future high-speed WLAN. It is desire to design a resource management mechanism so that the wireless resources could be effectively utilized.

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t	w1	w2	w3	w4	L1	L2	L3	L4	function value	gradient of w1	gradient of w2	gradient of w3	gradient of w4	gradient of L1	gradient of L ₂	gradient of L3	gradient of L4	randa (step size)
0	32	32	32	32	41	41	41	41	0.22945	0.000241	-3.31E-05	-0.00023	-0.0003	0.001088	0.001253	0.001358	0.001388	0
1	374.05	32	32	32	1587.7	1822.1	1971.3	2013.9	2.8893	0.000352	0.021292	-0.00898	-0.0184	-7.19E-05	-0.00022	0.000231	0.000359	1.42E+06
2	389.7	977.56	32	32	1584.5	1812.5	1981.6	2029.9	4.1078	0.00106	7.39E-05	0.002655	-0.02417	-0.00023	-3.08E-05	0.000248	0.000615	44409
3	766.42	1003.8	975.07	32	1503.3	1801.6	2069.8	2248.5	4.9766	0.000689	0.0002	2.49E-05	-0.03907	-0.00031	-8.95E-05	5.68E-06	0.001127	3.55E+05
4	797.04	1012.7	976.17	32	1489.4	1797.6	2070	2298.5	5.0593	0.000648	0.000208	3.61E-05	-0.03888	-0.00031	-9.09E-05	4.62E-06	0.001108	44409
5	798.83	1013.3	976.27	32	1488.5	1797.3	2070	2301.6	5.0643	0.000645	0.000202	3.16E-05	-0.03887	-0.00031	-9.09E-05	4.71E-06	0.001107	2775.6
6	799.73	1013.6	976.32	32	1488.1	1797.2	2070.1	2303.1	5.0667	0.000643	0.000206	3.21E-05	-0.03887	-0.00031	-9.11E-05	4.62E-06	0.001107	1387.8
7	800.18	1013.7	976.34	32	1487.9	1797.1	2070.1	2303.9	5.068	0.000644	0.000203	3.22E-05	-0.03887	-0.00031	-9.09E-05	4.71E-06	0.001106	693.89
8	800.23	1013.7	976.34	32	1487.9	1797.1	2070.1	2304	5.0681	0.000646	0.000203	3.46E-05	-0.03887	-0.00031	-9.09E-05	4.62E-06	0.001107	86.736

Appendix. Gradient-based method for /	3=8, initial penalty multiplier=0.1, initia	ll point=(32, 32, 32, 32, 41, 41, 41, 41)
	Iteration 1	

								Sound 1	cration 2	1 11 60	6							
t	w1	w2	w3	w4	L1	L2	L3	L4	function value	gradient of w1	gradient of w ₂	gradient of w3	gradient of w4	gradient of L1	gradient of L ₂	gradient of L3	gradient of L4	randa (step size)
0	800.24	1013.7	976.34	32	1487.9	1797.1	2070.1	2304	4.9565	0.000582	0.000176	1.45E-05	-0.03609	-0.00031	-9.09E-05	4.71E-06	0.001107	0
1	800.24	1013.7	976.34	32	1487.9	1797.1	2070.1	2304	4.9565	0.000583	0.000177	1.55E-05	-0.03609	-0.00031	-9.11E-05	4.53E-06	0.001106	7.89E-11
2	800.24	1013.7	976.34	32	1487.9	1797.1	2070.1	2304	4.9565	0.000584	0.000178	1.67E-05	-0.03609	-0.00031	-9.10E-05	4.62E-06	0.001106	2.52E-09
3	800.24	1013.7	976.34	32	1487.9	1797.1	2070.1	2304	4.9565	0.000583	0.000174	1.72E-05	-0.03609	-0.00031	-9.09E-05	4.71E-06	0.001106	1.26E-09
4	826.15	1021.4	977.11	32	1474.3	1793.1	2070.3	2304	4.9766	0.000551	0.000175	1.92E-05	-0.03579	-0.0003	-9.15E-05	4.26E-06	0.001108	44409
5	832.26	1023.4	977.32	32	1471	1792.1	2070.3	2304	4.9812	0.000543	0.000178	2.03E-05	-0.03573	-0.0003	-9.16E-05	4.35E-06	0.001109	11102

Iteration 2

(833.77	1023.9	977.38	32	1470.2	1791.8	2070.3	2304	4.9824	0.000543	0.000181	2.11E-05	-0.03571	-0.0003	-9.16E-05	4.26E-06	0.001109	2775.6
	833.95	1023.9	977.38	32	1470.1	1791.8	2070.3	2304	4.9825	0.000542	0.000176	1.95E-05	-0.03571	-0.0003	-9.15E-05	4.35E-06	0.001109	346.94
8	834.05	1024	977.39	32	1470	1791.8	2070.3	2304	4.9826	0.000541	0.000179	2.10E-05	-0.0357	-0.0003	-9.15E-05	4.35E-06	0.001109	173.47
9	834.1	1024	977.39	32	1470	1791.8	2070.3	2304	4.9826	0.000539	0.000177	1.72E-05	-0.0357	-0.0003	-9.15E-05	4.44E-06	0.001109	86.736
10	834.12	1024	977.39	32	1470	1791.8	2070.3	2304	4.9826	0.000538	0.000177	2.14E-05	-0.03571	-0.0003	-9.16E-05	4.35E-06	0.001109	43.368
11	834.12	1024	977.39	32	1470	1791.8	2070.3	2304	4.9826	0.000539	0.000175	2.11E-05	-0.0357	-0.0003	-9.17E-05	4.26E-06	0.001109	10.842

								and spectra	teration 2	SUM-								
t	w1	w2	w3	w4	L1	L2	L3	L4	function value	gradient of w1	gradient of w ₂	gradient of w3	gradient of w4	gradient of L1	gradient of L ₂	gradient of L3	gradient of L4	randa (step size)
0	834.12	1024	977.39	32	1470	1791.8	2070.3	2304	4.0681	0.000118	-2.47E-05	-9.92E-05	-0.01386	-0.0003	-9.17E-05	4.26E-06	0.001109	0
1	1002	988.91	836.4	32	1047.1	1661.5	2076.4	2304	4.1184	5.66E-05	-1.24E-05	-0.00013	-0.01325	-0.00027	-0.00011	2.66E-06	0.001119	1.42E+06
2	1022.1	984.49	790.05	32	951.05	1624.1	2077.3	2304	4.1297	5.01E-05	-1.07E-05	-0.00014	-0.01307	-0.00027	-0.00011	2.22E-06	0.001119	3.55E+05
3	1023.2	984.26	786.85	32	945.09	1621.8	2077.4	2304	4.1305	5.07E-05	-1.00E-05	-0.00014	-0.01306	-0.00027	-0.00011	2.31E-06	0.001119	22204
4	1023.8	984.15	785.25	32	942.11	1620.6	2077.4	2304	4.1308	4.93E-05	-9.68E-06	-0.00015	-0.01305	-0.00027	-0.00011	2.40E-06	0.001119	11102
5	1023.9	984.12	784.84	32	941.36	1620.3	2077.4	2304	4.1309	4.99E-05	-9.77E-06	-0.00015	-0.01305	-0.00027	-0.00011	2.58E-06	0.001119	2775.6
6	1024	984.11	784.64	32	940.99	1620.1	2077.4	2304	4.131	4.94E-05	-9.59E-06	-0.00015	-0.01305	-0.00027	-0.00011	2.40E-06	0.001119	1387.8
7	1024	984.1	784.59	32	940.89	1620.1	2077.4	2304	4.131	4.92E-05	-9.95E-06	-0.00015	-0.01305	-0.00027	-0.00011	2.31E-06	0.001119	346.94
8	1024	984.1	784.58	32	940.87	1620.1	2077.4	2304	4.131	4.96E-05	-9.24E-06	-0.00015	-0.01305	-0.00027	-0.00011	2.31E-06	0.001119	86.736
9	1024	984.1	784.57	32	940.86	1620.1	2077.4	2304	4.131	4.91E-05	-1.01E-05	-0.00015	-0.01305	-0.00027	-0.00011	2.22E-06	0.001119	43.368
10	1024	984.1	784.57	32	940.86	1620.1	2077.4	2304	4.131	4.96E-05	-1.05E-05	-0.00015	-0.01305	-0.00027	-0.00011	2.31E-06	0.001119	10.842
11	1024	984.1	784.57	32	940.86	1620.1	2077.4	2304	4.131	4.85E-05	-1.08E-05	-0.00015	-0.01305	-0.00027	-0.00011	2.40E-06	0.001119	0.67763
12	1024	984.1	784.57	32	940.86	1620.1	2077.4	2304	4.131	4.89E-05	-1.01E-05	-0.00015	-0.01305	-0.00027	-0.00011	2.31E-06	0.001119	0.67763

Iteration 3

t	w1	w2	w3	w4	L1	L2	L3	L4	function	gradient	gradient	gradient	gradient	gradient	gradient	gradient	gradient	Randa
		ł	ł		ił	µl	ił		value	Of W1	Of W2	Of W3	OT W4	of L1	of L ₂	of L ₃	ot L4	(step size)
0	1024	984.1	784.57	32	940.86	1620.1	2077.4	2304	-4.3635	-0.00171	-0.00169	-0.00165	0.13924	-0.00027	-0.00011	2.66E-06	0.001119	0
1	1014.5	974.73	775.43	804.95	939.37	1619.5	2077.4	2304	0.55651	0.020355	0.013771	0.019455	0.023605	-0.00019	1.81E-05	0.000195	0.000235	5551.1
2	1014.5	974.73	775.43	804.95	939.37	1619.5	2077.4	2304	0.55651	0.01322	0.00945	0.016714	0.007076	-0.00019	1.82E-05	0.000194	0.000235	4.14E-05
3	1014.5	974.73	775.43	804.95	939.37	1619.5	2077.4	2304	0.55651	-0.00252	-0.00218	-0.00329	-0.01872	-0.00019	1.82E-05	0.000194	0.000235	2.58E-06
4	1014.5	974.73	775.43	804.95	939.37	1619.5	2077.4	2304	0.55651	-0.00832	-0.01748	-0.01445	-0.0137	-0.00019	1.83E-05	0.000195	0.000235	1.01E-08
5	645.24	198.39	133.81	196.57	931.01	1620.3	2086.1	2304	2.6723	-0.00409	-0.00636	-0.00702	-0.00039	-0.0002	-0.00014	0.00028	0.000284	44409
6	622.54	163.11	94.842	194.41	929.89	1619.5	2087.6	2304	2.8092	-0.00901	0.004237	-0.00316	-0.00029	-0.00018	-0.00016	0.000323	0.000236	5551.1
7	572.5	186.63	77.327	192.8	928.9	1618.6	2089.4	2304	2.8887	-0.00848	0.001662	0.003384	-0.00613	-0.0002	-0.00015	0.000363	0.000224	5551.1
8	384.22	223.53	152.47	56.67	924.47	1615.3	2097.5	2304	3.1759	0.004272	0.004491	0.006112	-0.06789	-0.00038	-0.0002	0.000123	0.00068	22204
:	:	:	:	:	:	:	all all	1	AN :	$ \langle i$	× 8) :	:	:	:	:	:	:	:
145	386.96	230.59	63.265	32.053	913.04	1608.6	2100.6	2304	4.2671	0.000178	0.000294	0.000648	-0.01298	-0.0003	-0.00018	0.000126	0.000729	43.368
146	386.96	230.59	63.267	32.018	913.04	1608.6	2100.6	2304	4.2675	0.00017	0.000278	0.000627	-0.01272	-0.0003	-0.00018	0.000126	0.00073	2.7105
147	386.96	230.59	63.267	32.001	913.04	1608.6	2100.6	2304	4.2678	0.000166	0.00027	0.000617	-0.01259	-0.0003	-0.00018	0.000126	0.00073	1.3553

Iteration 4

Iteration 5

t	w1				T 1	T D	13	T 4	function	gradient	gradient	gradient	gradient	gradient	gradient	gradient	gradient	randa
L	W I	WZ	WJ	W4	LI	LZ	LJ	L4	value	of W1	of W2	of W3	of W4	of L1	of L ₂	of L ₃	of L4	(step size)
0	386.96	230.59	63.267	32.001	913.04	1608.6	2100.6	2304	4.0434	-0.00484	-0.00959	-0.00457	0.13646	-0.0003	-0.00017	0.000126	0.000731	(
1	386.86	230.38	63.168	34.96	913.03	1608.6	2100.6	2304	4.2012	0.0002	0.000363	0.00163	-0.01447	-0.0003	-0.00017	0.000141	0.000698	21.684
2	386.87	230.4	63.239	34.332	913.02	1608.6	2100.6	2304	4.203	-0.00079	-0.00155	0.002002	0.0097	-0.0003	-0.00018	0.000138	0.000705	43.368

3	386.83	230.33	63.326	34.753	913	1608.6	2100.6	2304	4.2037	-0.00011	-0.00022	0.00178	-0.00723	-0.0003	-0.00018	0.00014	0.000701	43.368
4	386.83	230.32	63.403	34.439	912.99	1608.6	2100.6	2304	4.2043	-0.0006	-0.00118	0.001887	0.005012	-0.0003	-0.00018	0.000138	0.000704	43.368
5	386.8	230.27	63.485	34.657	912.98	1608.6	2100.6	2304	4.2046	-0.00024	-0.00048	0.001767	-0.00387	-0.0003	-0.00018	0.000139	0.000702	43.368
6	386.79	230.25	63.562	34.489	912.96	1608.6	2100.6	2304	4.2048	-0.0005	-0.00099	0.001785	0.002701	-0.0003	-0.00018	0.000138	0.000704	43.368
:	:		:	:	:	:	:	:	:	:	•••	:	••	:		:		:
20	386.53	229.73	64.605	34.475	912.75	1608.4	2100.7	2304	4.2069	-0.00042	-0.00082	0.001238	0.001464	-0.0003	-0.00018	0.000136	0.000708	43.368
21	386.49	229.66	64.713	34.602	912.73	1608.4	2100.8	2304	4.2069	-0.0002	-0.00039	0.001218	-0.00416	-0.0003	-0.00018	0.000137	0.000707	86.736

								W That	teration c	JAN 1								
t	w1	w2	w3	w4	L1	L2	L3	3.14	function value	gradient of wi	gradient of w2	gradient of w3	gradient of w4	gradient of L1	gradient of L2	gradient of L3	gradient of L4	randa (step size)
0	386.49	229.66	64.713	34.602	912.73	1608.4	2100.8	2304	4.1463	-0.00787	-0.01504	0.003262	0.18226	-0.0003	-0.00018	0.000137	0.000707	0
1	386.45	229.58	64.73	35.59	912.73	1608.4	2100.8	2304	4.1699	0.005106	0.009669	-0.00293	-0.12342	-0.0003	-0.00018	0.000142	0.000697	5.421
2	386.48	229.63	64.714	34.921	912.72	1608.4	2100.8	2304	4.1876	-0.00358	-0.00684	0.002996	0.075856	-0.0003	-0.00018	0.000138	0.000703	5.421
3	386.46	229.6	64.731	35.332	912.72	1608.4	2100.8	2304	4.1926	0.001797	0.003409	0.000163	-0.05049	-0.0003	-0.00018	0.000141	0.000699	5.421
4	386.47	229.61	64.731	35.059	912.72	1608.4	2100.8	2304	4.1952	-0.00175	-0.00336	0.002331	0.032033	-0.0003	-0.00018	0.000139	0.000702	5.421
5	386.46	229.6	64.744	35.232	912.72	1608.4	2100.8	2304	4.1962	0.000516	0.000974	0.001084	-0.02123	-0.0003	-0.00018	0.00014	0.0007	5.421
6	386.46	229.6	64.75	35.117	912.72	1608.4	2100.8	2304	4.1966	-0.00098	-0.00187	0.001946	0.013555	-0.0003	-0.00018	0.000139	0.000701	5.421
7	386.46	229.59	64.76	35.191	912.72	1608.4	2100.8	2304	4.1968	-9.59E-06	-2.18E-05	0.001409	-0.00907	-0.0003	-0.00018	0.00014	0.000701	5.421
8	386.46	229.59	64.768	35.142	912.71	1608.4	2100.8	2304	4.1969	-0.00064	-0.00123	0.001763	0.005727	-0.0003	-0.00018	0.000139	0.000701	5.421
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
20	386.42	229.53	64.898	35.146	912.69	1608.4	2100.8	2304	4.1972	-0.00046	-0.00089	0.001596	0.001652	-0.0003	-0.00018	0.00014	0.000702	5.421
21	386.42	229.52	64.915	35.164	912.69	1608.4	2100.8	2304	4.1972	-0.00021	-0.00041	0.001462	-0.00421	-0.0003	-0.00018	0.00014	0.000702	10.842

Iteration 6

t	w1	w2	w3	w4	L1	1.2	L3	I.4	function	gradient	gradient	gradient	gradient	gradient	gradient	gradient	gradient	randa
e					5.	22	110	5	value	of W1	of W2	of W3	of W4	of L1	of L ₂	of L ₃	of L4	(step size)
0	386.42	229.52	64.915	35.164	912.69	1608.4	2100.8	2304	4.1895	-0.00802	-0.01526	0.005801	0.17754	-0.0003	-0.00018	0.00014	0.000702	C
1	386.41	229.51	64.919	35.284	912.69	1608.4	2100.8	2304	4.1932	0.004588	0.008713	-0.00151	-0.11504	-0.0003	-0.00018	0.00014	0.000701	0.67763
2	386.42	229.52	64.918	35.206	912.69	1608.4	2100.8	2304	4.1948	-0.00355	-0.00677	0.003427	0.073352	-0.0003	-0.00018	0.000139	0.000701	0.67763
3	386.41	229.51	64.921	35.256	912.69	1608.4	2100.8	2304	4.1955	0.001654	0.003139	0.000367	-0.04745	-0.0003	-0.00018	0.00014	0.000701	0.67763
4	386.41	229.51	64.921	35.224	912.69	1608.4	2100.8	2304	4.1958	-0.00169	-0.00324	0.00238	0.03025	-0.0003	-0.00018	0.000139	0.000701	0.67763
5	386.41	229.51	64.923	35.245	912.69	1608.4	2100.8	2304	4.1959	0.00045	0.000852	0.001101	-0.01966	-0.0003	-0.00018	0.00014	0.000701	0.67763
6	386.41	229.51	64.923	35.231	912.69	1608.4	2100.8	2304	4.1959	-0.00093	-0.00178	0.001925	0.012443	-0.0003	-0.00018	0.00014	0.000701	0.67763
7	386.41	229.51	64.925	35.24	912.69	1608.4	2100.8	2304	4.196	-4.56E-05	-8.87E-05	0.001398	-0.00819	-0.0003	-0.00018	0.00014	0.000701	0.67763
8	386.41	229.51	64.926	35.234	912.69	1608.4	2100.8	2304	4.196	-0.00062	-0.00117	0.001739	0.005081	-0.0003	-0.00018	0.00014	0.000701	0.67763
:		:	:	:	•••	:	LI STE	27	AN :	KI	1	:	••	:		:	:	:
20	386.41	229.5	64.943	35.233	912.68	1608.4	2100.8	2304	4.196	-0.00057	-0.00109	0.001702	0.004095	-0.0003	-0.00018	0.00014	0.000701	1.3553
21	386.41	229.5	64.944	35.236	912.68	1608.4	2100.8	2304	4.196	-0.00027	-0.00053	0.001529	-0.00282	-0.0003	-0.00018	0.00014	0.000701	0.67763
								Con Bergin	- 0 V	MORE				•				

Iteration 7

Iteration 8

	+	···1				Т 1	T 2	13	T 4	function	gradient	gradient	gradient	gradient	gradient	gradient	gradient	gradient	randa
	l	W I	WZ	w3	W4	LI	LZ	LS	L4	value	of W1	of w ₂	of W3	of W4	of L1	of L ₂	of L ₃	of L4	(step size)
	0	386.41	229.5	64.944	35.236	912.68	1608.4	2100.8	2304	4.1949	-0.00848	-0.01615	0.006444	0.18806	-0.0003	-0.00018	0.000141	0.000701	0
ſ	1	386.41	229.5	64.945	35.252	912.68	1608.4	2100.8	2304	4.1955	0.004816	0.009171	-0.00154	-0.12101	-0.0003	-0.00018	0.00014	0.000701	0.084703
ſ	2	386.41	229.5	64.944	35.242	912.68	1608.4	2100.8	2304	4.1957	-0.00374	-0.00711	0.003598	0.077425	-0.0003	-0.00018	0.000139	0.000701	0.084703
ſ	3	386.41	229.5	64.945	35.248	912.68	1608.4	2100.8	2304	4.1958	0.001756	0.003332	0.000312	-0.0499	-0.0003	-0.00018	0.00014	0.000701	0.084703

4	386.41	229.5	64.945	35.244	912.68	1608.4	2100.8	2304	4.1958	-0.00177	-0.00337	0.002428	0.031833	-0.0003	-0.00018	0.00014	0.000701	0.084703
5	386.41	229.5	64.945	35.247	912.68	1608.4	2100.8	2304	4.1958	0.000491	0.000935	0.001073	-0.02062	-0.0003	-0.00018	0.00014	0.000701	0.084703
6	386.41	229.5	64.945	35.245	912.68	1608.4	2100.8	2304	4.1958	-0.00096	-0.00183	0.001944	0.013049	-0.0003	-0.00018	0.00014	0.000701	0.084703
7	386.41	229.5	64.945	35.246	912.68	1608.4	2100.8	2304	4.1959	-2.91E-05	-5.71E-05	0.001385	-0.00856	-0.0003	-0.00018	0.00014	0.000701	0.084703
8	386.41	229.5	64.945	35.245	912.68	1608.4	2100.8	2304	4.1959	-0.00063	-0.00119	0.001743	0.005308	-0.0003	-0.00018	0.00014	0.000701	0.084703
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
20	386.41	229.5	64.948	35.245	912.68	1608.4	2100.8	2304	4.1959	-0.00058	-0.0011	0.001711	0.004152	-0.0003	-0.00018	0.00014	0.000701	0.16941
21	386.41	229.5	64.948	35.246	912.68	1608.4	2100.8	2304	4.1959	-0.00027	-0.00052	0.001532	-0.00285	-0.0003	-0.00018	0.00014	0.000701	0.084703

Iteration 9

t	w1	w2	w3	w4	L1	L2	L3	L4	function value	gradient of w1	gradient of w ₂	gradient of w ₃	gradient of w4	gradient of L1	gradient of L2	gradient of L3	gradient of L4	randa (step size)
0	386.41	229.5	64.948	35.246	912.68	1608.4	2100.8	2304	4.1957	-0.00849	-0.01614	0.006472	0.18773	-0.00031	-0.00018	0.000139	0.000701	0
1	386.41	229.5	64.948	35.248	912.68	1608.4	2100.8	2304	4.1958	0.004812	0.009141	-0.00151	-0.12064	-0.0003	-0.00018	0.00014	0.000701	0.010588
2	386.41	229.5	64.948	35.246	912.68	1608.4	2100.8	2304	4.1958	-0.00374	-0.0071	0.003593	0.077216	-0.0003	-0.00018	0.00014	0.000701	0.010588

t	w1	w2	w3	w4	L1	L2	L3	L4	function value	gradient of wi	gradient of w ₂	gradient of w ₃	gradient of w4	gradient of L	gradient of L2	gradient of La	gradient of L4	randa (step size)
0	386.41	229.5	64.948	35.247	912.68	1608.4	2100.8	2304	4.1958	-0.0085	-0.01619	0.006457	0.18796	-0.0003	-0.00018	0.000139	0.000701	0
1	386.41	229.5	64.948	35.247	912.68	1608.4	2100.8	2304	4.1958	0.004814	0.009151	-0.00152	-0.12079	-0.0003	-0.00018	0.00014	0.000701	0.001324
2	386.41	229.5	64.948	35.247	912.68	1608.4	2100.8	2304	4.1958	-0.00374	-0.0071	0.003616	0.077309	-0.0003	-0.00018	0.000141	0.000701	0.001324
3	386.41	229.5	64.948	35.247	912.68	1608.4	2100.8	2304	4.1958	0.001749	0.003321	0.000315	-0.04979	-0.0003	-0.00018	0.00014	0.000701	0.001324

Iteration 10

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出生地:台北市

出 生 日:中華民國六十四年十月二十六日

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