國立台灣大學資訊管理研究所碩士論文

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考慮群播成員變動性下之最低成本路由群播樹演算法

A Minimum Cost Multicast Routing Algorithm with the Consideration of Dynamic User Membership

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中 華 民 國 九 十 三 年 七 月

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本論文係提交國立台灣大學
資訊管理學研究所作為完成碩士
學位所需條件之一部份

研究生：葉榮耀　撰

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中華民國九十三年七月二十三日通過

謝　　詞

本論文能夠順利完成，首先要感謝指導教授林永松老師，在引領我們一窺數學奧妙的過程中，循循善誘、因材施教，不但將強大的數學工具傳授給我們，更能以此為立足點，體驗學海之浩瀚；而旭成學長熱情的教導以及精闢的講解也對緊要關頭時的我助益匪淺。我最要感謝的是我親愛的父母親多年來的教養以及無怨無悔的付出，還有大姊慧嫺、二姊恆芬在我生活以及求學各方面的照顧還有小梅的對我的鼓勵，使我能在無後顧之憂的情況下，完成學業。

另外，多虧了好友仲輝、佳蘊、明憲等好友的陪伴，讓我有充滿回憶的兩年。也感謝所上的老師們在教學上的不遺餘力，營造了一個優良的學習環境，以及實驗室的優秀夥伴們，讓我的研究生涯不虛此行。

葉榮耀　謹誌

于台大資訊管理學研究所

民國九十三年七月

論 文 摘 要

論文題目：考慮群播成員變動性下之最低成本路由群播樹演算法

作　　者：葉榮耀　　　　　　　　　　　　　　　　　　　　**民國九十三年七月**

指導教授：林永松博士

本研究的目的在於建立最小成本的的群播樹。跟一般群播樹建立演算法不同的是，本問題的模型包含一組固定的群播接收者，而群組中的成員具有的變動性是以觀察而來的活動機率 q_d 來含括。由於此模型是用來評估與預測群播樹所需的成本，所以不考慮節點實際的加入跟退出。因為模型近似於史坦那樹問題，所以利用拉格蘭日鬆弛法來加快求解的過程。

關鍵詞：群播樹，史坦那樹，拉格蘭日鬆弛法

THESIS ABSTRACT

A Minimum Cost Multicast Routing Algorithm Adapted to Subscriber Behavior

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In this thesis, we try to model the problem of constructing a multicast tree with minimum cost. Unlike the other minimum cost multicast tree algorithms, this model consists of one multicast group of fixed members. Each destination member d is dynamic and has a probability of being active as q_d which was gathered by observation over some period of time. With the omission of node join/leave handling, this model is suitable for prediction and planning purpose than for online maintenance of multicast trees. Lagrangean relaxation method is applied to speed up the solution procedure.

Keywords: Multicast, Steiner tree problem, Lagrangean relaxation

Table of Contents

List of Tables

List of Figures

Chapter 1 Introduction

1.1 Background

The power of Internet comes from its openness that interconnects computers around the world as long as they follow the protocols. After about one decade of development, this global data network has somehow revolutionized the way people communicate and the way businesses are done. Consequently more and more contents are digitalized and are transmitted over data network today, including video and voice streams. However the application involving online video and voice require higher quality of transmission and may consume much more bandwidth over its transmission path, therefore it's worthwhile that we pay more attention to the problems that were aroused by such applications.

A very common scenario is that a source may try to send data to a specific group of destinations, for example a server of video streaming service sending its video stream to all of its service subscribers. Such traffic group communication is called multicast, as opposed to unicast and broadcast. The multicast traffic over IP often follows the route of a spanning tree over the existing network topology, called a multicast spanning tree, taking advantages of sharing common links over paths destined for different receivers.

From the viewpoint of network planning, each link in the network can be assigned with a cost, and the problem of constructing a multicast spanning tree with its cost minimized is called Steiner tree problem, which is known to be NP-complete.

1.2 Motivation

From the multicast protocols surveyed in 1.4, we can see that most complexity of these protocols comes from dealing with the changing of group members, that is, the joining and leaving of nodes. The motivation of this thesis would be creating a mechanism for finding and evaluating the cost-efficiency of a multicast tree with a given network and fixed set of group members. Also the group members are dynamic in that they might shut-off for a while, and turn on later. However we do not deal with the complexity node joining and leaving in our heuristic, instead, the activity for a node in summarized as a probability. Therefore, the model proposed here tends to be of analytical and planning use. Still, the problem of multicasting has strong connection with the Steiner tree problem [1], which is a NP-complete problem, the approach of Lagrangean relaxation is taken to achieve accurate approximation with significantly reduced computation time.

1.3 Literature Survey

1.3.1 Steiner tree problem

In a communication network, multicast routing takes advantage of multicast routing trees, over the network topology for transmissions to minimize resource usage such as cost and bandwidth by sharing links when transmitting data from one node to many destination nodes. A minimum cost multicast tree is also referred to as a Steiner tree [1]. In a sense, a Steiner tree is to construct a minimum cost tree for a set of destination nodes in a network with costs on the corresponding network links. Yet the problem of constructing a Steiner tree is known to be a NP-complete problem.

1.3.2 Lagrangean Relaxation Method

Lagrangean relaxation method [2] [11]is one of the handiest tools available for solving optimization problems, such as integer programming problems, linear and non-linear programming problems. By relaxing complicating constraints and by placing them in the objective function with added Lagrangean multipliers, the problem is thus decomposed into sub-problems that are actually stand-alone and optimally solvable.

Also the way of how decomposition is done is not unique, there's always freedom for the modeler to exploit the underlying structure of optimization problem. For minimization problems, the optimal value of the relaxed problem is always a lower bound of objective function value. Different minimization multipliers can be experimented with to get the tightest bound. By examining the decision variables and Lagrangean multipliers yielded during iterations, we may derive a heuristic algorithm for getting primal solution. Therefore, Lagrangean relaxation has been applied to acquire adequate heuristics for many well-known NP-complete, e.g., traveling salesmen problem [5]. The power and flexibility makes it an ideal choice for approximating to a good solution or providing clues for heuristic designing when the problem is computationally hard.

1.3.3 Multicast Protocols

There have been so many protocols proposed to support the functionality of multicasting such as MOSPF, CBT [7] and QoSMIC [8]. Several important rules that may help to characterize multicast protocols are aggregation rule, tree construction and tree updating [6]. The aggregation rule is the strategy that node is to join the multicast group. The joining node may connect to the closest node in the group, or to a path shortest to the source node. As for the tree construction, unicast, BFS and pruned spanning tree may be used. The updating of multicast tree is required when extensive join/leave occurs or change of the network state. Here is a table adopted from J. I. Alvarez-Hamelin, P. Fraigniaud, and Alberto Dams's work [6]:

	Aggregation Rule	**Tree Construction**	**QoS Support**
MOSPF	Shortest path to source	OSPF	No
CBT	Shortest path to source	Unicast	No
QoSMIC	Closest node	Explore Multiple paths and keep good ones	Yes

Table 1 Properties of multicast protocols.

Some more heuristics proposed in [9][10] aims at the creation of multicast with consideration of QoS.

1.4 Thesis outline

- This formulation describes the model that consists of one multicast group with an origin trying to multicast traffics to all destination members of the group.

- The states of receivers are undeterministic. Each destination d has a probability of being active as q_d which was gathered by observation over some period of time.

- As a network service provider, constructing a multicast tree with the cost as low as possible is a primary concern.

- For every link, the cost consists of a fixed link maintenance cost b_l and transmission cost a_l proportional to link utilization.

Chapter 2 Problem Formulation

2.1 Problem Description

In this thesis we consider, for a network service provider, the problem of constructing a multicast spanning tree that sends traffic to receivers (destinations), while at the same time, the total cost resulted by the multicast tree is minimized.

Each destination $d \in D$ has a given probability Q_d that indicated the fraction of time that the destination is active, and thus the traffic is to be routed to that node. Such probability may be acquired by observation of user behavior over a certain a period of time. The cost associated with a link consists of two parts: 1) fixed cost of connection setup and 2) transmission cost proportional to link utilization. At the determination of the multicast tree, utilizations for all links may be computed, which are used to estimate the total cost.

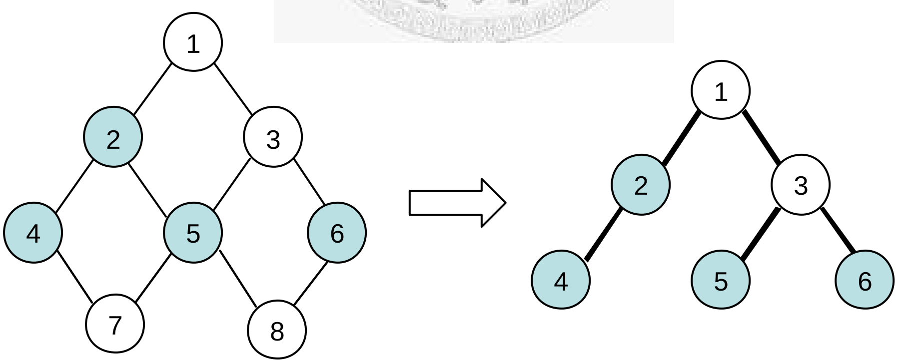

Figure 1 An example of multicast tree construction

Node	D1(root)	D2	D3	D4	D5	D6	D7	D8
Qd	1.00	0.30	0.00	0.70	0.10	0.20	0.00	0.00

Table 2 The Qd table of the example above

2.2 Problem Formulation

2.2.1 Notations

Given parameters：	
D	The set of all destinations in the multicast tree.
N	The set of all nodes in the network.
L	The set of all links in the network.
I_i	The set of all incoming links to node i.
q_d	The probability that the destination d is active.
a_ℓ	The transmission cost associated with link ℓ.
b_ℓ	The connection maintenance cost associated with link ℓ.
P_d	The set of all elementary paths from r to $d \in D$.
$d_{p\ell}$	The indicator function which is 1 if link ℓ is on path p.

Table 3 Given parameters

Decision variables：	
y_ℓ	1 if link ℓ is included in the multicast tree and 0 otherwise.
x_p	1 if path p is included in the multicast tree and 0 otherwise.
g_ℓ	The fraction of time that the link ℓ is active on the multicast tree.
$f_{d\ell}$	1 if link ℓ is used by destination $d \in D$

Table 4 Decision variables

2.2.2 Objective function

$ZIP = \min \sum_{\ell \in L} (b_\ell y_\ell + a_\ell g_\ell)$

Table 5 Objective Function

The objective is to minimize the total cost associated with the multicast tree, including the accumulated transmission costs (pay per time unit) and setup cost (pay per connection) on each link used.

2.2.3 Constraints

$g_\ell \ge 1 - \prod_{d \in D} (1 - q_d f_{dl})$	$\forall \ell \in L$	(1)
$\sum_{\ell \in I_i} y_\ell \le 1$	$\forall i \in N$	(2)
$\sum_{\ell \in I_r} y_\ell = 0$		(3)
$\sum_{p \in p_d} d_{p\ell} x_p \le f_{d\ell}$	$\forall \ell \in L, \forall d \in D$	(4)
$\sum_{p \in P_d} x_p = 1$	$\forall d \in D$	(5)
$f_{d\ell} \le y_\ell$	$\forall \ell \in L \; \forall d \in D$	(6)
$f_{dl} = 0 or 1$	$\forall \ell \in L \; \forall d \in D$	(7)
$y_\ell = 0 or 1$	$\forall \ell \in L$	(8)
$x_p = 0 or 1$	$\forall p \in P_d, \;\; \forall d \in D$	(9)
$0 \le g_\ell \le 1 - \prod_{d \in D} (1 - q_d)$	$\forall \ell \in L$	(10)

Table 6 Constraints

Constraint 1	Defines the link utilization as a function of q_d and f_{dl}.
Constraint 2	Every node cannot have more than one incoming link. To make sure that the result of routing would be a tree.
Constraint 3	The root node cannot have any incoming link. To make sure that the result of routing would be a tree.
Constraint 4	If link ℓ is not used by destination d, then for all paths $p \in p_d$ passing through ℓ, $x_p = 0$.
Constraint 5	For each destination, exactly one path may be chosen to form the multicast tree.
Constraint 6	If link l is not included in the multicast tree, then it won't be used by any destination.
Constraint 7	In the multicast tree if link l is used by destination, then f_{dl} = 1, otherwise f_{dl}=0.
Constraint 8	If link l is included in the multicast tree, then y_ℓ=1, otherwise y_ℓ=0.
Constraint 9	If path p in included in the multicast tree, then x_p = 1, otherwise x_p =0.
Constraint 10	Redundant constraint.
Constraint 11	The costs associated with all links are all greater than zero.

Table 7 Description of constraints

Furthermore, here is an example of many possible extensions that could be made to this problem but not discussed in this thesis. Say the dependency among destinations, e.g., the members of the group can be further divided into subgroups such that the group members within each subgroup behave identically. The link utilization can be modeled as follows:

$$g_\ell = 1 - \prod_{m \in G} (1 - q_m (1 - \prod_{i \in M_m} (1 - f_{i\ell}))) \quad \text{G is the set of subgroups.}$$

As you may notice that the structure of this formula resembles the constraint for link utilization of (C1), with its $f_{d\ell}$ replaced with $(1 - \prod_{i \in M_m} (1 - f_{i\ell}))$.

Chapter 3 Solution Approach

3.1 Lagrangean relaxation and Subproblems

In order to apply the Lagrangean relaxation, we must first identify a set of complicating constraints such that the removal of them would simplify the solution procedure. The relaxed constraints are added to the objective function of the primal problem, multiplied with Lagrangean multipliers.

In our problem, we relax constraints (C1), (C4), (C6) to form the following Lagrangean relaxation problem (LR).

$$Z_{LR}(a,b,q) = \min \sum_{\ell \in L} (b_\ell y_\ell + a_\ell g_\ell) + \sum_{\ell \in L} a_\ell \Big(\sum_{d \in D} \log(1 - q_d \cdot f_{dl}) - \log(1 - g_\ell) \Big)$$
$$+ \sum_{\ell \in L} \sum_{d \in D} b_{d\ell} \Big(\sum_{p \in P_d} d_{p\ell} \cdot x_p - f_{d\ell} \Big) + \sum_{\ell \in L} \sum_{d \in D} q_{d\ell} (f_{d\ell} - y_\ell)$$

such that (C2), (C3), (C5), (C7), (C8), (C9) and (C10).

The Z_{LR} can be further decomposed into four sub-problems:

SP1 (In terms of decision variable x_p):

$$\min \sum_{d \in D} \sum_{p \in P} \Big(\sum_{\ell \in L} b_{d\ell} \cdot d_{p\ell} \Big) \cdot x_p \quad \text{s.t. (C5), (C9)}$$

This sub-problem is equivalent to finding the sum of shortest paths from source to all $d \in D$, taking $b_{d\ell}$ as the arc weight of the link.

SP2 (In terms of decision variable y_ℓ):

$$\min \sum_{\ell \in L} (b_\ell - \sum_{d \in D} q_{d\ell}) \cdot y_\ell \qquad \text{s.t. (C2)}$$

To find the optimal value for this sub-problem, for all $\ell \in L$ set $y_\ell = 1$ whenever $(b_\ell - \sum_{d \in D} q_{d\ell}) \leq 0$ and (C2) holds.

SP3 (In terms of decision variable g_ℓ):

$$\min \sum_{\ell \in L} (a_\ell g_\ell - a_\ell \cdot \log(1 - g_\ell)) \quad \text{s.t. (C10)}$$

This subproblem of minimization can be solved by substituting g_ℓ with its lower and upper bound because the minimum of this function appears at endpoints.

SP4 (In terms of decision variable $f_{d\ell}$):

$$\min \sum_{\ell \in L} \sum_{d \in D} (a_\ell \log(1 - q_d \cdot f_{d\ell}) + (q_{d\ell} - b_{d\ell}) f_{d\ell}) \quad \text{s.t. (C7)}$$

Simply substitute $f_{d\ell}$ with 0 and 1 and keep the one that yields the minimum.

3.2 Lagrangean Dual problem and Subgradient method

According to the weak Lagrangean duality theorem [4], as long as the LR problem can be optimally solved, its value of objective function value always provides a lower bound on primal problem for any LR multiplier $m = (a_{\ell}, b_{d\ell}, q_{d\ell})$. Therefore, in order to tighten the lower bound acquired from the solving LR problem, we must experiment with different sets of LR multipliers iteration by iteration, and the problem of finding the tightest lower bound of LR problem is referred to as a dual problem, which is defined as below:

$$Z_D = \max_{a_{\ell} \le 0, b_{d\ell} \ge 0, q_{d\ell} \ge 0} Z_{LR}(a_{\ell}, b_{d\ell}, q_{d\ell})$$

The subgradient method is used to solve the dual problem Z_d. Let a vector s be a subgradient of Z_D. In iteration k of subgradient method, the multiplier vector is updated as follows:

$$m^{k+1} = m^k + t^k m^k$$

The step size t^k of update is determined by: $t^k = d\dfrac{Z_{IP} - Z_D(m^k)}{\left\|s^k\right\|^2}$, where $\left\|s^k\right\|^2$ is the square of the Euclidean norm of the subgradient s^k and Z_{IP} is the objective function of the getting primal feasible solution heuristic and $0 < d \le 2$.

3.3 Simple Algorithm

Before starting to solve the dual problem, a simple algorithm is needed to provide an adequate initial upper bound of the primal problem ZIP. Dijkstra [3] algorithm is used to generate a minimum cost spanning tree over the given network, using the connection setup cost bl as the arc weight of link l. The result yielded thereby is feasible and expected to give solution of better quality than a random guess.

3.4 Getting primal feasible solution

3.4.1 Heuristic One

By solving the dual problem optimally we get a set of decision variables that may be appropriate for being the inputs of getting primal heuristics. However that solution might not be feasible and thus takes some more modifications. Still another way of devising getting primal heuristics is to take the hints of LR multipliers.

In this thesis, two of our getting primal heuristics are created based upon the simple algorithm proposed in 3.3, by taking the LR multiplier $b_{d\ell}$ as the source of arc weight in Dijkstra algorithm. For heuristic one, each ℓ_i in $b_{d\ell}$ has all its corresponding $b_{d\ell_i}$ $d \in D$ summed up and taken as the arc weight of link ℓ_i. The action is illustrated below.

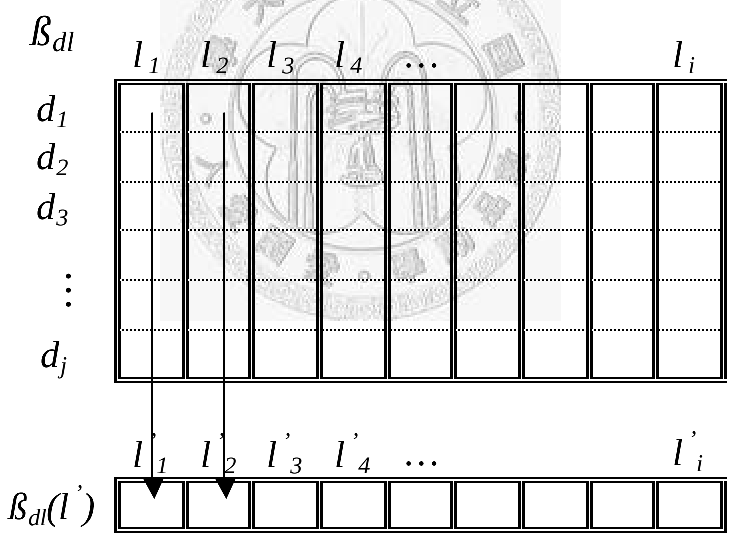

Figure 2 LR multiplier summation as arc weights in heuristic one

3.4.2 Heuristic Two

Just like heuristic one, getting primal feasible solution heuristic two takes LR multiplier $b_{d\ell}$ as the arc weights used in the Dijkstra algorithm [3]to generate minimum cost spanning tree However to make sure that tree topology is always resulted, we manipulate $b_{d\ell}$ in a different way as shown below. Whenever an element in $b_{d\ell}$ is selected as and arc weight and the link ℓ_i is added to the multicast tree, all the other elements $b_{d\ell_i}$ $d \in D$ in the same column are set to zero, making ℓ_i easier to be selected by other destinations. Also the results of both heuristics will be listed in 4.1.

$ß_{dl}$	l_1	l_2	l_3	l_4	…				l_i
d_1	X	0							
d_2	0	X							
d_3	0	0							
	0	0							
⋮	⋮	⋮							
	0	0							
	0	0							
	0	0							
d_j	0	0							

Figure 3 LR multiplier manipulation in heuristic two

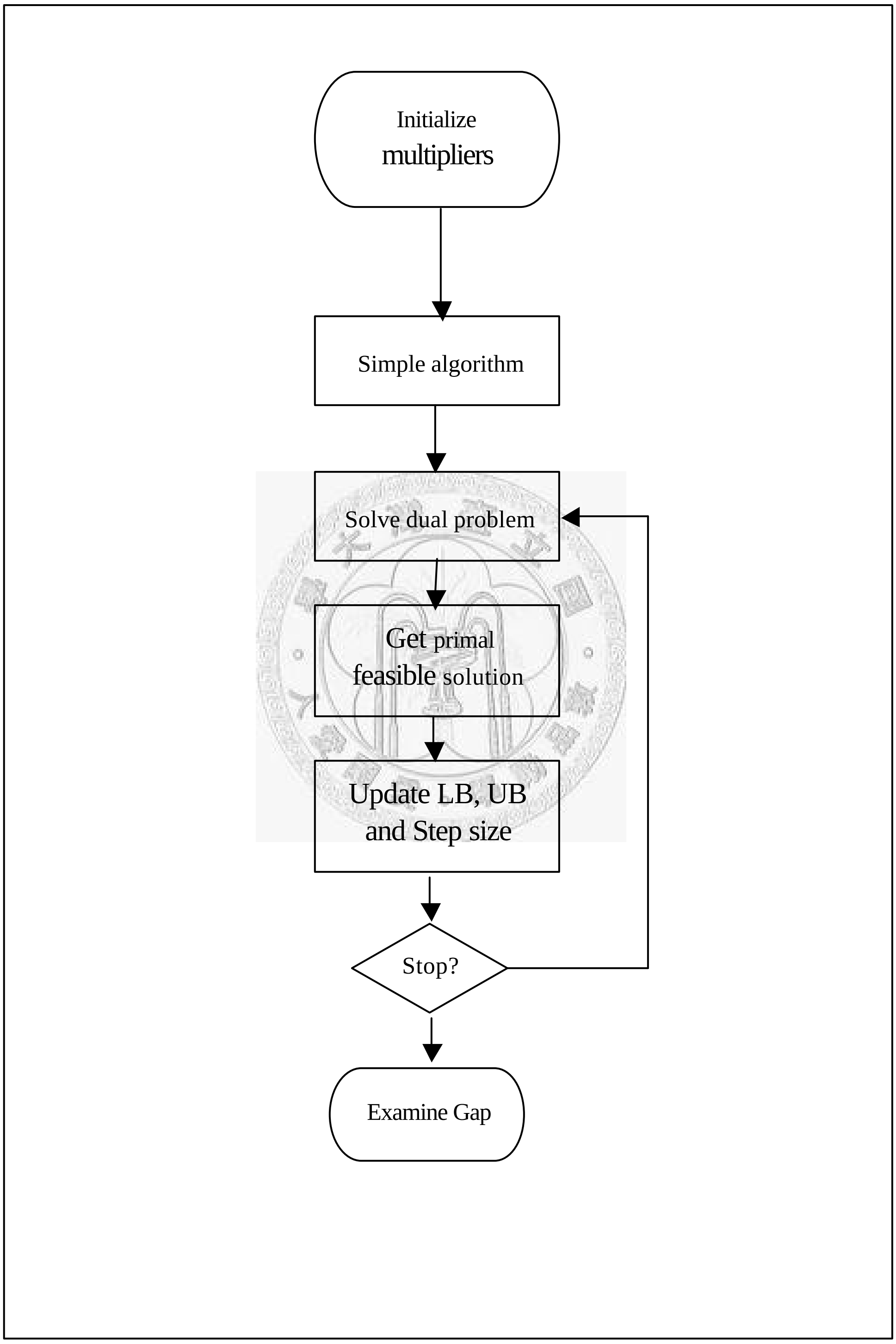

Figure 4 Flowchart of the steps of solution procedure

Chapter 4 Computational experiment

4.1 Computational experiment results

The computational tests are based on two kinds of networks: random network and grid network. Each is provided with instances of size 5, 10, 15 and 25. To verify the optimality of the solution, we use the gap between LB and UB, respectively yielded by solving dual problem and solving getting primal feasible solution heuristic. The gap is computed as follows:

$$GAP = \frac{(UB - LB)}{LB} * 100\%$$

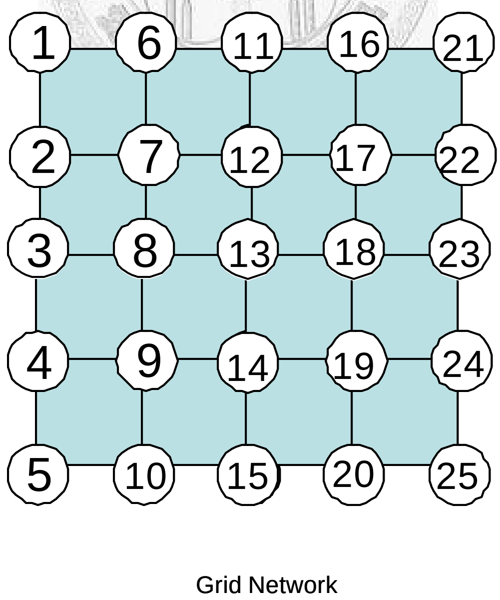

Figure 5 A sample grid network

T

Number of iterations	2000
Improvement count	15
Delta	2
Optimality condition	GAP <1%

Table 8 Parameters for Lagrangean relaxation

Getting Primal Heuristic One				
#Nodes	Topology	LB	UB	GAP
5	Random	7.75	7.75	0.00%
5	Grid	25.99	26.048	0.22%
10	Random	39.894	42.472	6.46%
10	Grid	53.832	54.777	1.75%
15	Random	37.407	40.703	8.81%
15	Grid	47.043	48.01	2.05%
25	Grid	144.608	145.433	0.57%

Table 9 Results of Heuristic One

Getting Primal Heuristic Two				
#Nodes	Topology	LB	UB	GAP
5	Random	7.484	7.75	3.55%
5	Grid	25.991	26.048	0.22%
10	Random	39.8926	42.472	6.47%
10	Grid	53.832	54.776	1.73%
15	Random	37.102	37.104	0.01%
15	Grid	38.094	47.311	24.19%
25	Grid	146.081	146.998	0.62%

Table 10 Results of Heuristic Two

4.2 Summary and conclusion

According to the computational experiments shown in 4.1, we may be sure that our LR-multiplier-based getting primal feasible solution heuristics generate adequate solutions since the gap eventually converges to the acceptable value. To summarize, by relaxing constraints in the primal problem and optimally solving dual problem, the set of LR multipliers revealed iteration by iteration became unique sources for improving our solutions in getting primal heuristics.

The contribution of this research would be quite academic, with the innovative idea of constructing a multicast tree that adapts to the activity of end users in a minimization problem, making the model itself aware of the phenomenon of dynamic user join and leave without all the fuss of dealing with it in our heuristic. For this reason, this model is ideal for network planning purpose. Still the computational results show that the structure of the problem is suitable for the methodology of Lagrangean relaxation. However this model is still in a simple form and interested people may come up with quite a few extensions to this simple model with ease.

4.3 Future works

Some additional topics to this problem might be 1) Multiple groups of users and the behaviors of the members within one group are identical or somewhat correlated. 2) Multiple trees may be constructed over the network at the same time, with different data-rate demands. 3) Quality-of-service constraints may be added such as: link capacity, hop count and delay constraints. 4) Different getting primal feasible heuristics can be invented to produce solutions with better optimality.

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