# 國立臺灣大學資訊管理研究所碩士論文 

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## 異質性光波長多工光網路之交換器配置演算法

# Switch Placement Algorithms in Optical WDM Heterogeneous Networks 

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中 華 民國 九十四 年 t 月


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# Switch Placement Algorithms in Optical WDM Heterogeneous Networks 

本文係提交國立台灣大學
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張孝澤 謹 識
于台大資訊管理研究所九十四年七月


## 論文摘要

論文題目：異質性光波長多工光網路之交換器配置演算法作者：張孝澤
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隨著光波長多工光網路的不斷發展，一條光纖所能攜带的光波長數目不斷以倍數增加，光網路交換器（OXC）的複雜度與成本也隨之增加。GMPLS 定義了三種光交換的方式：光纖交換，光波段交换與光波長交換。而在異質性網路下，允許每個節點有其中一種光交换的能力。光網路建置成本最大的來源即是光網路交換器，而光網路交換器的成本又與其所使用的埠數目直接相關，因此當一條光纖能攜带數百條光波長時，光波長交換器的成本也將提高數百倍，且交换器埠的數目至今仍有設計上的數量瓶頸存在

此篇論文的目的即是希望在異質性網路下，妥當的規劃各節點，以最低的成本而能滿足網路上所需的静態流量要求。我們將這個問題建立成一個數學模型，透過目標函式與限制式來適當的描述此問題，是一個整數規劃的問題，問題的本身具有高度的複雜性和困難度。因為光網路路由與光波長配置的問題（RWA）已知為一個NP－hard的問題，而此問題隱含了RWA問題，因此此問題也是一個 NP－hard問題，無法在有限的時間内以已知有效的演算法解決。因此我們採用最佳化領域中的拉格蘭日鬆弛法（Lagrangean Relaxation）來解決此問題。

另外，我們根據［8］中的RWA問題發展出一個簡易的交換器配置演算法，我們設計數項實驗在不同的網路拓撲下測試所提出演算法與簡易演算法相比，實驗結果顯示都有較佳的結果。

關鍵詞：光波長多工，光網路規劃，光纖交換，光波段交換，光波長交換，最佳化，拉格蘭日鬆弛法，數學規劃。


## THESIS ABSTRACT

# GRADUATE INSTITUTE OF INFORMATION MANAGEMENT <br> NATIONAL TAIWAN UNIVERSITY <br> NAME : HSIAO-TSE CHANG MONTH/YEAR:JUL, 2005 ADVISOR: YEONG-SUNG LIN <br> HONG-HSU YEN 

## SWITCH PLACEMENT ALGORITHMS IN OPTICAL WDM HETEROGENEOUS NETWORKS

With the rapid development of Wavelength Division Multiplexing (WDM), a fiber can carry more and more wavelengths, but the complexity and the cost of Optical Cross-connects (OXCs) also increase. To deal with the problem, General Multi-Protocol Labeling Switching (GMPLS) defines three kind of switching methods: fiber switch capable, waveband switch capable, and lambda switch capable. In a heterogeneous optical network, we allow each node to have one of the switching capabilities. OXCs contribute most to the planning cost of optical networks, and the cost of OXCs is in proportion to the number of ports. Therefore, while a fiber can carry hundreds of wavelengths, the cost of OXCs increases proportionally. Furthermore, there is still a shortage of ports in the OXC design.

In this thesis, we allocate the switch nodes properly based on the lowest cost, and satisfy all the static traffic demand in a heterogeneous network. We model this problem as an integer programming problem with an objective function and several
constraints, which is very complicated. Since the routing and wavelength assignment problem (RWA) is a well known NP-hard problem, and our problem contains the RWA problem, our problem is also NP-hard. As we cannot solve it in polynomial time by well known algorithms, we adopt Lagrangean relaxation as the solution approach.

In addition, we propose a simple heuristic algorithm modified from an RWA problem, and conduct several experiments on different network topologies. We find that the experiment results of Lagrangean Relaxation are better then those of the simple heuristic algorithm.

Keywords: Fiber Switch, Lagrangean Relaxation Method, Lambda Switch, Mathematical Programming, Network Planning, Optimization, Waveband Switch, WDM.

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## Chapter 1 Introduction

### 1.1 Background

### 1.1.1 DWDM Technology

WDM (Wavelength Division Multiplexing) technology or DWDM (Dense Wavelength Division Multiplexing) technology increases the carrying capacity of fibers. It can assign incoming optical signals to specific frequencies of light within a certain frequency band, called lambdas or channels. We can multiplex these lambdas all together and send them through the fiber simultaneously. Then, at the receiving end, we can separate the lambdas by using a demultiplexer. Figure 1-1 shows the architecture of WDM, in which the fiber carries n lambdas.


Figure 1-1 Wavelength Division Multiplexing Architecture

DWDM spaces the wavelengths closer than WDM. In the Figure 1-2, the $1^{\text {st }}$ window is the most cost-efficient band, but due to the high loss rate, we can only utilize this band for short distance transmissions. We find that the $3^{\text {rd }}, 4^{\text {th }}$, and $5^{\text {th }}$ windows, called "C band", "L band", and "S band", respectively, have relatively low loss rates. For example, the loss rate of C band is even lower than $0.2 \mathrm{~dB} / \mathrm{KM}$, which means that the optical signal with C band wavelength will only decline by half after sixty kilometers of transmission. Therefore, we can use these three bands for DWDM.


Figure 1-2 Fiber windows and their loss rates [13]

Nowadays, DWDM technology divides a fiber into more than 200 lambdas, and each lambda can carry more than 10 Gbps. In Figure 1-3, each fiber cable carries more than 100 fibers, each of which carries more than 200 lambdas, and each lambda carries more than 10 Gbps of traffic. Commercial products can easily achieve 2 Tbps per fiber.


Figure 1-3 Ultimate fiber capacity [13]

In 2002, NEC demonstrated a single fiber with 273 lambdas, each with 40 Gbps data rate, transmitting across 117 kilometers, so the total capacity of the fiber was 10.92 Tbps. Figure $1-4$ shows the evolution of the fiber capacity in WDM and TDM from 1993 to 2003. Note that in TDM, there is no wavelength division multiplexing. Hence, there is only one lambda on each fiber link. In WDM, we can see that the lambdas divided by a single fiber increase from 8 to 273 .


Figure 1-4 Evolution of the fiber link capacity [14]

### 1.1.2 OADM and OXC

An OADM (optical add-drop multiplexer) can drop lambdas from-or add lambdas to-a fiber. Fixed OADMs can only drop and add specific lambdas, while reconfigurable OADMs (see figure 1-5) can dynamically set the lambda to be dropped or added. An optical add-drop multiplexer can be only connected to one fiber and each lambda needs a $2 \times 2$ switch. When the 2 X 2 switch setting is parallel, the incoming lambda bypasses this OADM. When the switch setting is crossable, the switch drops the incoming lambda, adding a new lambda to the fiber.


Figure 1-5 A Reconfigurable Optical Add-Drop Multiplexer [13]


Figure 1-6 A Optical Cross-Connect [13]

An OXC (optical cross-connect) not only drops and adds lambdas, but also switches lambdas from an input fiber to a different output fiber. In Figure 1-6, there are 3 fibers connected to the switch, each with four lambdas. Therefore, we need four four-port lambda switches and a total of sixteen ports for input fibers inside this

OXC. Any lambda from the three input fibers can be switched to a different output fiber. However, note that that each lambda can only be directed to any output fiber once, which means that the same lambda from different input fibers must be switched to an output fiber exactly once.

The lambda switch inside the OXC can be a space switch, such as a 2-D MEMS (micro-electro-mechanical) space switch, as shown in Figure 1-7. Each wavelength can be directed to a different output port by the active mirror. A 2-D MEMS needs $\mathrm{N}^{2}$ mirrors (switches), where N is the number of input ports. In the latest technology of DWDM, each fiber can carry 273 lambdas. Therefore, the OXC has become extremely complex with 74,529 mirrors inside each lambda switch. As the complexity of the 2-D MEMS switch design is proportional to $\mathrm{N}^{2}$, it is not scalable to switches with a large number of ports.


Figure 1-7 2-D MEMS Space switch


Figure 1-8 3-D MEMS Switch
In recent research, another switch technology, shown in Figure 1-8, uses two arrays of mirrors that rotate on two axes. Each input fiber directs light to a mirror in the input array. The input mirror steers the input optical beam to any output mirror, and each output mirror directs light to an output fiber. This topology is called a 3-D MEMS switch, because the optical beams are switched in the three-dimensional space between the two MEMS die [15]. A 3-D switch configuration needs only 2 N mirrors, which is far fewer than the $\mathrm{N}^{2}$ mirrors required by 2-D MEMS. However, due to the difficulty of building flat, thin-film mirrors, which would make the optical loss lower while the mirrors rotates frequently, the number of ports of 3-D MEMS is still limited to around 50-200.

In future, photonic switches will have the potential to change the way optical networks are built and used, because large photonic switches can be built with a 3-D topology and bulk-silicon MEMS structures. Photonic switches with 256 ports and a mean optical loss of 1.3 dB have already been demonstrated, so multi-stage switches could form the basis of larger switches [15].

### 1.1.3 GMPLS

GMPLS (General Multi-Protocol Label Switching) [1], shown in Figure 1-9, is a unified, open, standard control plane. It is an extension of MPLS (Multi-Protocol Label Switching) and defines a more general control plane. GMPLS defines the Lambda switch capable (LCS) interface and fiber switch capable (FSC) interface that support fiber and wavelength layer transmission.

Due to the rapid development of DWDM, the optical cross connect may not afford the intense traffic of fibers by wavelength routing. A waveband switch capable (WBSC) interface is proposed by [2], whereby several wavelengths are grouped as a waveband. For example, if a fiber can carry 200 lambdas, we let 10 wavelengths group as a waveband so that there are 20 wavebands. If we deploy a waveband switch at this OXC node, we only need 20 input ports, instead of 200. The waveband label should be inserted between the wavelength routing layer and the fiber routing layer, as shown in Figure 1-9. Although wavelength routing is the most flexible routing method when compared with waveband routing and fiber routing, it incurs the most complex and most expensive OXC (most number of ports).


Figure 1-9 GMPLS Control Plane

### 1.2 Motivation

Due to the tremendous increase of fiber link capacity, each fiber can carry hundreds of lambda. However, the optical cross-connect tends to be much more complicated, and sometimes the OXC can not even handle the traffic.

GMPLS and its extension, described in Section 1.1.3, define three kinds of switching device: FSC, WBSC, and LSC. The FSC (fiber switch capable) is the most cost effective, but it is inflexible because all the traffic within the same fiber must be switched all together. In Figure 1-10, the first fiber is switched to the $\mathrm{n}^{\text {th }}$ fiber. The WBSC (waveband switch capable) is also much cheaper than LSC, and has better flexibility than FSC, because the traffic is switched by waveband allowing wavelengths in the same waveband to be switched simultaneously. For example, in Figure 1-11, the first waveband of the first fiber is switched to the first waveband of second fiber. Although the LSC (lambda switch capable) is the most expensive, because a lot of ports are needed, it is the most flexible switching device. Each wavelength can be switched to a different output fiber. For example, in Figure 1-12, the $\lambda_{0}$ from Fiber ${ }_{0}$ is switched to the same output fiber ${ }_{0}$. However the $\lambda_{99}$ from Fiber $_{0}$ is switched to the Fiber ${ }_{9}$.

Since the cost of a lambda switch is extremely high, it's not feasible to let all the switch nodes be lambda switches like all the traditional routing and wavelength assignment problems in the literature (see Section 1.3.1). Therefore, how to effectively allocate the three kinds of switches in a heterogeneous WDM optical network becomes important. To the best of my knowledge, there has not been any research into this problem, but the issue should become increasingly important as

WDM technology improves. This thesis contributes a great deal to network design and planning in heterogeneous networks.


Figure 1-10 Fiber switch


Figure 1-11 Waveband switch


Figure 1-12 Wavelength (lambda) switch

### 1.3 Literature Survey

### 1.3.1 RWA Related Issues

Although many papers discuss the RWA (routing and wavelength assignment) problem in a WDM optical network, most of them focus on a wavelength switching capable interface. Hence, these works are not applicable in DWDM technology today.

In [3], the RWA related issue is classified in three dimensions as follows:

## ©Type of switch node:

is Homogenous network: In the optical WDM network, all the nodes are hierarchical, which means that a waveband can be set up at any switch node, and can be decomposed at any switch node.
is Heterogeneous network: In the optical WDM network, not all the nodes are hierarchical, which means that the fiber traffic can only be switched by waveband at specific nodes with a waveband switching capable interface. The traffic only can be switched by fiber at specific nodes with a fiber switching capable interface.

## ©Routing Models

in Integrated routing: The routes are computed for both the wavelength and waveband paths simultaneously. This routing model is more effective, but the algorithm or formulation tends to be more complicated.
is Separate routing: First, the routes for the wavelength are computed, and then the routes for the waveband are computed. It is basically a two-stage computation. Therefore, the algorithm of this model is much simpler than integrated routing, but it is not as effective.

Online routing: The traffic of a network is dynamic. Each time a new demand for any source-destination pair occurs, the route and the assigned wavelength of this demand are computed at once. This can handle dynamic traffic well.

Offline routing: Given the traffic demand of all the source-destination pairs, the optimal routes are computed at the beginning of the process. This assumes that the traffic of the network will be stable for a period of time.


Figure 1-13 Three dimensions of the RWA related issues. [3]
Figure 1-13 shows the diagram of the three dimensions. In the next section, we focus on homogeneous and heterogeneous networks, and discuss existing works. Note that since the traditional RWA problem has already been proved to be an NP-hard problem, related issues containing the RWA problem are also NP-hard problems.

### 1.3.2 Homogeneous Network

All the switch nodes in a homogeneous network are hierarchical, and can set up wavebands or decompose wavebands to wavelengths. Hence, [4] developed the
architecture of MG-OXC (Multigranularity Optical Cross-Connect), shown in the Figure 1-14. If the input fiber goes through the fiber cross-connect, it can be switched to any output fiber. Some input fibers might go through the FTB (fiber to waveband) port, and be demultiplexed as wavebands. If the waveband goes through the waveband cross-connect, it can be switched and multiplexed as a fiber and then go to the BTF (waveband to fiber) port. Again, some wavebands might go through the BTW (waveband to wavelength) port, and be demultiplexed as wavelengths. If the wavelength goes through the wavelength cross-connect, it can be switched and multiplexed as a waveband and then go to the WTB (wavelength to waveband) port. Note that in wavelength cross-connects, some wavelengths might be dropped, and some might be added.


Figure 1-14 Multigranularity Optical Cross-Connect [4]

In [3], the authors develop both an online and an offline routing and wavelength assignment heuristic algorithm by separate routing methods.

In [4] with static traffic (offline routing), in order to minimize the total cost, the authors try to minimize the total port counts or minimize the maximum port counts of MG-OXC (including FXC, BXC, and WXC layer), shown in Figure 1-14. They propose an integer linear programming formulation, but they do not solve it. Instead, they propose a separate routing heuristic algorithm to deal with this problem, and claim a near-optimal solution quality. The heuristic algorithm has three stages: balanced path routing, wavelength assignment, and waveband switching. Since the proposed integer linear programming formulation is too complicated, the heuristic algorithm still uses a separate routing method, but it still has some limitations compared to the integrated routing method. However, an integrated routing algorithm is relatively difficult to design.

In [5], the authors develop a heuristic algorithm with dynamic traffic in order to minimize the port counts of MG-OXC and keep the blocking probability at an acceptable level. Note that the authors assume that the demultiplexing proportion of fiber-to-waveband and waveband-to-wavelength is fixed at each node, so it is not a very flexible method. In addition, they only conduct an experiment with a simple network with one fiber incident to every node; however the algorithm should be tested in a more complicated network.

The authors of [6] also propose an MG-OXC architecture to reduce costs and effectively utilize resources. They also designed two RWA heuristic algorithms, namely, dynamic tunnel allocation (DTA) and capacity-balanced static tunnel allocation (CB-STA) that take both online routing and offline routing into consideration.
[16] extends the work of [6]. The authors develop a formulation for a situation where traffic increases in order to effectively design MG-OXCs. This formulation can be simplified as two integer programming problems, and solved by optimization techniques. This is the first paper that tries to solve this problem with mathematical models. However, here the authors divide the formulation into two IP problems, which weakens to the original optimal solution.

### 1.3.3 Heterogeneous Network

There are very few related works on heterogeneous networks. [8] focuses on a heterogeneous network with static traffic. It proposes an integer programming formulation for routing and wavelength assignment and solves it by Lagrangean relaxation with a heuristic. The objective function is to minimize the most congested link. It assumes there are three kinds of switches: fiber capable, waveband capable, and lambda capable.

To the best of my knowledge, this work is the only paper that discusses RWA in heterogeneous networks. Therefore, our work contributes a great deal to network design and planning in heterogeneous networks.

### 1.3.4 Lagrangean Relaxation

In the 1970s, Lagrangean Relaxation was first proposed to deal with the large scale linear programming problem. Later, it was found that Lagrangean Relaxation also performed excellently in integer programming problems and even nonlinear programming problems.

When applying Lagrangean relaxation, the complicated constraints of the original hard problems can be properly relaxed. After dualizing the side constraints with multipliers to the objective function, the original problem becomes an easier Larangean problem. This new problem can then be divided into several independent subproblems with its constraints. Therefore, we only need to solve each subproblem by specific algorithms or exhaustive search within a smaller space. Since we relax some constraints of the original problem, the optimal value we find in the Lagrangean problem is a legitimate lower (upper) bound for a minimization problem (maximization problem).

However, the bound we find in the Lagrangean problem might violate some relaxed constraints in the original problem. Hence, it may not be a feasible solution after all, as we need to develop a heuristic algorithm to find an upper (lower) bound of the original problem. The bound should be a feasible solution to the original problem. By taking advantage of the multipliers originated in the Lagrangean problem, we can develop a relatively good heuristic algorithm to find a tighter bound in the feasible region.

In each iteration after finding the lower and upper bounds, we hope to find the tightest lower bound and upper bound. In order to find the tightest bounds, the multipliers solved in the Lagrangean problem are important. Although there are many ways to solve it, we adopt the subgradient method because it properly adjusts the parameters used for each iteration. The Lagrangean relaxation process is shown as in Figure 1-15.


Figure 1-15 Lagrangean Relaxation

### 1.4 Thesis Organization

In Chapter 2, we propose the problem formulation. In addition, the related notations and the graph transformation are also described. In Chapter 3, the solution approach to this problem is presented. In chapter 4, the getting primal feasible solution is proposed. We also develop a simple heuristic algorithm here for comparison with LR solution quality. The numerical experiment is described in

Chapter 5. Finally, in chapter 6, we present our conclusions and indicate the direction of future work.


## Chapter 2 Problem Formulation

### 2.1 Problem Description

This is an optical network planning and capacity management problem. In a heterogeneous WDM network with three different capable OXC (FSC, WBSC, LSC), how to allocate the three kinds of OXCs to minimize the total cost is the task of network planning. The output result of the problem must determine the routing and wavelength assignment of all the source-destination traffic demands. This is referred to as capacity management. Therefore, the RWA problem is part of our problem. Since the RWA problem has been proved to be NP-hard problem, the problem is also NP-hard.

In [3], this problem belongs to integrated offline routing in a heterogeneous network. We focus on a heterogeneous network because it iss not necessary for each node be hierarchical which is more expensive. We believe that compared to a homogeneous network, a heterogeneous network can utilize resources better and is
more cost effective.

Table 2-1 The assumptions, objective, and constraints

## Assumptions

1. There are three kinds of switches: fiber switch capable, waveband switch capable, and lambda switch capable.
2. All the traffic demands between source-destination pairs are static.
3. There is no wavelength or waveband conversion capability.

## Given

1. Network topology, including node set and link set.
2. The traffic demand between each source-destination.
3. The number of wavelengths a fiber can carry.
4. The number of wavebands a fiber can be divided into.
5. The cost functions of the three switches.

## Objective

To minimize the total cost.

## Subject to

1. Wavelength continuity.
2. Traffic demands.
3. Each link carries every lambda only once.
4. Each node installs only one kind of switch.

## Output Results

1. The kind of switch installed at each node.
2. All the paths with specific wavelengths between source-destination pairs are chosen.
3. Whether all the links with specific wavelength are chosen, or not.

### 2.2 Graph Transformation

Before formulating this problem, we need to apply a graph transformation (shown in Figure 2-1) in order to model the problem as an integer programming problem properly. First, we split each node into two-layer phantom nodes. The left side first layer phantom nodes accept the incoming fibers (IF), and each node accepts one fiber link. The right side first layer phantom nodes send the traffic to outgoing fibers (OF), and each node sends to one fiber link. There are three kinds of second layer phantom nodes which are for fiber switches, waveband switches and lambda switches.

Figure 2-1 shows that if a fiber switch is installed, $a_{l}$ is set to 1 , and the second layer phantom links, $d_{l}$, between second layer phantom nodes must be appropriately assigned in order to match all the second layer input and output phantom nodes. Similarly, if a waveband switch is installed, $b_{l}$ is set to 1 , and the second layer phantom links, $e_{l}$, between second layer waveband phantom nodes (each waveband layer) must be appropriately assigned in order to match all the second layer input and output phantom nodes. If a lambda switch is installed, all the first layer phantom nodes construct the link to the second layer lambda switch phantom nodes, which
means $c_{l}$ is set to 1 . In Figure 2-1, there are three input fibers and three output fibers. After the graph transformation, we can appropriately model this problem as an integer programming problem, as shown in section 2.4.


Lambda Switch

Figure 2-1 Graph Transformation

### 2.3 Notation

Here we give notations of the formulation, including the input parameters and decision variables shown in Table 2-2 and table 2-3:

Table 2-2 Input Parameters

| Input Parameters |  |
| :--- | :--- |
| Notation | Description |
| $N$ | the set of switch nodes in the network |
| $L$ | the set of physical optical links |
| $\bar{F}$ | the maximum number of fibers incident to a switch node |
| $L^{F_{a}}$ | the set of first layer FSC phantom links within a switch node |
| $L^{F_{b}}$ | the set of first layer WSC phantom links within a switch node |
| $L^{F_{c}}$ | the set of first layer LSC phantom links within a switch node |
| $L^{S_{a}}$ | the set of second layer FSC phantom links within a switch node |
| $L^{S_{b}}$ | the set of second layer WSC phantom links within a switch node |
| $Y_{n}^{\text {in }}$ | the set of first layer phantom input nodes of switch node $n$ |
| $Y_{n}^{\text {out }}$ | the set of first layer phantom output nodes of switch node $n$ |
| $V_{n}^{\text {in }}$ | the set of second layer phantom input nodes for switch node $n$ |
| $V_{n}^{\text {out }}$ | the set of second layer phantom output nodes for switch node $n$ |
| $J$ | the set of wavelengths on each link(assumed to be the same for all links) |
| $\mid \boldsymbol{J}$ | the number of wavelengths |
| $W$ | the set of origin-destination (OD) pairs requesting lightpath set-up |
| $W_{n}$ | the number of wavelengths in a waveband |
| the set of OD pairs originating at source node n |  |


| $S_{n}$ | the set of OD pairs where node $n$ is the source node |
| :--- | :--- |
| $B_{n}$ | the set of wavebands of WSC node $n$ |
| $P_{w}$ | The candidate path set of an OD-pair $w$ |
| $\lambda_{w}$ | lightpath demand of an OD-pair $w$ |
| $\delta_{p l}$ | 1, if path $p$ includes link $l ; 0$, otherwise |
| $\tau_{l j}$ | 1, if wavelength $j$ belongs to the phantom link $l ; 0$, otherwise |
| $\theta_{l b}$ | 1, if link $l$ is belongs to waveband $b ; 0$, otherwise |
| $\sigma_{l v}$ | 1, if link $l$ is incident to node $v ; 0$, otherwise |
| $\Psi_{n}\left(F_{n}\right)$ | the cost function of installing FSC at switch node $n ;$ |
| $\Omega_{n}\left(Z_{n}\right)$ | the cost function of installing WSC at switch node $n ;$ |
| $\Phi_{n}\left(U_{n}\right)$ | the cost function of installing LSC at switch node $n ;$ |

Table 2-3 Decision Variables

| Decision Variables |  |
| :--- | :--- |
| Notation | Description |
| $x_{p j}$ | 1, if lightpath $p$ uses wavelength $\mathrm{j} ;=0$, otherwise |
| $a_{l}$ | 1, if the first layer phantom FSC link, $l$, is selected; $=0$, otherwise |
| $b_{l}$ | 1, if the first layer phantom WSC link, $l$, is selected; $=0$, otherwise |
| $c_{l}$ | 1, if the first layer phantom LSC link, $l$, is selected; $=0$, otherwise |
| $F_{n}$ | 1, if FSC is installed at node $n ;=0$, otherwise |
| $Z_{n}$ | 1, if WSC is installed at node $n ;=0$, otherwise |
| $U_{n}$ | 1, if LSC is installed at node $n ;=0$, otherwise |
| $d_{l}$ | 1, if the second layer phantom FSC link $l$ is selected; $=0$, otherwise |
| $e_{l}$ | 1, if the second layer phantom WSC link $l$ is selected; $=0$, otherwise |

### 2.4 Problem Formulation

## Problem (P):

$\operatorname{Min} \sum_{n \in N}\left(\Psi_{n}\left(F_{n}\right)+\Omega_{n}\left(Z_{n}\right)+\Phi_{n}\left(U_{n}\right)\right)$
subject to:
$\sum_{p \in P_{w}} \sum_{j \in J} x_{p j}=\lambda_{w} \quad \forall w \in W$
$\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l} \leq 1 \quad \forall j \in J, l \in L$
$x_{p j}=0$ or 1
$\forall j \in J, p \in P_{w}, w \in W$
$F_{n}+Z_{n}+U_{n}=1$
$\forall n \in N$
$F_{n}=0$ or 1
$\forall n \in N$
$Z_{n}=0$ or 1
$\forall n \in N$
$U_{n}=0$ or 1
$\forall n \in N$
$\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l} \leq a_{l}$
$\forall j \in J, l \in L^{F_{a}}$
$\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l} \leq b_{l}$
$\forall j \in J, l \in L^{F_{b}}$
$\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l} \leq c_{l}$
$\forall j \in J, l \in L^{F_{c}}$
$a_{l} \sigma_{l v}=F_{n}$
$\forall l \in L^{F_{a}}, v \in Y_{n}^{\text {in }} \bigcup Y_{n}^{\text {out }}, n \in N$
$b_{l} \sigma_{l v}=Z_{n}$
$\forall l \in L^{F_{b}}, v \in Y_{n}^{\text {in }} \bigcup Y_{n}^{\text {out }}, n \in N$
$c_{l} \sigma_{l v}=U_{n}$
$\forall l \in L^{F_{c}}, v \in Y_{n}^{\text {in }} \bigcup Y_{n}^{\text {out }}, n \in N$
$a_{l}=0$ or 1
$\forall l \in L^{F a}$
$b_{l}=0$ or 1
$\forall l \in L^{F_{b}}$
$c_{l}=0$ or 1
$\forall l \in L^{F_{c}}$
$\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l} \leq d_{l}$
$\forall j \in J, l \in L^{S_{a}}$

$$
\begin{array}{ll}
\sum_{l \in L^{S_{a}}} d_{l} \sigma_{l v}=F_{n} & \forall v \in V_{n}^{i n}, n \in N \\
\sum_{l \in L^{S_{a}}} d_{l} \sigma_{l v}=F_{n} & \forall v \in V_{n}^{\text {out }}, n \in N \\
\sum_{w \in V^{2}} \sum_{p \in P_{v l}} x_{p j} \delta_{p l} \leq e_{l} \tau_{l j} & \forall j \in J, l \in L^{S_{b}} \\
\sum_{l \in L^{S_{b}}} e_{l} \theta_{l b} \sigma_{l v}=Z_{n} & \forall b \in B_{n}, v \in V_{n}^{i n}, n \in N \\
\sum_{l \in L^{S_{b}}} e_{l} \theta_{l b} \sigma_{l v}=Z_{n} & \forall b \in B_{n}, v \in V_{n}^{\text {out }}, n \in N \\
d_{l}=0 \text { or } 1 & \forall l \in L^{S_{a}} \\
e_{l}=0 \text { or } 1 & \forall l \in L^{S_{b}} \\
\sum_{p \in P_{w}} x_{p j} \delta_{p l} \leq 1 & \forall w \in W, j \in J, l \in L \cup L^{F_{a}} \cup L^{F_{b}} \cup L^{F_{c}} \cup L^{S_{a}} \cup L^{S_{b}}
\end{array}
$$

The objective function is to minimize the total cost of the switch nodes.

## Constraint (1):

The capacity constraint: Each source-destination traffic demand must be satisfied.

Constraint (2):
Each wavelength can be used at most once on each link.
Constraints (8), (9), (10), (17), (20):
They impose the limit that if the link is chosen, then the wavelength at each chosen link can be selected at most once.

Constraint (4):
Each switch node must use one of the three OXCs.
Constraint (3):
Each link can only either select the wavelength or not.

Constraints (5), (6), (7):
At each node n , the OXC can either be selected or not.
Constraints (14), (15), (16):
In the routing decision, the first layer phantom links can either be selected or not. Constraints (23), (24):

In the routing decision, the second layer phantom links can either be selected or not.

Constraints (11), (12), (13):
If an OXC is selected, the corresponding first layer phantom links must also be selected (set as 1).

Constraints (18), (19):
If a fiber switch is installed at the switch node, the second layer phantom links $\left(d_{l}\right)$ between the fiber switch's second layer phantom nodes must be properly chosen to match all the input and output phantom nodes. In other words, exactly one link will be incident to either the input or output phantom nodes if the fiber switch is installed. Otherwise, no phantom links should be selected.

Constraints (21), (22):
If a waveband switch is installed at the switch node, the second layer phantom links $\left(e_{l}\right)$ between the waveband switch's second layer phantom nodes must be properly chosen to match all the input and output phantom nodes. In other words, exactly one link will be incident to either the input or output phantom nodes if the fiber switch is installed. Otherwise, no phantom links should be selected.

Constraint (25):
This is a redundant constraint that does not influence the result; however it can limit the solution space when solving the subproblems.

## Chapter 3 Solution Approach

### 3.1 Lagrangean Relaxation

We relax Constraints (2), (8), (9), (10), (17), and (20) constraints and multiply them by the multiplier vectors vectors, $\mathrm{q}, \mathrm{r}, \mathrm{s}, \mathrm{t}, \mathrm{u}$, and v respectively, which adds to the objective function as follows:
$Z_{d}(q, r, s, t, u, v)$
$=\operatorname{Min}$

$$
\begin{aligned}
& 〔 \sum_{n \in N}\left(\Psi_{n}\left(F_{n}\right)+\Omega_{n}\left(Z_{n}\right)+\Phi_{n}\left(U_{n}\right)\right) \\
& +\sum_{l \in L} \sum_{j \in J} q_{l j}\left(\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l}-1\right)+\sum_{l \in L^{L_{a}}} \sum_{j \in J} r_{l j}\left(\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l}-a_{l}\right) \\
& +\sum_{l \in L^{r_{b}}} \sum_{j \in J} s_{l j}\left(\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l}-b_{l}\right)+\sum_{l \in L^{t_{c}}} \sum_{j \in J} t_{l j}\left(\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l}-c_{l}\right) \\
& \left.+\sum_{l \in L^{S_{a}^{a}}} \sum_{j \in J} u_{l j}\left(\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l}-d_{l}\right)+\sum_{l \in L^{s_{b}}} \sum_{j \in J} v_{l j}\left(\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l}-e_{l} \tau_{l j}\right)\right]
\end{aligned}
$$

subject to (1), (3)~(7), (11)~(16), (18), (19), (21)~(25).

### 3.2 Subproblem

After a proper rearrangement of the formulation, it becomes:
$Z_{D}(q, r, s, t, u, v)$

$$
\begin{aligned}
=\operatorname{Min}[ & \sum_{j \in J} \sum_{w \in W} \sum_{p \in P_{w}}\left(\sum_{l \in L} q_{l j} \delta_{p l}+\sum_{l \in L^{r_{a}}} r_{l j} \delta_{p l}+\sum_{l \in L^{L_{i}}} s_{l j} \delta_{p l}+\sum_{l \in L^{F_{c}}} t_{l j} \delta_{p l}+\sum_{l \in L^{S_{a}}} u_{l j} \delta_{p l}+\sum_{l \in L^{S_{b}}} v_{l j} \delta_{p l}\right) x_{p j} \\
& +\sum_{n \in N}\left(\Omega_{n}\left(Z_{n}\right)+\Psi_{n}\left(F_{n}\right)+\Phi_{n}\left(U_{n}\right)\right) \\
& -\left(\sum_{j \in J}\left(\sum_{l \in L^{R_{a}}} r_{l j} a_{l}+\sum_{l \in L^{r_{b}}} s_{l j} b_{l}+\sum_{l \in L^{r_{b}}} t_{l j} c_{l}+\sum_{l \in L^{s_{a}}} u_{l j} d_{l}+\sum_{l \in L^{S_{b}}} v_{l j} e_{l} \tau_{l j}\right)\right]-\sum_{j \in J} \sum_{l \in L} q_{l j}
\end{aligned}
$$

subject to (1), (3)~(7), (11)~(16), (18), (19), (21)~(25).

We can divide $Z_{D}(q, r, s, t, u, v)$ into two independent subproblems including their constraints.

### 3.2.1 Subproblem 1

$Z_{\text {sub } 3.1}(q, r, s, t, u, v)$
$=\operatorname{Min} \sum_{j \in J} \sum_{w \in W} \sum_{p \in P_{w}}\left(\sum_{l \in L} q_{l j} \delta_{p l}+\sum_{l \in L^{r^{R}}} r_{l j} \delta_{p l}+\sum_{l \in L^{r_{b}}} s_{l j} \delta_{p l}+\sum_{l \in L^{L^{t}}{ }^{c}} t_{l j} \delta_{p l}+\sum_{l \in L^{S_{a}}} u_{l j} \delta_{p l}+\sum_{l \in L^{S_{b}}} v_{l j} \delta_{p l}\right) x_{p j}$
subject to

$$
\begin{array}{lc}
\sum_{p \in P_{p}} \sum_{j \in J} x_{p j}=\lambda_{w} & \forall w \in W \\
x_{p j}=0 \text { or } 1 & \forall j \in J, p \in P_{w}, w \in W \\
\sum_{p \in P_{w}} x_{p j} \delta_{p l} \leq 1 & \forall w \in W, j \in J, l \in L \cup L^{F_{a}} \cup L^{F_{b}} \cup L^{F_{c}} \cup L^{S_{a}} \cup L^{S_{b}} .
\end{array}
$$

For $x_{p j}$, it can be decomposed into $|W|$ independent subproblems, and each of which can be solved by the shortest path algorithm and the Surballe's Link Disjoint K-shortest path algorithm [9], [10]. For each OD-pair, we calculate the shortest path cost of each wavelength. For the lightpath demand of each OD-pair, we calculate all the marginal costs of $k+1$ link disjoint shortest path of each wavelength ( $k$ is the current lightpath we find), and pick the smallest one as this lightpath demand cost.

The algorithm stops once we find enough traffic demand. Our algorithm is shown as Table 3-1, and the proof is given in Table 3-2.

The complexity of the k -shortest path algorithm for each iteration (to find one more lightpath) is $n^{2} \log _{n}$. Hence, the complexity of the k -shortest path algorithm is $n^{2}$ (shortest path algorithm) plus $(k-1) n^{2} \log n$. In the worst case, where the k lightpaths are chosen by the same wavelength, we must run $[1+2+3 \ldots+(k-1)]$ iterations. Therefore, for each OD-pair, $w$, the time complexity becomes $\mathrm{O}\left(|J| n^{2}+\mathrm{k}^{2} n^{2} \log n\right)$. Note that $n$ is the number of phantom nodes in the topology, and is proportional to the number of physical links; $|J|$ is the number of wavelengths a fiber can carry; and $k$ is the number of lighpaths of this OD-pair's total traffic demand.

Table 3-1 Find the minimum cost of K lightpaths for each OD-pair

```
Algorithm Min_K_Lightpath
begin
    for each wavelength \(j \in J\) do
    begin
        run Dijkstra's-shortest-path on each wavelength layer \(x_{j}^{1}\);
        do \(n p_{j}:=0 ; \quad / *\) num-disjoint-path on each wavelength set to zero */
    end;
    for each wavelength \(j \in J\) do
        find the minimum \(x_{j}^{1}, n p_{j}=1 ;\)
    end;
    repeat
        for wavelenth \(j:=1\) to \(J\) do
        begin
            if \(n p_{j}>0\) then
            begin
                run Link-Disjoint-k-shortest-path on wavelength \(j\) to
                calculate \(x_{j}^{n p_{j}+1}\);
            end;
            for each wavelength \(j \in J\) do
            begin
            find the minimum \(x_{j}^{n p_{j}+1}-x_{j}^{n p_{j}}\)
            \(n p_{j}:=n p_{j}+1 ;\)
            end;
    until all traffic demand of OD pair \(w, \lambda_{w}\) is satisfied
end.
```

Table 3-2 The proof of Min_K_Lightpath Algorithm

## Proof Min_K_Lightpath

1. First, we let $x_{j}^{t}$ be the t -link-disjoint shortest path for wavelength j , shown as following.

|  | 1-shortest | 2-shortest | 3-shortest | 4-shortest | $\ldots \ldots \ldots$. | t-shortest |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\lambda_{\mathbf{1}}$ | $x_{1}^{1}$ | $x_{1}^{2}$ | $x_{1}^{3}$ | $x_{1}^{4}$ |  | $x_{1}^{t}$ |
| $\lambda_{\mathbf{2}}$ | $x_{2}^{1}$ | $x_{2}^{2}$ | $x_{2}^{3}$ | $x_{2}^{4}$ |  | $x_{2}^{t}$ |
| $\lambda_{\mathbf{3}}$ | $x_{3}^{1}$ | $x_{3}^{2}$ | $x_{3}^{3}$ | $x_{3}^{4}$ |  | $x_{3}^{t}$ |
| $\ldots \ldots \ldots .$. | $\ldots \ldots \ldots .$. | $\ldots \ldots \ldots .$. | $\ldots \ldots \ldots .$. | $\ldots \ldots \ldots \ldots$ | $\ldots \ldots \ldots .$. | $\ldots \ldots \ldots \ldots$ |
| $\lambda_{\mathbf{j}-1}$ | $x_{j-1}^{1}$ | $x_{j-1}^{2}$ | $x_{j-1}^{3}$ | $x_{j-1}^{4}$ |  | $x_{j-1}^{t}$ |
| $\lambda_{\mathbf{j}}$ | $x_{j}^{1}$ | $x_{j}^{2}$ | $x_{j}^{3}$ | $x_{j}^{4}$ |  | $x_{j}^{t}$ |

2. While the traffic demand of od-pair $w$ is one, we choose the smallest $x_{s}^{1}$ as the shortest path, and $n p_{s}$ is set to 1 . While the traffic demand of od-pair $w$ is two, we choose the smallest among $x_{j}^{n p_{j}+1}-x_{j}^{n p_{j}}$ for all the wavelength $j$.

We can show that it is impossible to choose $x_{p}^{2}$, while $p \neq s$, because

$$
x_{p}^{2} \geqq 2 x_{p}^{1} \geqq x_{p}^{1}+x_{s}^{1} .
$$

3. Since the traffic demand from 1 to 2 , we choose the smallest $x_{j}^{n p_{j}+1}-x_{j}^{n p_{j}}$. We assume the traffic demand from $k$ to $k+1$, we can choose the smallest $x_{j}^{n p_{j}+1}-x_{j}^{n p_{j}}$. If the traffic demand is $k+2$, we can show that it is impossible to choose $x_{j}^{n p_{j}+M}{ }_{-}$ $x_{j}^{n p_{j}}$, while $M \geqq 2$, and sacrifice some original selected path by Lemma 3.1 which is given in Table 3-3.
4. Since we already know that from $k$ to $k+1$, it is held. Then we prove that from $k+1$ to $k+2$ traffic demands, it is still held. According to the mathematical induction, for all $n$ to $n+1$ traffic demands, we can always choose the smallest $x_{j}^{n p_{j}+1}-x_{j}^{n p_{j}}$ as the $n+1$ smallest cost lightpaths.

Table 3-3 Lemma 3.1

## Lemma 3.1

1 While $M=2, x_{j}^{n p_{j}+2}-x_{j}^{n p_{j}} \geqq 2\left(x_{j}^{n p_{j}+1}-x_{j}^{n p_{j}}\right) \geqq\left(x_{j}^{n p_{j}+1}-x_{j}^{n p_{j}}\right)+\left(x_{i}^{n p_{i}}-x_{i}^{n p_{i}-1}\right)$, where $i$ is the biggest $x_{i}^{n p_{i}}+x_{i}^{n p_{i}-1}$ among all the wavelength.

2 We assume while $M=m$, it is not possible to choose $x_{j}^{n p_{j}+m}-x_{j}^{n p_{j}}$, and sacrifice some original selected path.

While $M=m+1, \quad x_{j}^{n p_{j}+m+1}-x_{j}^{n p_{j}} \geqq\left(x_{j}^{n p_{j}+m+1}-x_{j}^{n p_{j}+m}\right)+\left(x_{j}^{n p_{j}+m}-x_{j}^{n p_{j}}\right)$, where
$\left(x_{j}^{n p_{j}+m+1}-x_{j}^{n p_{j}+m}\right) \geqq\left(x_{i}^{n p_{i}}-x_{i}^{n p_{i}-1}\right)$, where $i$ is the biggest $x_{i}^{n p_{i}}+x_{i}^{n p_{i}-1}$ among all the wavelength.

Hence, we find that While $M=m+1$, it is not possible to choose $x_{j}^{n p_{j}+m+1}-$ $x_{j}^{n p_{j}}$, and sacrifice some original selected path.

3 According to the mathematical induction, for all $\mathrm{M} \geqq 2$, traffic demand from $k+1$ to $k+2$, it is impossible to choose any $x_{j}^{n p_{j}+M}-x_{j}^{n p_{j}}$, and sacrifice some original selected path.

### 3.2.2 Subproblem 2

$Z_{\text {sub } 3.2}(q, r, s, t, u, v)$
$=\operatorname{Min} 〔 \sum_{n \in N}\left(\Psi_{n}\left(F_{n}\right)+\Omega_{n}\left(Z_{n}\right)+\Phi_{n}\left(U_{n}\right)\right)-$

$$
\begin{equation*}
\left.\sum_{j \in J}\left(\sum_{l \in L^{r_{l}}} r_{l j} a_{l}+\sum_{l \in L^{r_{b}}} s_{l j} b_{l}+\sum_{l \in L^{r_{b}}} t_{l j} c_{l}+\sum_{l \in L^{S_{a}}} u_{l j} d_{l}+\sum_{l \in L^{s_{l}}} v_{l j} e_{l} \tau_{l j}\right)\right] \tag{SUB3.2}
\end{equation*}
$$

subject to
$\begin{array}{ll}F_{n}+Z_{n}+U_{n}=1 & \forall n \in N \\ F_{n}=0 \text { or } 1 & \forall n \in N \\ Z_{n}=0 \text { or } 1 & \forall n \in N \\ U_{n}=0 \text { or } 1 & \forall n \in N\end{array}$
$a_{l} \sigma_{l v}=F_{n}$
$\forall l \in L^{F_{a}}, v \in Y_{n}^{\text {in }} \bigcup Y_{n}^{\text {out }}, n \in N$
$b_{l} \sigma_{l v}=Z_{n} \quad \forall \quad \forall l \in L^{F_{b}}, v \in Y_{n}^{\text {in }} \bigcup Y_{n}^{\text {out }}, n \in N$
$c_{l} \sigma_{l v}=U_{n}$
$\forall l \in L^{F_{c}}, v \in Y_{n}^{\text {in }} \bigcup Y_{n}^{\text {out }}, n \in N$
$a_{l}=0$ or 1
$\forall l \in L^{F_{o}}$
$b_{l}=0$ or 1
$\forall l \in L^{F_{b}}$
$c_{l}=0$ or 1
$\forall l \in L^{F_{c}}$
$\sum_{l \in L^{L^{a}}} d_{l} \sigma_{l v}=F_{n}$
$\forall v \in V_{n}^{i n}, n \in N$
$\sum_{l \in L^{S_{a}^{a}}} d_{l} \sigma_{l v}=F_{n} \quad \forall v \in V_{n}^{\text {out }}, n \in N$
$\sum_{l \in L^{s_{b}}} e_{l} \theta_{l b} \sigma_{l v}=Z_{n}$
$\forall b \in B_{n}, v \in V_{n}^{i n}, n \in N$
$\sum_{l \in L^{S_{b}}} e_{l} \theta_{l b} \sigma_{l v}=Z_{n}$
$\forall b \in B_{n}, v \in V_{n}^{\text {out }}, n \in N$
$d_{l}=0$ or 1
$\forall l \in L^{S_{a}}$
$e_{l}=0$ or 1
$\forall l \in L^{S_{b}}$.
$U_{n}, F_{n}, Z_{n}, a_{l}, b_{l}, c_{b}, d_{l}$, and $e_{l}$, they can be decomposed into $|N|$ independent subproblems, each of which can be solved by an exhaustive search on $U_{n}, F_{n}, Z_{n}$ to determine $a_{l}, b_{l}, c_{l}, d_{l}$, and $e_{l}$. While determining $d_{l}$ and $e_{l}$, we need to implement Bipartite Weighted Matching algorithm [11] to decide the second layer phantom links that match the second layer phantom nodes.

For each node $n$, we only need to run a bipartite matching algorithm $|J||B|+1$ times $(|J /|B|$ times for waveband switch, 1 time for fiber switch). Hence, the time complexity of subproblem 2 for each node is $\left.\mathrm{O}\left[(|J||B|)^{*}(\operatorname{Inc})_{n}\right)^{3}\right]$, where Inc ${ }_{n}$ is the number of incident fibers to node $n$.

### 3.3 The Dual Problem and the Subgradient Method

Based on the algorithms for solving the subproblems, we can find the optimal solution of the Lagrangean problem effectively. According to the weak Lagrangean duality theorem, for any given nonnegative Lagrangean multiplier, the optimal solution of the Lagrangean relaxation problem's objective function is the lower bound of the original problem. We use the subgradient method to solve this problem as follows:
$Z_{D}=\min Z_{D 1}(q, r, s, t, u, v)$
subject to:
$q, r, s, t, u, v \geqq 0$

Let S be the subgradient vector of $Z_{D l}(q, r, S, t, u, v)$. During the $\mathrm{k}^{\text {th }}$ iteration of the subgradient optimal procedure, the multiplier vector $m^{k}=\left(q^{l k}, r^{2 k}, s^{3 k}, t^{4 k}, u^{5 k}, v^{6 k}\right)$ is renewed by the function $m^{k+1}=m^{k}+\alpha^{k} S^{k} . \quad S^{k}(q, r, s, t, u, v)=\left(\sum_{w \in V} \sum_{p \in P_{w}} x_{p j} \delta_{p l}-1\right.$, $\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l}-a_{l}, \quad \sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l}-b_{l}, \quad \sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l}-c_{l}, \quad \sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l}-d_{l}$, $\left.\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l}-e_{l} \tau_{l j}\right)$. The quantity of $\alpha^{k}$ is decided by the value: $\delta \frac{Z_{I P}{ }^{k}-Z_{D 1}\left(m^{k}\right)}{\left\|S^{k}\right\|^{2}}$, where $Z_{I P}{ }^{k}$ is the lower bound value of the original problem's objective function at the $\mathrm{k}^{\text {th }}$ iteration, and $\delta$ is a constant $(0 \leq \delta \leq 2)$.

### 3.4 Lagrangean Relaxation with Heuristics

We have already described the Lagrangean relaxation process in Section 1.3.4. The Lagrangean relaxation algorithm for this optimization problem is shown in Table 3-3.

Table 3-4 Lagrangean Relaxation with Heuristics Algorithm

```
Algorithm LRH
begin
    initialize the Lagrangean multiplier vector \(\mathbf{q}:=\mathbf{0 , r}:=\mathbf{0 , s}:=\mathbf{0}, \mathbf{t}:=\mathbf{0}, \mathbf{u}:=\mathbf{0}, \mathbf{v}:=\mathbf{0}\);
    \(U B:=\) cost of lambda switches at every node and \(L B:=-\infty\);
    quiescence_age:=0;
    stepsize_scalar \(\delta:=2\);
    for each \(k:=1\) to Iteration_Number do
    begin
        solve sub-problem S1; /*described in Section 3.2.1*/
        solve sub-problem S2; /*described in Section 3.2.2*/
        \(Z_{\text {dual }}=Z_{S 1^{\prime}}+Z_{S 2^{-}} \sum_{j \in J} \sum_{l \in L} q_{l j} ;\)
        if \(Z_{\text {dual }}>\) LB
            then \(L B:=Z_{\text {dual }}\) and quiescence_age \(:=0\);
            else quiescence_age:= quiescence_age+1;
        run Primal Heuristic Algorithm /*described in section 4.1*/
        if quiescence_age:=improvement_counter then
        \(\delta:=\delta / 2 ;\)
        quiescence age: \(=0\);
        if \(u b<U B\) then \(U B:=u b ; \quad / * u b\) is the newly computed upper bound */
        update the step size and the multiplier vector /*by the subgradient
                                    method described
                                    in Section 3.2.3*/
    end;
end.
```



## Chapter 4 Getting Primal Feasible Solutions

### 4.1 Getting Primal Algorithms

First, we take the recommended switch nodes from the subproblem 2 and the multiplier value as the link weight. For each OD-pair's traffic demand, we run Dijkstra shortest path algorithm on each wavelength layer. We then choose the smallest wavelength layer as the lightpath for this traffic demand and set the link weight all along the path to be infinite. If we cannot find any path for the OD-pair's demand, we calculate the average wasted lambda of every node, pick the node with highest average wasted lambda, and update it (from the fiber switch to the waveband switch or from the waveband switch to the lambda switch). After updating the node, we reroute all the traffic demands from the beginning. When we have satisfied all the traffic demands, that is the primal feasible solution. We then downgrade the updated nodes as the updated sequence, and reroute all the traffic demands again. If there is no feasible solution, we update the node to the original switch and downgrade next node. After trying downgrade every updated node, we will find a better feasible solution. If all switch nodes are lambda switches and we still cannot find a path for a demand, there is no feasible solution. The algorithm is shown in Table 4-1.

Table 4-1 Getting Primal Heuristic Algorithm

```
Algorithm Primal Heuristic
begin
    for each node \(n \in N\) do
    begin
        for each link \(l \in L, L^{F_{a}}, L^{F_{b}}, L^{F_{c}}, L^{S_{a}}, L^{S_{b}}\) do cost \(_{l j}:=\infty\);
        for each link \(l \in L\) do cost \(_{l j}:=q_{l j}\);
        if \(F_{n}:=1\) for each \(\operatorname{link} l \in L^{F_{a}} \mathbf{d o} \cos t_{l j}:=r_{l j}\); for each \(\operatorname{link} l \in L^{S_{a}} \mathbf{d o} \cos t_{l j}:=u_{l j}\);
        if \(Z_{n}:=1\) for each link \(l \in L^{F_{b}}\) do \(\cos _{l j}:=\mathrm{s}_{l j} ;\) for each \(\operatorname{link} l \in L^{S_{b}}\) do \(\cos t_{l j}:=v_{l j}\);
        if \(U_{n}:=1\) for each link \(l \in L^{F_{c}}\) do cost \(_{l_{j}}:=t_{l j}\);
    end
    for each OD pair \(s d:=1\) to \(|\mathrm{S}|\) do num-path-setup \(s d:=0\);
    repeat
        for each OD pair \(s d:=1\) to \(|\mathrm{S}|\) do
        begin
            if num-path-setup \({ }_{s d}<\lambda_{w}\) then
            begin
                run Dijkstra's-shortest-path on each wavelength layer;
                if the shortest path exists then
```

```
                    begin
                    designate the wavelength associated with the shortest path \(j^{*}\);
                    for all links \(l\) on the shortest path do
                    begin \(\operatorname{cost}_{l j^{*}:=\infty}\);
                            if \(l \in L^{S_{a}}, L^{S_{b}}\) then set links to other fibers in this node be infinite;
                    end;
                end;
                        else if all the nodes are lambda switches then
                        return "infeasible";
                        else for each node n do calculate the average wasted lambdas wasted \(_{n}\)
                            find the largest wasted \(_{n^{*}}\);
                            if \(F_{n} *=1\) then \(F_{n} *=0, Z_{n} *:=1\), update the corresponding link cost;
                            if \(Z_{n} *=1\) then \(Z_{n} *:=0, U_{n} *=1\), update the corresponding link cost;
                            end;
            end;
        end;
    until all OD pair demand are satisfied;
```

for each node updated
downgrade the node as the updating sequence;
if there is no feasible solution, then
upgrade the node to the original switch
else find a better feasible solution.
end;
update the upper bound;
end.

The average wasted lambda of each node is calculated as follows. At each node, we first calculate the wasted lambdas from each input fiber (waveband) to the corresponding output fiber (waveband). We then calculate the average number of values at every node and pick the largest node to update to the waveband switch (lambda switch).

In Fig.4-2, there are three fibers incident to this fiber switch node. The first input fiber is routed to second output fiber. Since this node is the destination of $\lambda_{1} \sim$ $\lambda_{16}$ and is the source of $\lambda_{21} \sim \lambda_{32}$, only $\lambda_{17} \sim \lambda_{20}$ are routed to the second output fiber. If we update this node to a waveband switch, $\lambda_{21} \sim \lambda_{32}$ can be routed to other output fibers, because these wavelengths are on other wavebands. Hence, the number of wasted lambdas of the first fiber at this node is 12 .


Figure 4-1 Wasted Lambda Calculation

### 4.2 Simple Heuristic Algorithms

In addition to using Lagrangean relaxation, we have also developed a simple heuristic algorithm, which is a modified version of the heuristic algorithm in [8]. If we cannot find a path for a traffic demand, we take the routing and wavelength assignment idea from [8], without seeing the multiplier information and the solutions of subproblems, as the RWA method in this heuristic and the switch update criteria from the getting primal heuristic algorithm in Section 4.1. The simple heuristic algorithm is shown in Table 4-2.

Table 4-2 Simple Heuristic Algorithm

```
Algorithm Simple Heuristic Algorithm
begin
    for each node \(n \in N\) do
    begin
        \(F_{n}:=1 ;\)
        for each link \(l \in L, L^{F_{a}}, L^{F_{b}}, L^{F_{c}}, L^{S_{a}}, L^{S_{b}}\) do \(\cos _{l j}:=\infty\);
        for each link \(l \in L\) do \(\operatorname{cost}_{l j}:=1\);
        for each link \(l \in L^{F_{a}}\) do cost \(_{l j}:=1\); for each link \(l \in L^{S_{a}}\) do \(\operatorname{coss}_{l j}:=1\);
    end
    for each OD pair \(s d:=1\) to \(|\mathrm{S}|\) do num-path-setup \({ }_{s d}:=0\);
    repeat
        for each OD pair \(s d:=1\) to \(|\mathbf{S}|\) do
        begin
            if num-path-setup \({ }_{s d}<\lambda_{w}\) then
            begin
            run Dijkstra's-shortest-path on each wavelength layer;
            if the shortest path exists then
            begin
                        designate the wavelength associated with the shortest path \(j^{*}\);
                        for all links \(l\) on the shortest path do
                    begin \(\operatorname{cost}_{l j}{ }^{*}=\infty\);
                            if \(l \in L^{S_{a}}, L^{S_{b}}\) then set links to other fibers in this node be infinite;
                    end;
                end;
                else if all the nodes are lambda switches then
                        return "infeasible";
                        else for each node n do calculate the average wasted lambdas wasted \(_{n}\)
                        find out the biggest wasted \(_{n^{*}}\);
                            if \(F_{n} *=1\) then \(F_{n} *=0, Z_{n} *=1\), update the corresponding link cost;
                            if \(Z_{n} *:=1\) then \(Z_{n} *:=0, U_{n} *:=1\), update the corresponding link cost;
                end;
            end;
        end;
    until all OD pair demand satisfied;
```

for each node updated
downgrade the node as the updating sequence;
if there is no feasible solution, then
upgrade the node to the original switch
else find a better feasible solution.
end;
update the upper bound;
end.

We will compare the solution quality of LR and this simple heuristic algorithm (SA) in Chapter 5. Getting primal heuristic algorithm of LR1 and simple algorithm (SA1) do not consider the node downgrading process after finding a feasible solution. Getting primal heuristic algorithm of LR2 and simple algorithm (SA2) take node downgrading into consideration.

The complexity of getting primal heuristic algorithm of LR1 for each iteration is $\mathrm{O}\left(|J| n k(p h n)^{2}\right)$ where $k$ is the number of lighpaths, $n$ is the number of nodes, and phn is the number of phantom nodes. The complexity of getting primal heuristic algorithm of LR1 for each iteration is also $\mathrm{O}\left(|J| n k(p h n)^{2}\right)$.

## Chapter 5 Computational Experiments

### 5.1 Experiment Environment

In this chapter, we conduct several computational experiments to test the solution quality and effectiveness of our solution approach. In the following, we conduct the following experiments: from a small network topology to a large network topology, and from a high connectivity network to a low connectivity network; Table $5-1$ shows the general parameter settings for the computational experiments.

Table 5-1 Parameters of the computational experiments

| Number of Nodes | $7 \sim 28$ |
| :--- | :--- |
| Number of Links | $28 \sim 90$ |
| Number of wavelengths | $16 \sim 32$ |
| Number of wavebands | 4 |
| Number of lighpaths | $120 \sim 280$ |
| Number of Iteration | 1000 |
| Improvement Counter | 20 |
| Initial Upper Bound | Cost of lambda switches at every <br> node |


| Initial Scalar of Step Size | 2 |
| :--- | :--- |
| Test Platform | CPU : Intel Pentium-4 2.4 GHz, <br> AMD K8 2.4GHz <br> OS : MS Windows 2000 <br> MS Windows XP |
|  | OS |

### 5.2 Seven-node Small Network

### 5.2.1 Network Topology

The seven-node small network topology is shown in Fig. 5-2, and general information about the network is given in Table 5-2.


Figure 5-1 Seven-node Small Network Topology

Table 5-2 Seven-node Small Network

| Number of Nodes | 7 |
| :---: | :---: |
| Number of Links | 28 |
| Number of wavelengths | 16 |
| Number of wavebands | 4 |
| Connectivity | 0.667 |
| Average node degree | 4 |
| OD-pairs number | 28 |
| Lightpath Demand of each OD-pair | $4.6 \sim 6.0$ |
| Total number of lightpaths | $130 \sim 168$ |

### 5.2.2 Solution Quality

In Fig. 5-2, we observe that the upper and lower bounds converge quite well when the average lightpath demand per OD-pair is 5.1 . We can find that the gap between the bounds is between $3 \%$ and $45 \%$, as shown in Fig. 5-3. As the lightpath demand increases, the gap between the upper bound lower bound also dramatically increases. The reason is that the cost structure of switches is proportional to the number of ports used. In this case, the waveband switch is four times more expensive than the fiber switch, while the lambda switch is sixteen times more expensive than the fiber switch. Therefore, as traffic demand increases, more waveband and lambda switches are needed, the cost may increase substantially. This is the reason that the upper bound increases dramatically, while the lower bound increases relatively slowly. Hence, the gap becomes larger.

In Fig. 5-4, as the traffic demand is low, it is easier to route the traffic, so the solutions of SA and LR are very close. However, as the lightpath demand increases, LRs are vastly superior to SAs. In addition, when traffic demand is very heavy, SA can not even find a feasible solution, whereas LR can still find a fairly good solution.

In Table 5-3 we observe that, on average, LRs at at least 100 percent more cost efficient than SAs when the the traffic demand is heavier. However, LR2 is not significantly better than LR1 because the connectivity of the network is too high, and it is hard to downgrade any node.

7 node small network with od-pair with avg. 5.1 lightpaths


Figure 5-2 The upper and lower bounds of the seven-node network


Figure 5-3 Gap of different traffic demand


Figure 5-4 The SA and LR solution quality in the 7-node small network
Table 5-3 The improvement ratio in 7-node network

| Avg. Lightpath <br> Demand of each <br> OD-pair | SA1 | LR1 | Improvement <br> ratio(\%) | SA2 | LR2 | Gap(\%) | Improvement <br> ratio(\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4.8 | 7400 | 5600 | 32.14 | 7400 | 5600 | 3.12 | 32.14 |
| 4.9 | 29600 | 5600 | 428.57 | 29600 | 5600 | 1.73 | 428.57 |
| 5 | 37400 | 5600 | 567.86 | 37400 | 5600 | 2.01 | 567.86 |
| 5.1 | 50600 | 16400 | 208.54 | 50600 | 16400 | 17.10 | 208.54 |
| 5.2 | 65600 | 23600 | 177.97 | 48800 | 23600 | 16.87 | 106.78 |
| 5.3 | 72800 | 31400 | 131.85 | 50600 | 29600 | 8.03 | 70.95 |
| 5.4 | 82400 | 33200 | 148.19 | 56600 | 31400 | 1.56 | 80.25 |
| 5.5 | 82400 | 33200 | 148.19 | 63800 | 33200 | 16.57 | 92.17 |
| 5.6 | 82400 | 37400 | 120.32 | 64400 | 37400 | 31.14 | 72.19 |
| 5.7 | No feasible | 40400 | $\infty$ | No feasible | 40400 | 46.51 | $\infty$ |
| 5.8 | No feasible | 55400 | $\infty$ | No feasible | 46400 | 74.11 | $\infty$ |
| 6 | No feasible | 70400 | $\infty$ | No feasible | 53600 | 117.63 | $\infty$ |

### 5.2.3 Computation Time

In Fig.5-5, we can see that as the traffic demand increases, the number of iterations also increases. In Fig. 5-6, the computation time also increases as the traffic demand increases, because more switch nodes are updated, and once a node is updated, all the traffic demand must be rerouted from the beginning.


Figure 5-5 The number of iterations in the 7-node network


Figure 5-6 Computation time of the 7-node network

### 5.3 GTE Network

### 5.3.1 Network Topology

The GTE network is a well known medium-sized network often used for computational experiments. The network's topology is shown in Fig. 5-7, and general information about the network is given in Table 5-4.

Table 5-4 GTE Network

| Number of Nodes | 12 |
| :---: | :---: |
| Number of Links | 50 |
| Number of wavelengths | 16 |
| Number of wavebands | 4 |
| Connectivity | 0.379 |
| Average node degree | 4.167 |
| Number of OD-pairs | $40-70$ |
| Lightpath Demand of each OD-pair | 4 |
| Total number of lightpaths | $160 \sim 280$ |



Figure 5-7 The GTE Network Topology

### 5.3.2 Solution Quality

The convergence between the upper and lower bounds of 35 OD-pairs with a traffic demand of four wavelengths is shown in Fig. 5-8. In Fig. 5-9, we find that as the number of OD-pairs increases, LR performs better than SA. Furthermore, the improvement ratios shown in Table 5-5 confirm that LRs are much more cost effective than SAs. LR2 is better than LR1 in about $10-20 \%$ because as the network size increases, there are potentially more nodes which were updated before can be downgraded after some other nodes updated later.


Figure 5-8 The upper and lower bounds in the GTE Network


Figure 5-9 The solution quality of SA and LR in the GTE network

Table 5-5 The improvement ratios of the GTE network

| OD pair | SA1 | LR 1 | improvement <br> ratio(\%) | SA2 | LR2 | improvement <br> ratio(\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 40 | 10000 | 10000 | 0 | 10000 | 10000 | 0 |
| 45 | 10000 | 10000 | 0 | 10000 | 10000 | 0 |
| 50 | 10000 | 10000 | 0 | 10000 | 10000 | 0 |
| 55 | 52600 | 23800 | 121.00 | 47800 | 20200 | 136.63 |
| 60 | 94600 | 71200 | 32.87 | 73000 | 55600 | 31.295 |
| 65 | 140800 | 89800 | 56.79 | 125800 | 72400 | 73.76 |
| 70 | 148000 | 95800 | 54.49 | 128200 | 74800 | 71.39 |
| 75 | No feasible | 138400 | $\infty$ | No feasible | 114800 | $\infty$ |

### 5.4 USA Network

### 5.4.1 Network Topology

The USA network topology is shown in Fig. 5-10, and general information about it is given in Table 5-6. Compared to the GTE network, the USA network is a much larger topology.

Table 5-6 The USA Network

| Number of Nodes | 28 |
| :---: | :---: |
| Number of Links | 90 |
| Number of wavelengths | 16 |
| Number of wavebands | 4 |
| Connectivity | 0.12 |
| Average node degree | 3.214 |
| Number of OD-pairs | $30-50$ |
| Lightpath Demand of each OD-pair | 4 |
| Total number of lightpaths | $120 \sim 200$ |



Figure 5-10 The USA Network topology

### 5.4.2 Solution Quality

Fig. 5-11 shows the convergence of the upper and lower bounds in the USA network with 32 OD-pairs. In Fig. 5-12 and Table 5-7, the LRs performance are between 50 percent to 100 percent better than that of SAs. LR2 is better than LR1 in about 5-10 \% because as the network size increases, there are potentially more nodes which were updated before can be downgraded after some other nodes updated later.

USA Network with 32 OD-pairs


Figure 5-11 The upper bound and lower bounds of the USA network


Figure 5-12 SA and LR solution quality in the USA network

Table 5-7 The improvement ratios of the USA network

| OD-pair | SA1 | LR1 | improvement <br> ratio(\%) | SA2 | LR2 | improvement <br> ratio(\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 30 | 29600 | 24400 | 21.31 | 18000 | 17600 | 2.27 |
| 32 | 45200 | 32400 | 39.51 | 36000 | 26600 | 35.34 |
| 34 | 68200 | 36000 | 89.44 | 42200 | 30800 | 37.01 |
| 36 | 84400 | 45400 | 85.90 | 70000 | 41000 | 70.73 |
| 38 | 77800 | 44200 | 76.02 | 68400 | 44200 | 54.75 |
| 42 | 99200 | 59000 | 68.14 | 82200 | 56400 | 45.74 |
| 44 | 117000 | 63800 | 83.39 | 84800 | 61000 | 39.02 |
| 48 | 122600 | 80000 | 53.25 | 91200 | 72200 | 26.32 |
| 50 | 144000 | 99600 | 44.58 | 102000 | 76200 | 33.86 |

### 5.4.3 Computation Time

As we can see in Figures 5-13 and 5-14, the number of iterations and the computation time both increase as the OD-pair traffic demand increases. The reason is that there are more updated nodes when the lightpath demand is heavy.


Figure 5-13 The number of iterations in the USA Network


Figure 5-14 The computation time of the USA network

### 5.5 Discussion

### 5.5.1 Number of Wavelengths

From a small 7 -node network to a large USA network, we find that our LR approach always outperforms the SA approach.

With a fixed traffic demand, as the wavelengths in a fiber increase, the cost decreases. In Fig. 5-15, there are 55 OD-pairs, each of which has four lightpaths. We believe that as DWDM technology improves, which means that the number of wavelengths carried in a fiber will increase, the overall cost of planning will decrease. This observation is shown in Fig. 5-15 with 8 wavelengths, 12 wavelengths, 16
wavelengths, and 20 wavelengths.


Figure 5-15 The cost structure of different numbers of wavelengths in a fiber

### 5.5.2 Number of Wavebands

With a fixed traffic demand, if the number of wavebands in a fiber is too many or too few, the cost increases. Since the number of wavebands that a fiber can be divided into is also an input parameter, we must decide the number of wavebands before switch allocation and RWA.

In Fig. 5-16, we can see that the cost is lowest when the number of wavebands is moderate in the GTE network with 24 lambdas. There are 55 OD-pairs, each of which has six lightpaths. . If there are too many wavebands, a waveband switch will be closer to a lambda switch, and the cost will be higher because there will be too many
ports in the waveband switch. On the other hand, if there are too few wavebands, a waveband switch will be closer to a fiber switch, and the cost is higher because too many waveband switches will be updated to lambda switches. This observation provides a good guideline for deciding the number of wavebands.


Figure 5-16 Cost structure of different number of waveband in a fiber

### 5.5.3 Scalability

As we can see in the time complexity analysis, we find that Subproblem 1 ( each OD-pair: $\left.\mathrm{O}\left(|J| n^{2}+\mathrm{k}^{2}(p h n)^{2} \log (p h n)\right)\right)$ or getting primal heuristic $\left(\mathrm{O}\left(|J| n k(p h n)^{2}\right)\right)$ dominates the complexity. We find that when the number of wavelengths increases, the computation time will increase linearly; when the number of phantom nodes increases, the computation time will at most increase proportionally to $(p h n)^{2} \log (p h n)$.

Note that the number of phantom nodes is proportional to the number of physical links. Therefore, even as DWDM technology improves or the network topology becomes larger, the computation time can still be handled in the acceptable range. If we have powerful computers, the scalability will be much better.


## Chapter 6 Conclusion and Future Work

### 6.1 Conclusion

As WDM networks have emerged as promising candidates for future networks with large bandwidth, efficient utilization of the limited and expensive components in networks becomes an import research issue. The cost of optical network planning results primarily from optical cross-connects (OXCs), while the cost of which is proportioned to the number of ports used. WDM technology has improved so rapidly that a fiber can carry more than 200 wavelengths, however, the cost associated with the lambda switch is extremely high when there are so many ports. Furthermore there are limitations to the switch design with so many ports.

In the past, most research has focused on the routing and wavelength assignment (RWA) problem with lambda switch nodes, but this is not applicable anymore. [8] first proposed a mathematical formulation to solve the RWA problem with different switches (i.e., a fiber capable switch or a lambda capable switch), but the switch nodes must be determined first. However, the authors do not propose a good way to
allocate the switch nodes.

In GMPLS, it is proposed that FSC (fiber switch capable), WBSC (waveband switch capable), and LSC (lambda switch capable) interfaces be used to generate a general control plane.

With these three kinds of switch, we can allocate switch nodes properly based on the lowest cost, and satisfy all the static traffic demand in a heterogeneous network. In order to solve the problem, we represent it as a mathematical formulation. To the best of our knowledge, this has not been done before. The waveband switch capability is also modeled as a mathematical formulation for the first time. As the integer programming problem itself is highly complicated, we adopt Lagrangean relaxation as the solution approach.

We also propose a simple heuristic algorithm modified from the RWA problem in [8], and conduct several experiments on different network topologies from small-scale to large-scale networks. We find that the experiment results of Lagrangean Relaxation are much better then those of the simple heuristic algorithm. In addition, since it is a planning problem, the computation time is also within a tolerable range. Therefore, it is an excellent approach for dealing with the optical network planning problem in heterogeneous networks.

Finally, we develop some simple principles so that network planners can effectively determine the number of wavelengths a fiber should carry and number of wavebands a fiber should be divided into in an optical planning situation.

### 6.2 Future Work

Currently, very little researches focuses on network planning in a heterogeneous network. We first model this problem as a mathematical formulation, and provide [8] a good switch allocation guideline for further RWA determination. Our work and [8] both focus on static traffic demand in a heterogeneous network. As for dynamic traffic demand, there is still a great need to conduct more research into network planning and RWA issues.

Some literature discusses optical cross-connects design issues in homogeneous networks [3], [4], [5], [6], [16]. Although a few develop a mathematical formulation, they do not try to solve it by optimization-based methods. Instead, they develop many heuristics to deal with both static and dynamic traffic OXC design issues. It would be worth reformulating this design problem as a solvable mathematical formulation, and solve it by an optimization-based approach. The solution quality would definitely be better then a heuristic-based approach. Therefore, we can use resources more effectively and efficiently.


## Reference

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