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博士論文

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有線電視網路規劃演算法之研究

Network Planning Algorithms

in CATV Networks

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Network Planning Algorithms in CATV Networks

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The seal of National Taiwan University is a circular emblem. It features a central figure, likely a scholar or a historical figure, surrounded by a ring of Chinese characters. The outermost ring contains the university's name in both Chinese and English. The seal is rendered in a light, semi-transparent style, serving as a background for the text.

July 2006



This dissertation is dedicated to my beloved wife,
Cathy Wu, my lovely children, Joyce, Yuming, and Yugar,
and my great parents, Jin-Sun Peng and Show-Chin Liao.



謝詞

刻意的把謝詞的撰寫時間留到最後，在九年的博士班生涯中，實在說『得之於人者太多，出之於己者太少。』漫長的九年當中，不知道有多少人的關心與付出，才能支持我走到現在。如果說成功是來自於百分之一的天才，與百分之九十九的努力，那麼我要說，我能夠成功的完成論文，實在是百分之一的自我堅持，與百分之九十九許許多多的人的支持與鼓勵。

最重要也是最感動的，是我的指導教授林永松博士。恩師的驚人的學識與智慧，我想是所有認識與不認識的人都會深深折服的，然而，我要說，恩師的為人處事之態度，才是讓學生永難忘懷。在這些年當中，自己在無邊的學海中載浮載沈，幾度覺得自己行將山窮水盡，甚至已經下定決心不再回來。是恩師一次次的伸出雙手，將我從窮山惡水之中拉出來，鼓勵我繼續前進。每每在我對自己失去信心之際，恩師以他的生命與智慧，給了我繼續努力的力量。『一日為師，終身為父。』我想這當是恩師最佳的寫照。

論文能夠如期完成，不能不感謝呂俊賢老師、林盈達老師、趙啟超老師、以及孫雅麗老師的建議與協助。在論文初稿與口試期間，能承蒙幾位老師在他們繁忙的生活當中，犧牲了許多時間來協助我，不論是在知識的領域或是學者應有的態度與風範上，都展現出令學生敬佩與效法的身影。在未來的學術生涯中，學生當繼續努力以不辜負諸位老師的提攜與期許。

在許多艱困的時刻，我永遠不會忘記是許多朋友鼓舞與安慰，支撐著我走下去。旭成，雖然你小我一屆，但是我總覺得你是學長，因為我總是跟著你的步伐，套用你的格式而成功的通過層層考驗。你與佩玲時時刻刻的盯著我不讓我放棄，相信此時此刻，你們一定也跟我一樣高興。許多的學長學弟，志浩、國忠、演福、政達、柏皓、文政、宏翕、勇誠、義倫，不管是畢業了還是還沒畢業的，這些年大家相互扶持，這份革命情感，我將永遠記得。特別感念是幾位同班同學，余平、照輝、敬仁，歷經千辛萬苦，雖然你們有些人有了不同的生涯規劃，然而，我始終覺得，這個學位是我們所共有的。

最後，僅以此論文獻給我的父母，彭金松先生與廖秀琴女士。沒有他們就沒有今天的我。陪伴我這些年的內人吳淑琴小姐，許多辛酸相信只有她能理解。而我的三個兒女，于瑄、昱銘、昱嘉，看到你們，就看到我的快樂與安慰。

要感謝的人實在太多，我想，只有謝天吧！

論文摘要

有線電視網路規劃演算法之研究

彭國維

中華民國九十五年七月

指導教授：林永松 博士

國立台灣大學資訊管理研究所

由於具有高頻寬、高普及率以及容易擴充等優點，有線電視網路已然成為國家資訊基礎建設中不可或缺的一部份。除了政府相關部門制訂有線電視法等相關法規來規範有線電視業者外，同時為滿足上行通路的系統信號品質要求，要建構一雙向傳輸有線電視網路所必須考慮的層面相當多且複雜。整體來說，有效的提升有線電視網路的管理與規劃，傳統的有線電視網路將不僅是能夠提供電視節目的傳送，更可能是一種經濟有效的通訊網路系統。然而，將原本單向傳輸的有線電視網路擴充為互動式存取網路卻必須面對許多問題。另一方面，由於有線電視網路的管理與規劃具有相當的複雜性，因此，今日有線電視網路的設計，仍依賴網路規劃人員的經驗，而無法透過可驗證與重複實施的程序來加以規劃。因此，所得到的設計其網路服務品質往往是無法預測的，在傳統的電視節目傳輸上，或許較低的服務品質只是帶來較差的畫面品質，但是在資料傳輸時，卻可能成為極大的問題。如何能經濟有效的提升網路的效能，成了有線電視網路的重要議題。

在此論文中，我們將針對有線電視網路規劃問題進行探討，使用數學模型來描述此類網路規劃問題，並使用幾何規劃法作為基礎，以最佳化的方式提出適合的演算法。本論文的研究內涵與成果簡述如下：

- 最小成本之有線電視網路規劃問題：考慮路由決策與訊號品質規範下所有給定有線電視用戶滿足有線電視服務的最小成本有線電視網路系統。我們成功的將此問題以數學模型進行描述，並提出適用幾何規劃法的修改數學模型，以進行最佳化為基礎演算法的發展。
- 有線電視網路規劃的單層解題程序：傳統的網路規劃方法僅考慮網路訊號品質的追蹤與計算，對於所規劃的網路成本並未能進行最佳化調整。即便有少

數研究嘗試進行網路規劃問題的成本最小化，所得的結果仍不盡理想。本研究發展了單層有線電視網路規劃演算法，考慮一次解決頭端至所有用戶端的有線電視網路成本最佳化。我們以修改式最陡梯度法改善了之前的結果，成本降低幅度在 51% 至 92% 之間。此外，為求進一步改善計算的效率，降低所使用的計算時間，我們針對了修改式最陡梯度法的步幅起始值，分析與網路問題特性的關係。我們提出了步幅起始值的設定調整機制，大幅減少了所需的計算時間，而能達到相同的解題品質。

- 有線電視網路規劃的多層解題程序：對於較大型有線電視網路規劃問題，採用單層解題程序無法於合理時間內解決時，我們需要將原問題拆解成數個較小的網路規劃問題，分層加以解決。根據我們的實驗結果，多層解題程序的計算時間平均為單層解題程序所花時間的 40%，隨著網路大小的增加，多層解題程序所花的計算時間與單層解題程序相較差異快速增加。雖然計算速度提升，然多層解題程序犧牲了全域調整的可能性，這影響了所得到的解題品質。多層解題程序所獲得的最小成本，相較單層解題程序要高出 2% 至 45%。

論文的最後，我們提出三個未來重要的延續研究議題，以供後續學者進行研究。這些議題包括：多層網路規劃法中各層間的調整機制、新應用環境下的有線電視網路規劃問題、以及混合式光纖電纜有線電視網路規劃問題。

關鍵詞：有線電視網路、史坦那樹、修改式最陡梯度法、多層解題程序、網路規劃、幾何規劃法、數學規劃、網路最佳化

Dissertation Abstract

Network Planning Algorithms in CATV Networks

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July 2006

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An increasing number of new services are now running on CATV networks. The earliest CATV (Community Antenna Television) systems were constructed in small towns or semi-rural areas, where off-air television reception was poor or unavailable [1]. Because of their popularity and high bandwidth, CATV such networks have become one of the most popular technologies for providing a “last-mile” communication platform. The quality of CATV network systems depends to a large extent on the experience of the designers who must consider the performance constraints mandated by standards and government regulations. Consequently, the quality of CATV network design may be unreliable, and in many cases poor.

In this dissertation, we study CATV networks planning problems. Mathematical formulations are used to model the planning problems, and geometric programming method, based on the proposed mathematical formulations, is adopted to solve the network planning problems. The scope and contributions of this dissertation are highlighted by the following.

For the min-cost CATV networks planning problem, we propose a mathematical model to describe CATV networks planning problem. Based on some mathematical features of the model, some reformulations are necessary to solve the problem. The surrogate functions are used to reformulate the objective function and some constraints.

By applying some nonlinear programming techniques, the single layer solution procedure for CATV network planning problems is developed. Some computational experiments are described and explained. From the experiment results, the solution

procedure we developed is better than previous works. The comparison showed that our solution procedure is better in most of cases. The improvements on minimum costs are ranged from 51% to 92%. Based on the experiment results, we get some important finding in this problem, especially about the parameters settings in solution procedure. By the setting rules presented in this chapter, the solution quality, both the minimum cost and the scalability of the problem, can be further improved.

From the analysis of the solution procedure, however, we still could not deal with problems with too many nodes. Therefore, a multilayer solution procedure is proposed in Chapter 4. By layering a large network into several smaller networks, we can divide the problem and conquer every sub problems in reasonable time. After that, we can treat each network as a macro user in upper layer, and construct the network planning problem for upper layer. By summation the costs of upper layer and every sub layers, we can get the total cost of the entire network. By the multilayer solution procedure, we can solve CATV network planning problems with more nodes. We have compared with the single-layer solution procedure and show that only 40% of time is needed in multi-layer solution procedure. On the other side, the minimum costs solved by multilayer solution procedure are ranged from 2% to 45% larger than single-layer solution procedure. By balancing the computation time and solution quality, the multilayer solution procedure still provides a way to solve a larger network in limited time.

Besides the costs and computing time, we have developed algorithms for placement of drop points. In order to improve the costs of CATV networks, the placement of drop points in clusters is adjusted by proposed globally adaptive placement algorithm. Based on experiment results, the reduced costs ranges from 9% to 13%. With tradeoff between computing time and costs, we propose partially adaptive placement algorithm, which only adjust the leave nodes on upper layer networks. Compared with globally adjustment, the computing time is reduced to 61.5% and only 4.88% cost increased.

Finally, we point out three challenging issues to be tackled in the future. These issues include adjustment procedure between layers in multiplayer solution procedure, how to apply the solution procedures to other kinds of application environments, and modifications for HFC (Hybrid Fiber/Coax) networks.

Keywords: CATV Network, Modified Steepest Descent Method, Multilayer Solution Procedure, Network Planning, Geometric Programming, Mathematical Modeling, Network Optimization.



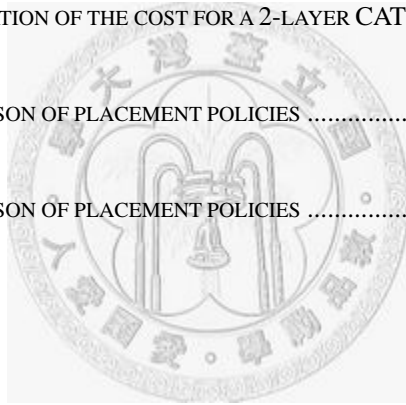
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CHAPTER 1 INTRODUCTION

1.1 Overview

An increasing number of new services are now running on CATV networks. The earliest CATV (Community Antenna Television) systems were constructed in small towns or semi-rural areas, where off-air television reception was poor or unavailable [1,2]. Because of their popularity and high bandwidth, CATV such networks have become one of the most popular technologies for providing a “last-mile” communication platform.

However, the quality of traditional CATV network systems may not be able to fully support new services, like Movies-On-Demand (MOD) and Voice-over-IP (VoIP). Thus the operators of CATV network systems need to improve their current network equipments and capacity to accommodate new network traffic.

The quality of CATV network systems depends to a large extent on the experience of the designers who must consider the performance constraints mandated by standards and government regulations. Obviously, the CATV network design process is quite complicated [3,4,5,6], and many kinds of CATV CAD software have

been developed to simplify the work of designers [7]. These CAD tools help designers track some parameters of the networks they develop and allow them to try different design strategies. Even so, the tools could not guarantee the best design and the quality of CATV networks still depends to a large extent the expertise of the designers. Consequently, the quality of CATV network design may be unreliable, and in many cases poor.

1.2 Research Scope

Our goal is to develop a near-optimal two-way CATV network design algorithms to minimize total installation costs, subject to the performance constraints [8,9,10]. From the perspective of network management, a mathematical design algorithm should be helpful to network planners. Our algorithm is based on mathematical programming, relaxation techniques, and heuristics [11,12,13,14,15,16,17,18]. Because of the nature of the problem, the nonlinear property is unavoidable and must therefore be dealt with.

In the next chapter, we introduce a mathematical model to describe CATV networks. Based on some mathematical features of the model, some reformulations are necessary to solve the procedure. In Chapter 3, by applying some nonlinear programming techniques, the single layer solution procedure for CATV network planning problems is developed. Some computational experiments are described and explained. From the experiment results, the solution procedure we developed is better than previous works. The comparison showed that our solution procedure is better in most of cases. The improvements on minimum costs are ranged from 51% to 92%. Based on the experiment results, we get some important finding in this problem, especially about the parameters settings in solution procedure. By the setting rules presented in this chapter, the solution quality, both the minimum cost and the scalability of the problem, can be further improved.

From the analysis of the solution procedure, however, we still could not deal with problems with too many nodes. Therefore, we propose a multilayer solution procedure in Chapter 4. By layering a large network into several smaller networks, we can divide the problem and conquer every sub problems in reasonable time. After that,

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Finally, we have concluded some results in Chapter 5. By comparing the results from single-layer solution procedure and general engineering procedure in CATV industry, we have shown that the single-layer solution procedure actually provides a better solution. However, the computational complexity of single layer solution procedure is too high to deal with large networks. Based on the analysis of single-layer solution procedure, we propose the multilayer solution procedure to solve problems with too many nodes in CATV networks. By clustering and layering the large network, we can solve the network planning problems of CATV networks in reasonable time. Although some quality is sacrificed, we can handle large networks that the single-layer solution procedure could not within reasonable time. At the end of this chapter, we conclude some directions that future research can be underneath.

1.3 Research Background

1.3.1 CATV Communication Network Architecture

In Figure 1-1, shows a traditional CATV system architecture, including the Head End (HE) and the distribution network. The function of the HE is to process and transmit programs received from satellites or self-developed content. Additionally, the HE is responsible for monitoring and controlling the whole CATV network. Typically, the network topology is a tree rooted at the HE, but it could be a star or beehive structure [19].

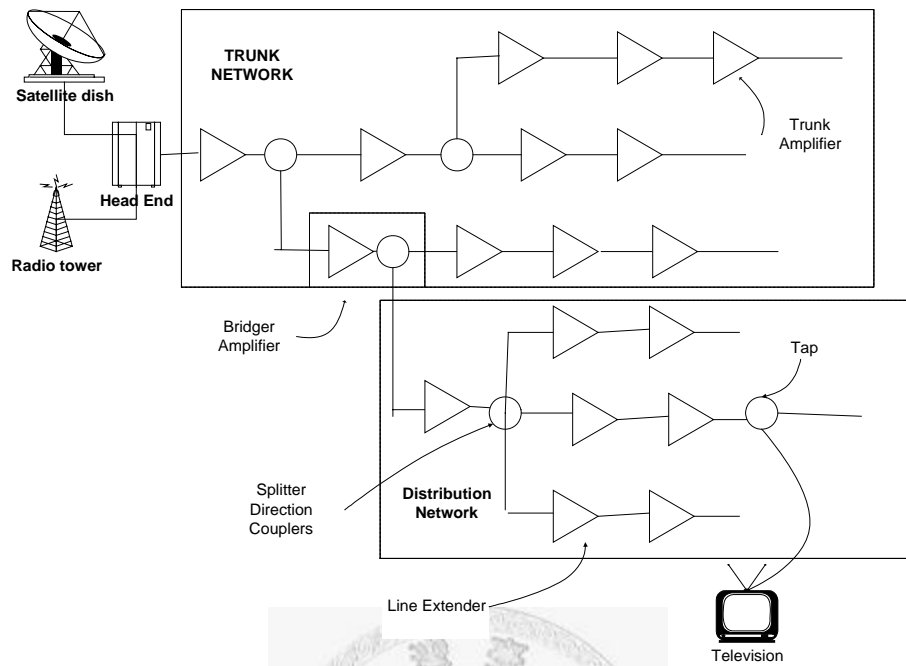


Figure 1-1: The network structure of CATV networks

A typical CATV network topology and a number of key transmission components are shown in Figure 1-2. In order to construct a high-quality CATV system, we must first explore the relationships among the components and their combined effect on end-to-end performance.

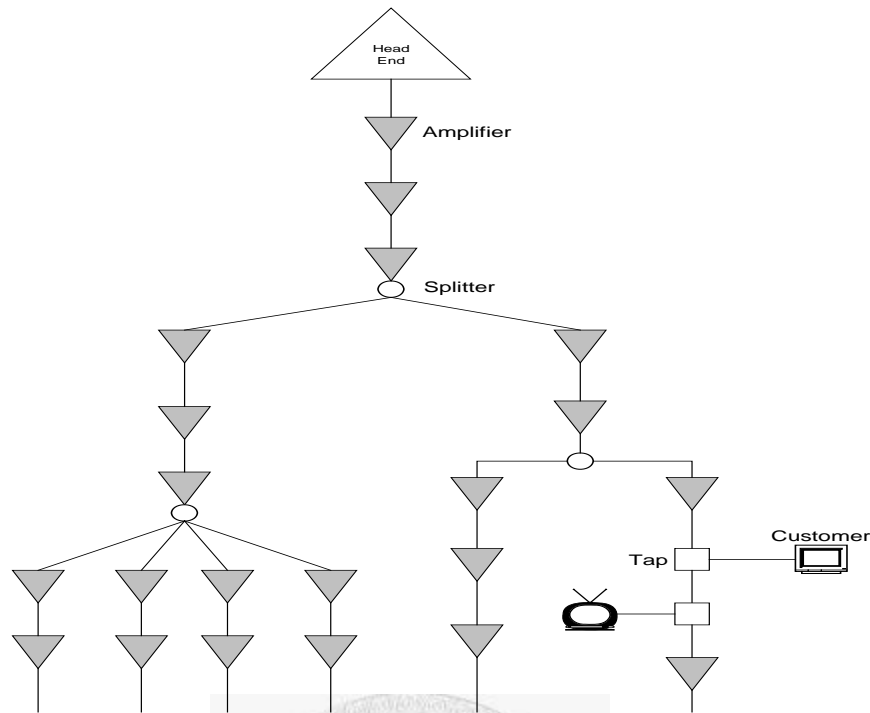


Figure 1-2: The Topology of a CATV Networks

From the figure, it is clear that a CATV system contains many physical parts and devices. Since the number of possible combinations of selection and allocation of components is very large, it is a very challenging task to build feasible CATV network systems. In the mean time, we hope to minimize total deployment costs. In Chapter 2, we will discuss this problem in details.

1.3.2 Internet Access over CATV Communication Networks

It is possible to carry two-way signals on a cable system. The coaxial cable is not itself directional, and sending signals on it in one direction does not preclude sending signals in the other direction. Enabling upstream transmission requires three types of technical changes to the network:

1. Spectrum must be allocated for the signals that travel in the upstream direction. Figure 1.3 shows a commonly used spectrum map for the signals traveling over cable network; the range 5 to 42 MHz is typically dedicated to upstream transmission [10].

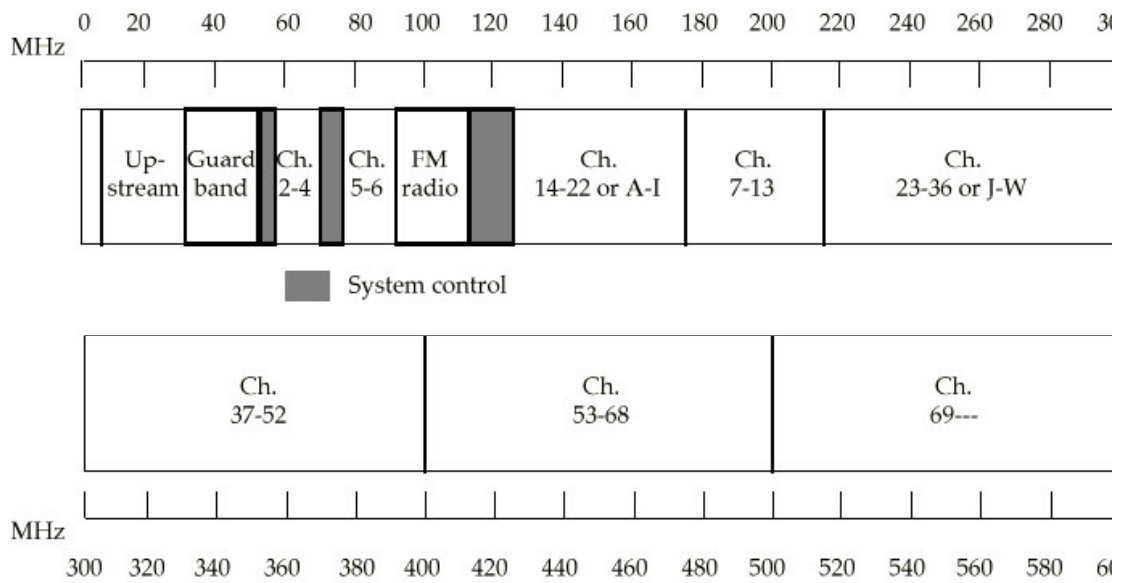


Figure 1-3: Frequency Spectrum of CATV Networks

2. In cable systems, the directionality of the network comes from the amplifiers in the systems that amplify signals from an incoming cable onto an outgoing cable. The tree topology along with directional amplifiers delivers signals only outward from the head end. Cable systems with two-way capabilities use amplifiers that work in both directions. The trick is that only one portion of the cable spectrum is amplified in each direction, so that signals in one frequency range are carried in one direction and signals in another range are carried in the other direction [5]. The direction that returns signals toward the head end is called the upstream path or the return path, as Figure 1.4.

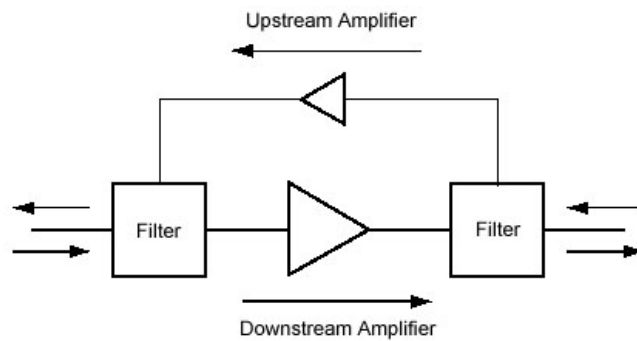


Figure 1-4: Two-way Amplifiers

3. The Techniques of Modulation: The most basic question for the physical layer in digital cable systems is what modulation schemes should be used on the network. The options available include well-understood techniques as Quadrature Phase Shift Keying (QPSK) and Quadrature Amplitude Modulation (QAM). The various solutions impose different equipment costs, achieve different coding efficiencies (the amount of capacity in a given amount of spectrum), and have different sensitivities to noise and distortion in the cable network. The choice of modulation schemes may depend on the frequency allocation for upstream and downstream communication, because different frequency ranges have different noise characteristics.

4. The share of upstream channels: Downstream transmission from the head end is broadcast, the same signals are sent on all wires. But upstream transmission is personal: each subscriber is trying to place a different signal onto the network. When going up the tree these different signals must eventually share the same piece of transmission spectrum. Since the stations have to share the upstream channels, it must implement a Medium Access Control (MAC) protocol. The first MAC interface specifications to be completed are the Data Over Cable Service Interface Specifications (DOCSIS) [3], started as a project of the Multimedia Cable Network System (MCNS) organization. In parallel to the MCNS activities, the Project P802.14, Cable-TV Protocol Working Group of the IEEE LAN/MAN Standards Committee has been working on producing a standard suitable for two-way transmission of multiple services over Cable-TV system [14][8]. In [20] C. Y. Huang and Y. D. Lin compare these two MAC interface standards in much more detailed.

5. The Noise Problem in Upstream Path: When designing CATV networks, the problem of noise on the upstream path must also be considered. The topology of cable systems puts the upstream path at an inherent disadvantage, because branching in a cable tree is carried out by passive devices, namely, splitters and taps. On the downstream path, a signal passed through a splitter is attenuated on the splitter's outputs, but the noise is also attenuated, therefore the splitter does not change the signal-to-noise ratio. On the upstream path, however, a splitter's outputs become the inputs, and the splitter simply combines the incoming signals and noise from branches. If a signal is only present on one input, but noise is present on both, the combination adds the noise together, resulting in a reduced signal-to-noise ratio as the signal is passed along the upstream path. The head end only receives an upstream signal from a single source, but it receives noise from all sources connected to the upstream path. The problem, called noise funneling [21], is illustrated in Figure 1-5. The upstream path is therefore inherently noisier than the downstream path. The size and shape of the cable system, the source of the noise, and how well the noise is controlled all affect this problem.

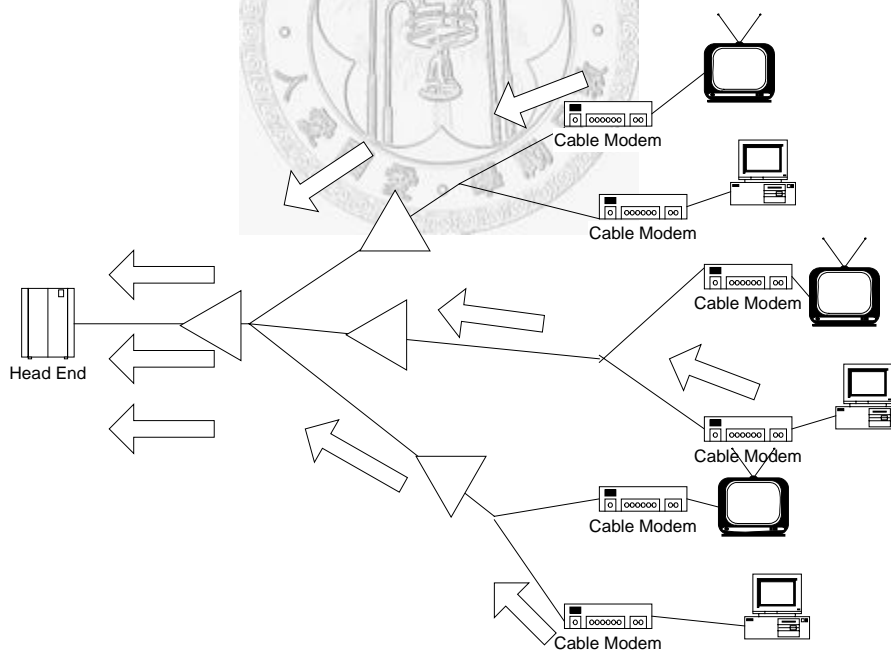


Figure 1-5: Noise-funneling effect

5. The inefficiency of TCP protocol in asymmetry links: Some researches [22,23] have shown that the performance of TCP protocol is poor in asymmetry links. Since the size of sliding windows is adjusted by the round-trip time, the congestion of

slower uplinks is the nature of CATV networks. Consequently, the utilization of faster downlink would be low. In the related papers, modifications of TCP protocol are suggested to accommodate the asymmetry property of CATV networks.

1.3.3 Network Planning of CATV Communication Networks

Normally the cable system is designed from the head end to the system ends, keeping track of the system losses and gains so each subscriber tap have a signal level within proper limits. The network planning of HFC networks are achieved by designing the head end, trunk networks, and distribution networks separately [19]. In the following paragraph, we will explain network-planning procedures, which are used by most of CATV network operators currently.

1. Trunk system design:

The design philosophy will be demonstrated by an example of a portion of a system shown in Fig. 1.6. Notice that the output of the head end into the trunk cable is at the same level as the desired amplifier output. Also notice that as a result of the high loss of 2500 ft of cable at 220 MHz the +32-dBmV signal level decreases to +12 dBmV and at channel 2 only to +19-dBmV. Obviously the amplifier needs to have an equalizer installed to equalize the difference.

Since the trunk amplifiers are spaced according to the unity gain building block theory their positions are basically fixed according to the accumulated losses. The other factor for the positions of amplifiers is power supply. Since amplifiers are active component, the positions for possible electrical power supply are also considered.

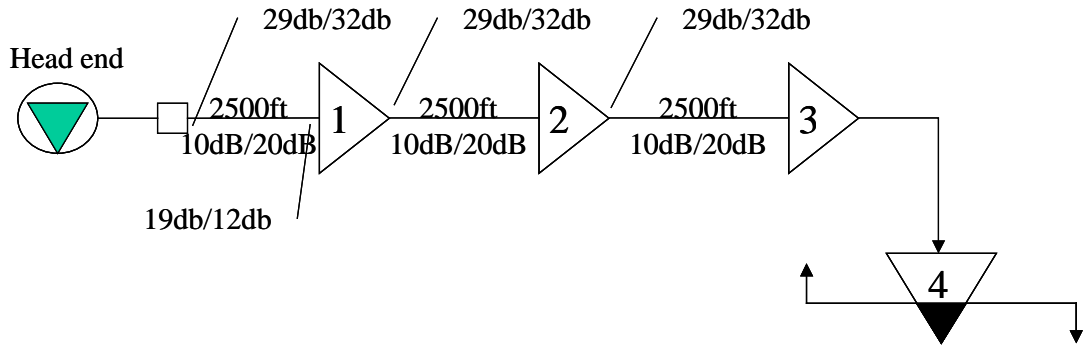


Figure 1-6: Head end to hub system.

2. Feeder system design

When doing design work on feeder systems the main goal is to provide the proper signal level at the subscriber's tap port. For example, the signal level at the subscriber's tap port in Figure 1-7 should be between 17 dBmV and 10 dBmV.

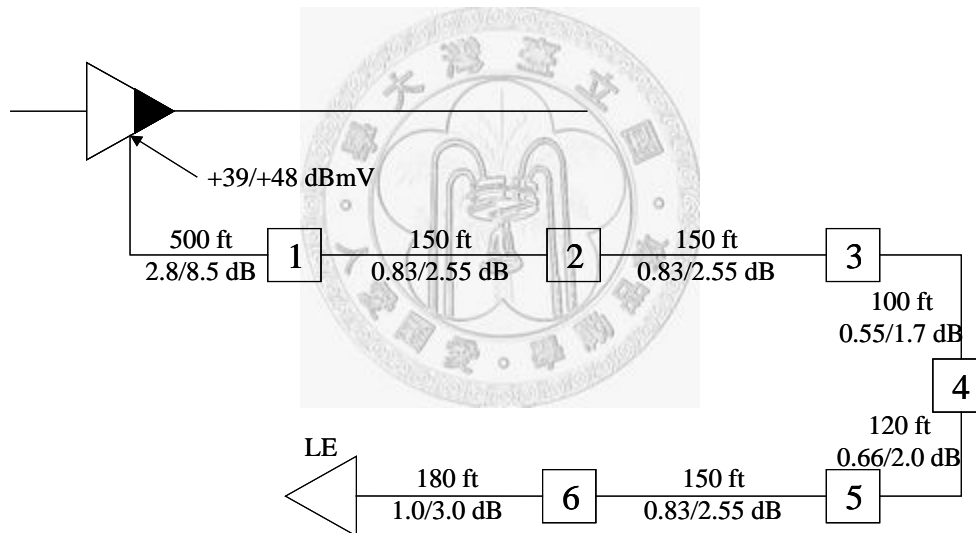


Figure 1-7: Span of taps example.

Most designs should allow a tap port for each dwelling unit with the exception of large apartment houses. From the calculations made in Table 1-1 the cable lengths with their losses decrease the signal into the subscriber taps. The signal into the tap is divided equally by the tap value to the four subscriber ports, with the signal being passed through to the output connector to the next tap through the following span of cable.

Table 1-1: System points versus signal level or signal loss

Point in system	CHANNEL 2, dB/450M Hz	Point in system CHANNEL 2, dB/450MH z	Point in system CHANNEL 2, dB/450M Hz
Bridger output level	39/48	Cable loss (150 ft)	Signal in taps 30.56/26.6
Cable loss (500 ft)	2.8/8.5	Signal in tap 3	Tap 5 value 14 dB
Signal in tap 1	36.2/39.5	Tap 3 value	Tap 5 port signal 16.56/12.6
Tap 1 value	23 dB	Tap 3 port signal	14-dB Tap through loss 1.4/2.0
Tap 1 port signal	13.2/16.5	20-dB Tap through loss	Signal out tap 5 29.16/24.6
23-dB Tap through loss	0.5/0.8	Tap 3 output signal	Cable loss (150 ft)
Tap output signal	35.5/38.7	Cable loss (100 ft)	Signal in tap 6 28.33/22.0
Cable loss (150 ft)	0.83/2.55	Signal in tap 4	Tap 6 value 11 dB
Signal in tap 2	34.67/36.15	Tap 4 value	Tap 6 port signal 17.33/11.0
Tap 2 value	20 dB	Tap 4 port signal level	11-dB Tap through loss 2.9/3.5
Tap 2 port signal level	14.67/16.15	17-dB Tap through loss	Signal out tap 6 25.43/18.5
20-dB Tap through loss	0.6/1.0	Tap 4 output level	Cable loss (180 ft)
Tap 2 output signal	34.1/35.15	Cable loss (120 ft)	Signal at end (180ft) 24.43/15.5

3. Reverse Cable systems

Since the frequency bands on a cable system can be separated by using appropriate filters, one band of frequencies can be assigned as the downstream or

forward system and the other band designated the upstream or reverse system. This capacity makes the cable system bidirectional. Amplifier modules are incorporated in the same housing and also use the same power supply module. Such an amplifier configuration is depicted in figure 1-8, which shows the forward and reverse signal paths.

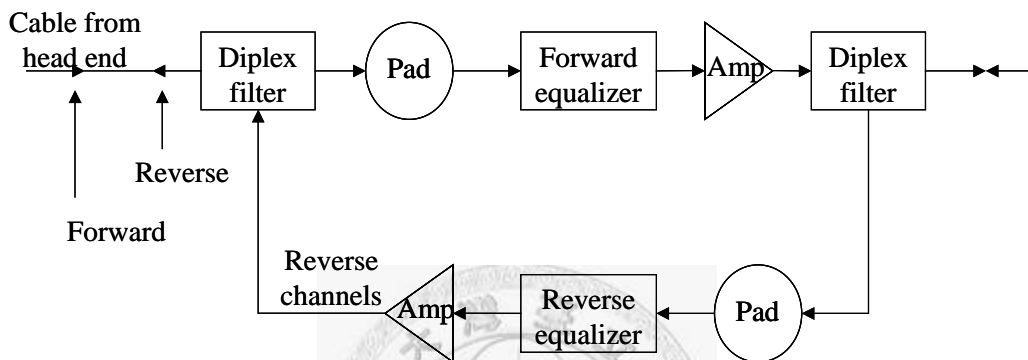


Figure 1-8: Forward-reverse amplifier configuration.

From the discussion in previous section, the noise funneling effect is an important issue when we design the reverse cable systems. The 5- to 30-MHz reverse bandwidth of the subsplit type of two-way cable system operates fairly well. The cable loss at these low frequencies chiefly produces low-gain, low-noise amplifiers in the reverse direction. Limiting the number of amplifier station decrease the amount of reverse noise. To limit noise increase, addressable bridger leg switching has to be used so the reverse signals are not buried in noise, which lessens the usefulness of the reverse system. If a completely instituted reverse system is needed, then careful design of the whole system, both forward and reverse, should be made to limit the number of amplifier stations, reverse amplifiers, etc. Bridger-leg switching may also be used to turn on the areas feeding upstream signals only as necessary by a controlling computer system.

E. R. Barlett has mentioned that the cable television design process is a lot like bookkeeping [19]. Keeping track of the signal level at the system branches, all the tap levels, as well as the carrier-to-noise and intermodulation figures is a difficult job at best. The computer-aided design (CAD) software of cable systems is a great benefit to

designers.

P. P. Yermolov has compared software packages in design of CATV networks [7]. Existing software for CATV design differs by its structure and functions. The most simple and functionally limited is the software called CATV Designer. It is intended for the calculation of television distribution networks and allows creating these network schemes using multitude of elements, to calculate characteristics and to show the report with obtained results.

Other software packages are more powerful and complex. For example, Symplex Suite of software from SpanPro, Inc. is a collection of applications designed for broadband cable systems. This suite of software both manages and models modern coaxial broadband networks. By using this suite of software, the designers can construct, test, and tune a virtual cable network. This package is allowed to produce cost reports, bills of materials, and track the status of designs.

With these software packages, the designers can reduce the calculation work like signal level tracking, cost accumulation, and network verification. However, the design quality, like cost minimization, is still relied on the experience and expertise of designers. Experienced designers can construct qualified CATV systems with less cost and ease to expand. Most designers can only do feasible solutions. However, they can not get optimized solutions. Since the complexity of this designing problem, the CATV operators still rely on their network planers.

In the past few years, there are few researches addressed on the development of CATV network designing algorithms [24,25,26]. This problem is of great practical value but difficult to solve. It consists several NP-Complete problems, like Steiner tree problem, and these hard problems are interacted with each other. Even though researchers construct a mathematical model in [25], they still did not provide an algorithm to solve this problem. To the best of our knowledge, the research is the few attempts to formulate the problem mathematically and to devise optimization-based solution procedures. We then apply the methods of nonlinear programming, linear programming relaxation, and geometric programming to solve the problem [13,15].

1.3.4 Research Methods (Geometric Programming, Mathematical programming)

In order to solve the network planning and capacity management problems, we propose the mathematical programming methodology to deal with. Currently widely used mathematical programming methods include Linear Programming, Multi-objective Programming, Network Programming, Integer Programming, Dynamic Programming, and Nonlinear Programming [11,12,13,14,15,16,17,18]. Mathematical programming methods can be seen as a systematical way to search the solution space. The general steps in mathematical programming methodology are as follow[18]:

1. Problem description: To describe the properties of problems and to analyze the core problems is the first step to set the scope of problems.
2. Building the decision model: Define decision variables, construct constraint formulations, and set up the objective functions.
3. Development of algorithms: Different mathematical models can be solved through different algorithms. How to choose an existing algorithm or development a new algorithm largely depends on the features of mathematical model. With proper selection and development of algorithms, the mathematical models can be solved effectively and efficiently.
4. Verifications of mathematical models: By comparing the result of solutions and original problems, the veridicality of mathematical models can be shown.

In the CATV network planning problems, the unconvex and nonlinear properties is difficult to handle. In general mathematical programming methodology, a problem with convex solution space is easier to solve. By local search methods using in many operation research problems, the minimum value of convex object function can be approached iteration by iteration. However, the linear programming techniques are not applicable to the un-convex and nonlinear CATV problems.

Therefore we convert the original CATV problems into convex problems by geometric programming method [15,16]. To treat the problem of minimizing problems, we employ the inequality that states that the arithmetic mean is at least as

great as the geometric mean. The simplest case of the geometric inequality is

$$\frac{1}{2}U_1 + \frac{1}{2}U_2 \geq U_1^{1/2}U_2^{1/2}$$

$$U_1 \geq 0, U_2 \geq 0$$

By using the inequality, we can get the minimum or maximum of some functions. For example, the minimum of the following equation $g(t)$

$$g(t) = 4t + \frac{1}{t}$$

can be solved by inequality

$$g(t) \geq (8t)^{1/2} \left(\frac{2}{t}\right)^{1/2} = 4$$

We get the minimum of $g(t)$ as 4. The general form of geometric programming method can be described as follow:

$$\min g_0(t) \tag{IP}$$

$$s.t.: t_1 > 0, t_2 > 0, \dots, t_m > 0 \tag{1}$$

$$g_1(t) \leq 1, g_2(t) \leq 1, \dots, g_p(t) \leq 1 \tag{2}$$

$$g_k(t) = \sum_{i \in J[k]} c_i t_1^{a_{i1}} t_2^{a_{i2}} \dots t_m^{a_{im}}, \quad k = 0, 1, \dots, p,$$

$$J[k] = \{ m_k, m_k + 1, m_k + 2, \dots, n_k \}, \quad k = 0, 1, \dots,$$

$$m_0 = 1, m_1 = n_0 + 1, m_2 = n_1 + 1, \dots, m_p = n_{p-1} -$$

$$n_p = n \tag{3}$$

The exponents a_{ij} are arbitrary real constants, but the coefficients c_i must be

positive constant. Besides, the design parameters t_i are taken to be positive variables. If a equation $g_k(t)$ with all elements conforms these properties, we will say $g_k(t)$ is a “posynomial” equation[15]. If all equations, including the objective function and constraints, are all posynomial equations, we can apply the geometric programming method and dualize the original problem as follow:

$$\max v(\delta) = \left[\prod_{i=1}^n \left(\frac{c_i}{\delta_i} \right)^{\delta_i} \right] \prod_{k=1}^p \lambda_k(\delta)^{\lambda_k(\delta)} \quad (\text{IP})$$

$$s.t. : \delta_1 \geq 0, \delta_2 \geq 0, \dots, \delta_n \geq 0 \quad (1)$$

$$\sum_{j \in J[0]} \delta_j = 1 \quad (2)$$

$$\sum_{i=1}^n a_{ij} \delta_i = 0 \quad j = 1, 2, \dots, p \quad (3)$$

$$\lambda_k(\delta) = \sum_{i \in J[k]} \delta_i, \quad k = 1, 2, \dots, p$$

$$J[k] = \{m_k, m_k + 1, m_k + 2, \dots, n_k\}, \quad k = 0, 1, \dots, p,$$

$$m_0 = 1, m_1 = n_0 + 1, m_2 = n_1 + 1, \dots, m_p = n_{p-1} + 1, n_p = n.$$

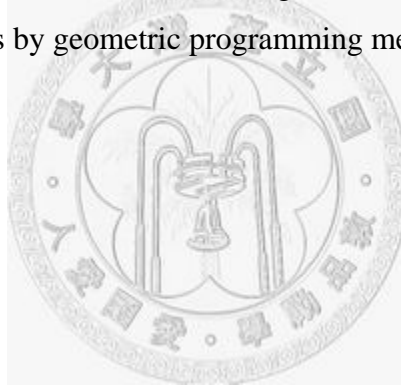
$v(\delta)$ is the dual objective function. The variables $\delta_1, \delta_2, \dots, \delta_n$ are dual variables. Constraint (1) is positivity condition. Constraint (2) is normality condition, and constraint (3) is orthogonality condition. Since the dual problem is a convex problem, we can solve by using many convex programming technique. After that, the solution of the primal problems can get from dual problems.

Comparing with other programming techniques, there are some advantageous features of geometric programming [15]:

1. Geometric programming provides a systematic method for solving this class of nonlinear optimization problems. For meaningful problems, either with or without constraints, the method always produces a global minimum, not just a relative minimum.

2. The minimum of a primal problem is equal to the maximum of the dual problem whose constraints are linear. If the primal problem has zero difficulty, the solution of the dual problem, hence the solution of the primal problem, is obtained by solving a system of linear equations.
3. Each value of the dual function provides a lower bound in the minimum value of the primal function. Moreover, a maximizing sequence for the dual variables produces a minimizing sequence for the primal variables. Because the minimum value of the primal function is equal to the maximum value of the dual function, this common optimum value can be approximated with given arbitrary accuracy.

For some nonlinear and non-convex problems, like CATV network planning problems, it is possible to adjust the form of equations and dualize through geometric programming method [24]. With these advantageous features, we can solve CATV network planning problems by geometric programming method.



CHAPTER 2 PROBLEM FORMULATION

2.1 Problem Descriptions

Below is a verbal description of the two-way transmission CATV network design problem considered. First, we have to collect the information of performance requirements, capacity requirements, network components' specifications/cost structures, etc. Then convert them into an objective function and relevant constraints in mathematical forms.

Given:

- (1) Downstream performance objectives
- (2) Upstream performance objectives
- (3) Specifications of network components
- (4) Cost structure of network components
- (5) Potential locations to build head end and the associated cost
- (6) Number and positions of end users
- (7) Terrain which networks will pass through and the associated cost

To determine:

- (1) Operational parameters (e.g., gain of each amplifier)
- (2) Allocation of network components
- (3) Routing

Objective:

To minimize the total installation cost

Subject to:

- (1) Downstream performance objective constraints
- (2) Upstream performance objective constraints
- (3) Tree network structure

In the description of CATV network design problem, we have to determine three kinds of configurations. The first is routing configuration. By given number and positions of end users, we have to construct a minimum cost tree to connect head end and all users. This is called the Steiner tree problem. For example, if we should

connect the head end and three users in Figure 2-1, what is the minimum cost tree?

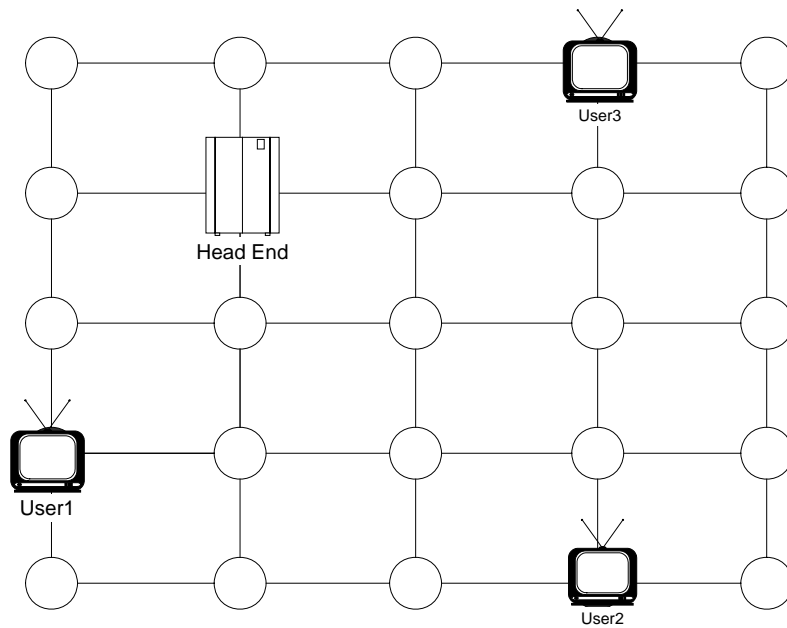


Figure 2-1: Example of a CATV design problem

In this example, if only link costs considered, we can find a minimum cost tree in Figure 2.2 by exhaustive searching.

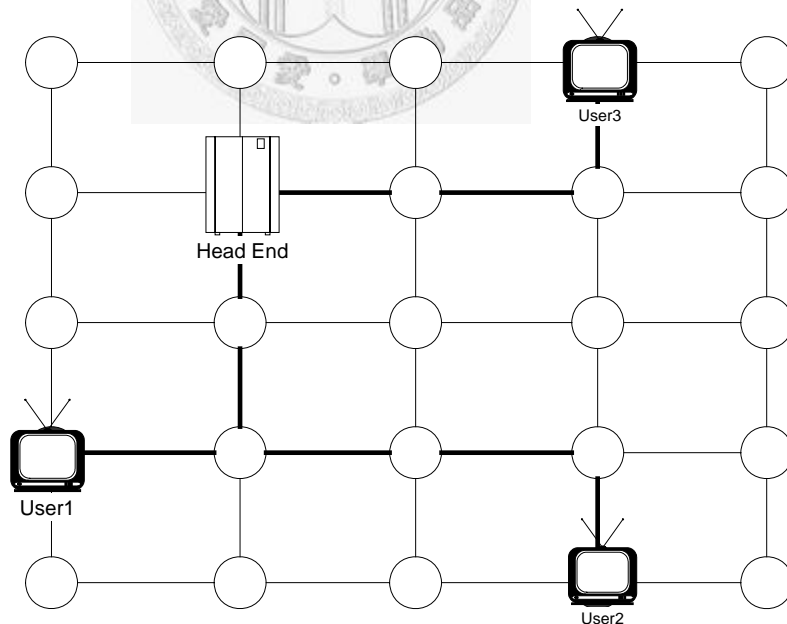


Figure 2.2: Example of Steiner tree problem

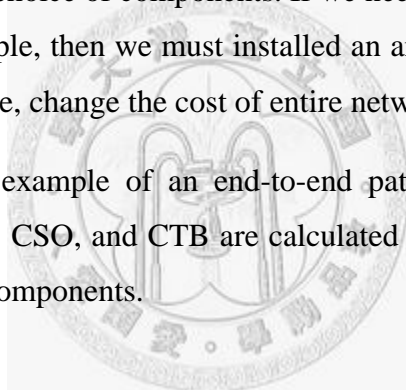
Since Steiner tree problem is known to be NP-complete [27], we can only find the near minimal cost trees by heuristics in limited time. For example, minimum cost

paths heuristic (MPH)[28] can construct a tree for a Steiner tree problem based on algorithms of minimum spanning trees.

The other configuration to be determined is allocation of network components. As discussed in chapter 1, there are many kinds of CATV network components to be installed on networks. The locations and number of components will determine the signal quality along the networks. Besides, more components installed means more cost spent. How to minimize the network cost in such a way the signal quality is satisfied need to be solved.

After the tree constructed and components installed, the final configuration need to be determined is the operational parameter. Some components, especially for active components like amplifiers, can be tuned to different configurations. In order to maintain the proper level of signal passed through, the components must be tuned properly. It also limits the choice of components. If we need a amplifier to amplify the signal to +30dB, for example, then we must installed an amplifier with the gain more than 30dB. It will, of course, change the cost of entire network.

We now present an example of an end-to-end path to demonstrate how the end-to-end CNR, X-MOD, CSO, and CTB are calculated using related parameters of the intermediate network components.



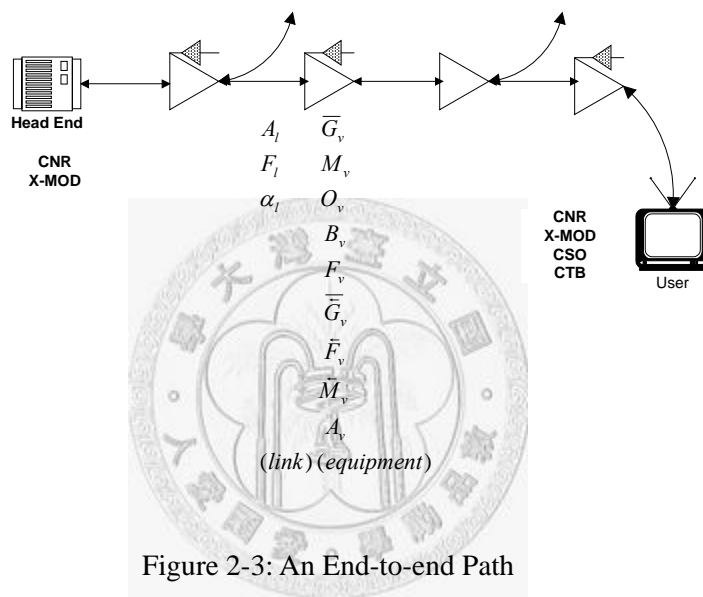
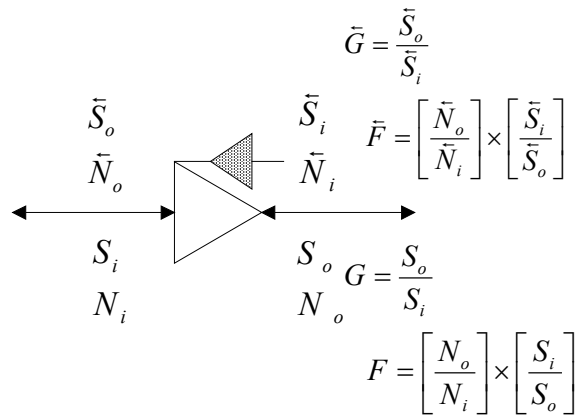


Figure 2-3: An End-to-end Path

In Figure 2-3, the components could be amplifiers or other devices. Let S_i be the input signal, N_i be the input noise from the head end, S_o be the output signal, and N_o be the output noise to the user. In addition, let G_v be the gain of the component, F_v be the noise figure of the component, α_1 be the cable splitting factor, and A_l, A_v be the attenuation factor.

As shown in Figure 2-3, four factors affect end-to-end performance on the downstream path. The first is the gain G_v of an amplifier, which typically increases the power levels of both the input signal and the input noise. Besides the end-to-end CNR, the gain of an amplifier also impacts on the end-to-end performance of X-MOD, CSO, and CTB. These parameters increase when the gain increases. The second factor

is the noise figure, F_v , of an amplifier and the noise figure, F_l , of a link, which indicate the amplitude of internal noise that the amplifier or link introduces. The third factor is the attenuation, A_l , of a link and/or the insertion loss, A_v , of a passive device, which reduce the power level of the input signal and the amount of noise. The fourth factor is the splitting factor, α_l , which monitors the effect of power reduction due to transmission cable branching.

Meanwhile, three factors affect end-to-end performance on the upstream path. The first is the gain, \bar{G} , of an upstream amplifier, which typically increases the power levels of both the upstream input signal and the upstream input noise. The second is the noise figure, \bar{F}_v , of an amplifier and the noise figure, F_l , of a link, which indicate the amplitude of internal noise that the amplifier or link introduces. The third factor is the attenuation, A_l , of a link and/or insertion loss, A_v , of a passive device, which reduce the power level of the input signal and the amount of noise respectively.

Besides the challenges above, the most difficult problem is the correlation between these configurations. The topology of Steiner trees will limits the possible locations to put network components. And the location of a component will influence its configuration. Since these problems are difficult to solve individually, Combination of these problems are even more difficult. It makes the design problems of CATV network to be a very challenge work.

Therefore, we construct mathematical expressions of the CATV network design problem. By leveraging mathematical programming methods, we can solve this problem effectively. We will show our mathematical expressions in the next section.

2.2 Mathematical Formulation of the CATV Network

Design Problem

Here we construct mathematical expressions of the CATV network design problem. A legend of the notations used is given in Appendix A. The following are

the proposed mathematical expressions of the CATV network design problem.

$$\begin{aligned} \min \quad & \sum_{l \in L} [y_l C_l + y_l \Phi_l(A_l, F_l)] + \\ & \sum_{v \in V} [z_v^\alpha \Phi_\alpha(F_v, \bar{G}_v, M_v, B_v, O_v) + z_v^{\bar{\alpha}} \Phi_{\bar{\alpha}}(\bar{F}_v, \bar{\bar{G}}_v, \bar{M}_v) + z_v^\beta \Phi_\beta(A_v)] \end{aligned} \quad (\text{IP1})$$

s.t.

$$(1.1) \quad \sum_{p \in P_w} x_p \sum_{n=1}^{H_{pc}} \left(\frac{z_{\lambda pn}^\alpha F_{\lambda pn}}{\prod_{i=1}^n G_{\lambda pn} A_{\lambda pn} \prod_{j=1}^n A_{\kappa pn} \alpha_{\kappa pn}} + \frac{F_{\kappa pn}}{\prod_{i=1}^n G_{\lambda pn} A_{\lambda pn} \prod_{j=1}^n A_{\kappa pn} \alpha_{\kappa pn}} \right) \leq N \times 10^{\frac{S - C_{sys}}{N} - N} \quad \forall w \in W$$

$$(1.2) \quad \sum_{p \in P_w} x_p \sum_{n=1}^{H_{pc}} [z_{\lambda pn}^\alpha 10^{\frac{M_{\lambda pn}}{20}} \prod_{i=1}^n G_{\lambda pi} A_{\lambda pi} \times \prod_{j=1}^n A_{\kappa pj} \alpha_{\kappa pj}] \leq \frac{10^{\frac{-M_{sys}}{20}}}{S} \quad \forall w \in W$$

$$(1.3) \quad \sum_{p \in P_w} x_p \sum_{n=1}^{H_{pc}} [z_{\lambda pn}^\alpha 10^{\frac{B_{\lambda pn}}{20}} \prod_{i=1}^n G_{\lambda pi} A_{\lambda pi} \times \prod_{j=1}^n A_{\kappa pj} \alpha_{\kappa pj}] \leq \frac{10^{\frac{-B_{sys}}{20}}}{S} \quad \forall w \in W$$

$$(1.4) \quad \sum_{p \in P_w} x_p \sum_{n=1}^{H_{pc}} [z_{\lambda pn}^\alpha 10^{\frac{O_{\lambda pn}}{10}} \prod_{i=1}^n G_{\lambda pi} A_{\lambda pi} \times \prod_{j=1}^n A_{\kappa pj} \alpha_{\kappa pj}] \leq \frac{10^{\frac{-O_{sys}}{10}}}{S} \quad \forall w \in W$$

$$(1.5) \quad \frac{\sum_{w \in W} \sum_{p \in P_w} x_p \sum_{n=1}^{H_{pc}} (z_{\lambda pn}^{\bar{\alpha}} \bar{F}_{\lambda pn} \prod_{i=1}^n \bar{G}_{\lambda pn} A_{\lambda pn} \prod_{i=1}^{n-1} A_{\kappa pn} + \bar{F}_{\kappa pn} \prod_{i=1}^n \bar{G}_{\lambda pn} A_{\lambda pn} \prod_{i=1}^{n-1} A_{\kappa pn})}{\sum_{p \in P_w} x_p \prod_{i=1}^{H_{pc}} \bar{G}_{\lambda pn} A_{\lambda pn} \prod_{i=1}^{H_{pc}} A_{\kappa pn}} \leq \bar{N} \times 10^{\frac{\bar{S} - \bar{C}_{sys}}{\bar{N}} - \bar{N}} - \bar{N} \times |W| \quad \forall w \in W$$

$$(1.6) \quad \sum_{p \in P_w} x_p \sum_{n=1}^{H_{pc}} [z_{\lambda pn}^{\bar{\alpha}} 10^{\frac{M_{\lambda pn}}{20}} \prod_{i=n}^{H_{pc}} \bar{G}_{\lambda pi} A_{\lambda pi} \times \prod_{j=n+1}^{H_{pc}} A_{\kappa pj} \alpha_{\kappa pj}] \leq \frac{10^{\frac{\bar{X}_{sys}}{20}}}{\bar{S}} \quad \forall w \in W$$

$$(1.7) \quad G_v \leq \bar{G}_v \quad \forall v \in V$$

$$(1.8) \quad \bar{G}_v \leq \bar{\bar{G}}_v \quad \forall v \in V$$

$$(1.9) \quad \sum_{l \in L_{out}^v} \alpha_l \leq 1 \quad \forall v \in V$$

$$(1.10) \quad 0 \leq \alpha_l \leq 1 \quad \forall l \in L$$

$$(1.11) \quad \sum_{l \in L_{in}^v} y_l = 1 \quad \forall v \in V$$

$$(1.12) \quad y_l = 0 \text{ or } 1 \quad \forall l \in L$$

$$(1.13) \quad \sum_{w \in W} \sum_{p \in P_w} \delta_{pl} x_p \leq y_l |W| \quad \forall l \in L$$

$$(1.14) \quad \sum_{p \in P_w} x_p = 1 \quad \forall w \in W$$

$$(1.15) \quad x_p = 0 \text{ or } 1 \quad \forall p \in P_w, w \in W$$

$$(1.16) \quad z_\alpha \leq A_v \leq 1 \quad \forall v \in V$$

$$(1.17) \quad 0 \leq A_l \leq 1 \quad \forall l \in L$$

$$(1.18) \quad F_v^L \leq F_v \leq F_v^U \quad \forall v \in V$$

$$(1.19) \quad \bar{F}_v^L \leq \bar{F}_v \leq \bar{F}_v^U \quad \forall v \in V$$

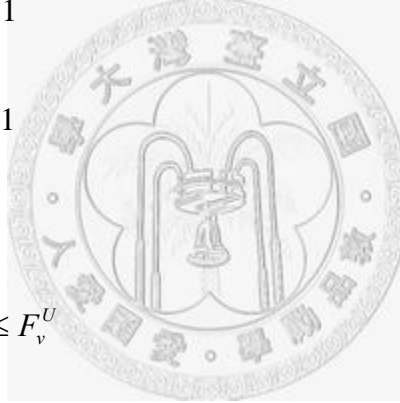
$$(1.20) \quad F_l^L \leq F_l \leq F_l^U \quad \forall l \in L$$

$$(1.21) \quad M_v^L \leq M_v \leq M_v^U \quad \forall v \in V$$

$$(1.22) \quad B_v^L \leq B_v \leq B_v^U \quad \forall v \in V$$

$$(1.23) \quad O_v^L \leq O_v \leq O_v^U \quad \forall v \in V$$

$$(1.24) \quad \bar{G}_v^L \leq \bar{G}_v \leq \bar{G}_v^U \quad \forall v \in V$$



$$(1.25) \quad \bar{G}_v^L \leq \bar{G}_v \leq \bar{G}_v^U \quad \forall v \in V$$

$$(1.26) \quad \bar{M}_v^L \leq \bar{M}_v \leq \bar{M}_v^U \quad \forall v \in V$$

$$(1.27) \quad (F_v, \bar{G}_v, M_v, B_v, O_v, A_v, \bar{G}_v, \bar{F}_v, \bar{M}_v) \in K_e \quad \forall v \in V$$

$$(1.28) \quad (A_l, F_l) \in K_c \quad \forall l \in L$$

$$(1.29) \quad F_v \in F_t^A \quad \forall v \in V$$

$$(1.30) \quad A_v \in A_t^A \quad \forall v \in V$$

$$(1.31) \quad \bar{G}_v \in \bar{G}_t^A \quad \forall v \in V$$

$$(1.32) \quad M_v \in M_t^A \quad \forall v \in V$$

$$(1.33) \quad B_v \in B_t^A \quad \forall v \in V$$

$$(1.34) \quad O_v \in O_t^A \quad \forall v \in V$$

$$(1.35) \quad \bar{G}_v \in \bar{G}_t^A \quad \forall v \in V$$

$$(1.36) \quad \bar{F}_v \in \bar{F}_t^A \quad \forall v \in V$$

$$(1.37) \quad F_l \in F_c^A \quad \forall l \in L$$

$$(1.38) \quad \bar{M}_v \in \bar{M}_t^A \quad \forall v \in V$$

$$(1.39) \quad A_l \in A_c^A \quad \forall l \in L$$



$$(1.40) \quad \frac{\bar{G}_v}{z_v^\alpha} \leq \frac{1}{\varepsilon} \quad \forall v \in V$$

$$(1.41) \quad \varepsilon \leq z_v^\alpha \leq 1 \quad \forall v \in V$$

$$(1.42) \quad \frac{\bar{G}_v}{z_v^{\bar{\alpha}}} \leq \frac{1}{\varepsilon} \quad \forall v \in V$$

$$(1.43) \quad \varepsilon \leq z_v^{\bar{\alpha}} \leq 1 \quad \forall v \in V$$

$$(1.44) \quad \varepsilon \leq z_v^\beta \leq 1 \quad \forall v \in V$$

$$(1.45) \quad z_v^\alpha + z_v^\beta \leq 1 \quad \forall v \in V$$

$$(1.46) \quad z_v^{\bar{\alpha}} \leq z_v^\alpha \quad \forall v \in V$$

The primal problem, IP1, is the objective function that contains the cost summation of all equipment and links installed in the CATV network. Constraint (1.1) enforces the downstream CNR performance requirement of each OD pair. Constraint (1.2) enforces the downstream X-MOD performance of each OD pair. Constraint (1.3) enforces the downstream CTB performance of each OD pair. Constraint (1.4) enforces the downstream CSO performance of each OD pair. Constraint (1.5) enforces the upstream CNR performance requirement of each OD pair. Constraint (1.6) enforces the upstream XMOD performance requirement of each OD pair. Constraint (1.7) ensures that the gain in downstream transmission is less than or equal to the selected full gain. Constraint (1.8) ensures that the gain in upstream transmission is less than or equal to the selected full gain. Constraints (1.9) and (1.10) ensure power conservation after splitting. Constraints (1.11) and (1.12) enforce the tree structure of CATV networks. Constraint (1.13) ensures that uninstalled links can not be selected by the OD pairs. Constraint (1.14) ensures that exactly one path is selected by each end user. Constraint (1.15) decides whether or not each OD pair can be selected. Constraint (1.16) ensures that when the equipment is installed as an amplifier, the attenuation factor is 1. Constraints (1.17) to (1.26) list the lower and upper bounds of the attenuation factor of the link, downstream noise figure, upstream noise figure,

X-MOD, CTB, CSO, downstream full gain, and upstream full gain. Although these constraints are redundant, they facilitate efficient solution procedures. Constraints (1.29) to (1.39) define the set of NF, full gain, X-MOD, CTB, CSO, and the attenuation factor whose value sets are discrete positive real numbers. Constraint (1.40) ensures that the value of the downstream gain is 1 when the component is not an amplifier. Constraint (1.42) ensures that the value of the upstream gain is 1 when the component is not an upstream amplifier. Constraints (1.41) and (1.43) list the lower and upper bounds of decision variables z_v^α and $z_v^{\tilde{\alpha}}$ respectively. Constraint (1.44) lists the lower and upper bounds of decision variable z_v^β . Constraint (1.45) ensures that one location is either an amplifier or a passive component. Constraint (1.46) requires that the upstream amplifier must be two-way.

When considering the signal quality in the upstream channel, we do not include the CTB and CSO signal quality constraints in our mathematical formulations. The effect of these constraints is not significant, because the bandwidth of the upstream channel is narrow and the number of channels is small [2].

2.3 Reformulation of the CATV Network Design

Problem

To solve the CATV network design problem, it is important to clearly define the formulation that can be solved by mathematical programming methods. We observe that the form of the original formulations is similar to the posynomial form [15]. If the formulations of the problem are all posynomials, then we can use the geometric programming method to effectively solve it.

However, in the original formulations, there are some equations that are not posynomials. In this section, we will show how to use surrogate function method to reformulate these equations. The first equation we need to reformulate is the objective function. We find that the cost of each element is related to a number of parameters, but the exact relationship among the parameters has not been explored. Although we can specify the relationship according to the backend cost database set, it is not

feasible for calculating solutions.

From the data collected from the equipment manufacturers, we find that when the values of the parameters for the full gain, X-MOD, CSO, CTB, and attenuation factor are larger, or the value of the noise figure is smaller, the cost will increase. Therefore, we define the surrogate function of the cost function as follows.

$$\min \sum_{l \in L} [d_1(F_l)^{-1} + d_2(A_l)^1] + \sum_{v \in V'} \{z_v^\alpha [d_3(F_v)^{-1} + d_4(\bar{G}_v)^1 + d_5(M_v)^1 + d_6(B_v)^1 + d_7(O_v)^1] + z_v^{\tilde{\alpha}} [d_8(\bar{F}_v)^{-1} + d_9(\bar{G}_v)^1 + d_{10}(\bar{M}_v)^1] + z_v^{\tilde{\beta}} [d_{11}(A_v)^1]\},$$

where d_i is the coefficient of the surrogate function of the cost function. When one kind of component is selected, it is added to our CATV network installation cost.

The other equations that are not posynomial are some of constraints. There are exponential elements in Constraints (1.2), (1.3), (1.4), and (1.6), that violate the characteristics of posynomials. In order to find the suitable surrogate functions for these constraints, we use the LINGO package [29] to transform these constraints into posynomial form. To meet the performance requirements, each value of the surrogate functions must be equal to or large than original functions. Table 2-1 lists the details of the surrogate functions, and in Figures 2-4 to 2-6 we compare the original functions and the surrogate functions.

Table 2-1: Surrogate functions of X-MOD, CTB, AND CSO

Name	Original Function	Value Set (dB)	Surrogate Function	Avg. Error (%)
X-MOD	$10^{(M/20)}$	(61, 88, 96, 97)	$(1.250386E-13) * M^{(8.935579)}$	0.009329198
CTB	$10^{(B/20)}$	(61, 90, 102, 110)	$(9.290291E-15) * B^{(9.567963)}$	0.022317717
CSO	$10^{(O/10)}$	(66.5, 79.5, 88, 90.5)	$(1.1106E-11) * O^{(10.22383)}$	0.062867807

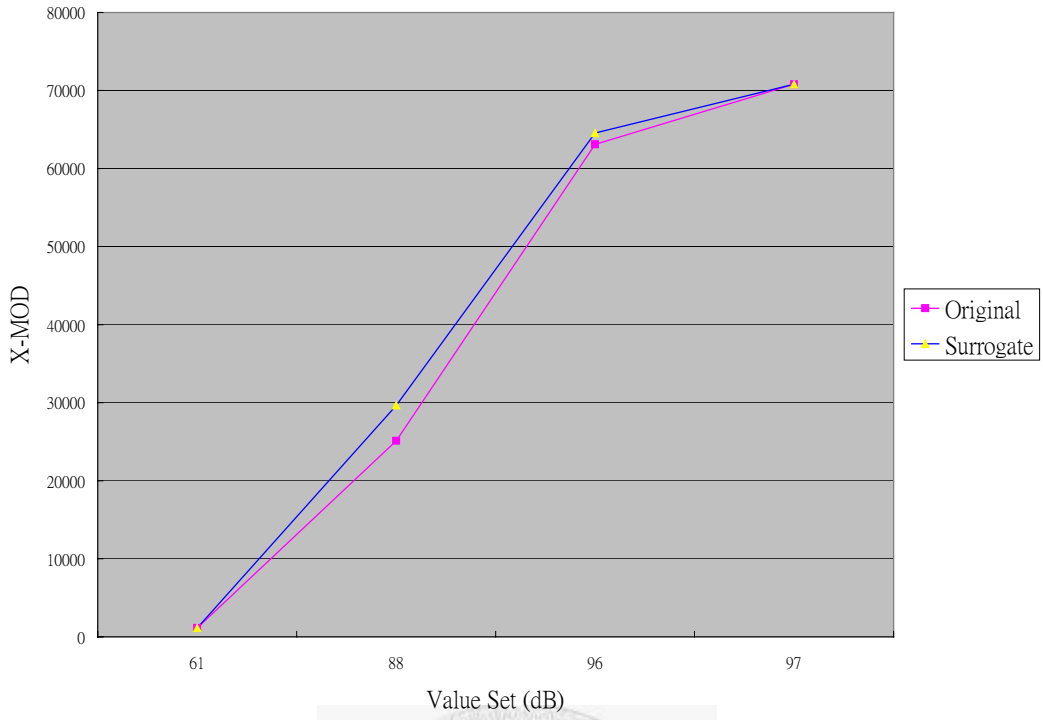


Figure 2-4: Comparison of functions for X-MOD.

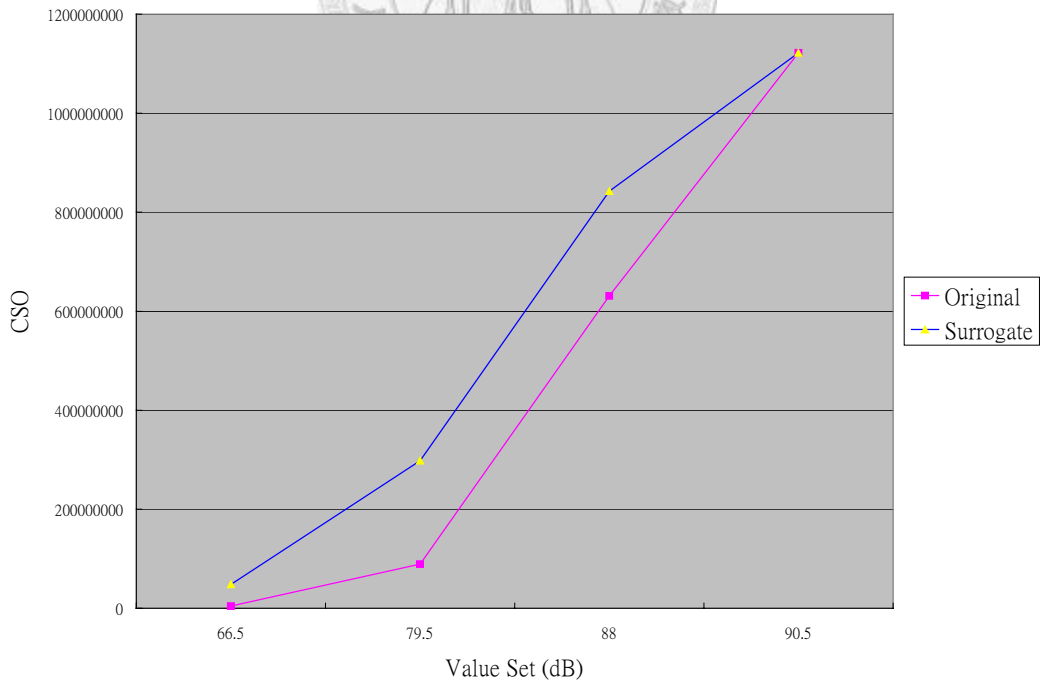


Figure 2-5 Comparison of functions for CSO.

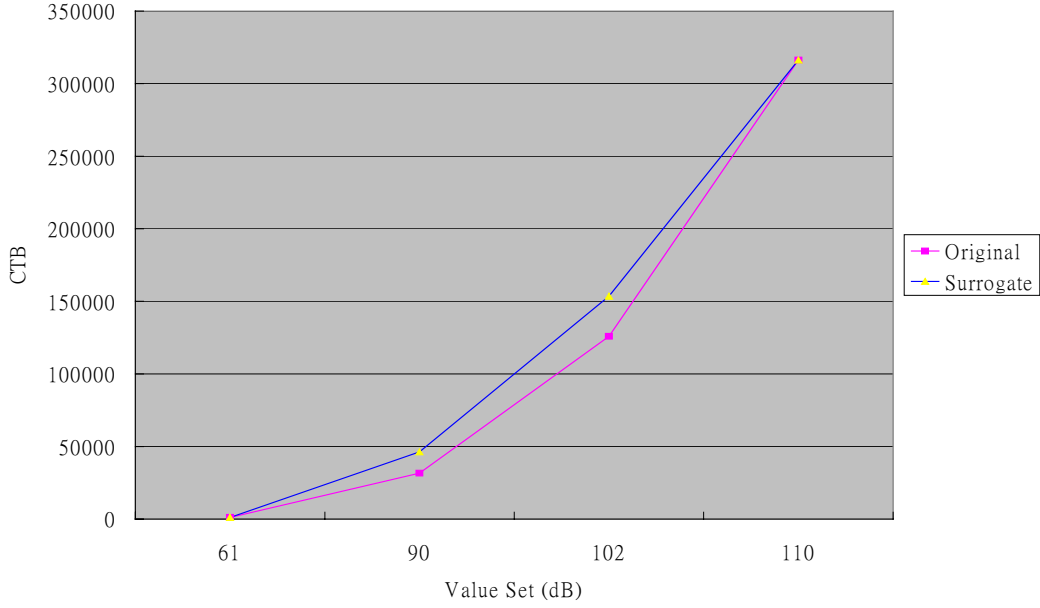


Figure 2-6 Comparison of functions for CTB.

In Table 2-1, we list the original function, the value set, the surrogate function, and the error. The error is the summation of the difference between the surrogate function and the original function at each point specified by the value set. We observe that the average percentage error for CSO is slightly larger than the others. This is because the exponent of CSO is larger than the others, so the LINGO package could not find the optimal solution. Instead, only a feasible solution can be found.

2.4 Reformulation of the CATV Network Design

Problem

In the previous section, we introduced the surrogate functions of the cost functions, X-MOD, CTB, and CSO constraints. In this section, we list part of the reformulated CATV network design problem:

$$\min \sum_{l \in L} [d_1(F_l)^{-1} + d_2(A_l)^1] + \sum_{v \in V'} \{z_v^\alpha [d_3(F_v)^{-1} + d_4(\bar{G}_v)^1 + d_5(M_v)^1 + d_6(B_v)^1 + d_7(O_v)^1] + z_v^{\bar{\alpha}} [d_8(\bar{F}_v)^{-1} + d_9(\bar{G}_v)^1 + d_{10}(\bar{M}_v)^1] + z_v^{\bar{\beta}} [d_{11}(A_v)^1]\} \quad (\text{IP2})$$

s.t.

$$(2.1) \quad \sum_{p \in P_w} x_p \sum_{n=1}^{H_{pc}} \left(\frac{z_{\lambda pn}^\alpha F_{\lambda pn}}{\prod_{i=1}^n G_{\lambda pn} A_{\lambda pn} \prod_{j=1}^n A_{\kappa pn} \alpha_{\kappa pn}} + \frac{F_{\kappa pn}}{\prod_{i=1}^n G_{\lambda pn} A_{\lambda pn} \prod_{j=1}^n A_{\kappa pn} \alpha_{\kappa pn}} \right) \leq N \times 10^{\frac{S - C_{sys}}{10}} - N \quad \forall w \in W$$

$$(2.2) \quad \sum_{p \in P_w} x_p \sum_{n=1}^{H_{pc}} [z_{\lambda pn}^\alpha (1.25 * 10^{-13}) M_{\lambda pn}^{8.94} \prod_{i=1}^n G_{\lambda pi} A_{\lambda pi} \times \prod_{j=1}^n A_{\kappa pj} \alpha_{\kappa pj}] \leq \frac{10^{\frac{-M_{sys}}{20}}}{S} \quad \forall w \in W$$

$$(2.3) \quad \sum_{p \in P_w} x_p \sum_{n=1}^{H_{pc}} [z_{\lambda pn}^\alpha (9.29 * 10^{-15}) B_{\lambda pn}^{9.57} \prod_{i=1}^n G_{\lambda pi} A_{\lambda pi} \times \prod_{j=1}^n A_{\kappa pj} \alpha_{\kappa pj}] \leq \frac{10^{\frac{-B_{sys}}{20}}}{S} \quad \forall w \in W$$

$$(2.4) \quad \sum_{p \in P_w} x_p \sum_{n=1}^{H_{pc}} [z_{\lambda pn}^\alpha (2.5 * 10^{-11}) O_{\lambda pn}^{11.23} \prod_{i=1}^n G_{\lambda pi} A_{\lambda pi} \times \prod_{j=1}^n A_{\kappa pj} \alpha_{\kappa pj}] \leq \frac{10^{\frac{-O_{sys}}{10}}}{S} \quad \forall w \in W$$

$$(2.5) \quad \frac{\sum_{w \in W} \sum_{p \in P_w} x_p \sum_{n=1}^{H_{pc}} (z_{\lambda pn}^\alpha \bar{F}_{\lambda pn} \prod_{i=1}^n \bar{G}_{\lambda pn} A_{\lambda pn} \prod_{i=1}^{n-1} A_{\kappa pn} + \bar{F}_{\kappa pn} \prod_{i=1}^n \bar{G}_{\lambda pn} A_{\lambda pn} \prod_{i=1}^{n-1} A_{\kappa pn})}{\sum_{p \in P_w} x_p \prod_{i=1}^{H_{pc}} \bar{G}_{\lambda pn} A_{\lambda pn} \prod_{i=1}^{H_{pc}} A_{\kappa pn}} \leq \bar{N} \times 10^{\frac{\bar{S} - \bar{C}_{sys}}{10}} - \bar{N} \times |W| \quad \forall w \in W$$

$$(2.6) \quad \sum_{p \in P_w} x_p \sum_{n=1}^{H_{pc}} [z_{\lambda pn}^\alpha (1.25 * 10^{-13}) M_{\lambda pn}^{8.94} \prod_{i=n}^{H_{pc}} \bar{G}_{\lambda pi} A_{\lambda pi} \times \prod_{j=n+1}^{H_{pc}} A_{\kappa pj} \alpha_{\kappa pj}] \leq \frac{10^{\frac{\bar{X}_{sys}}{20}}}{\bar{S}} \quad \forall w \in W$$

The primal problem IP2 is the surrogate function of the objective function IP1.

Most constraints are the same as those applied to IP1, where constraints (2.2), (2.3), (2.4), and (2.6) are surrogate functions of the original constraints.

2.5 Concluding Remarks

In this chapter, we first explain the nature of CATV network design problems. There are several problems to be solved. First is the Steiner tree problem. To design a CATV network, we first need a tree to connect head end and all users. Since it has been proved as NP-Complete problem, it is impossible to solve it in polynomial time. The second problem is where to place the network components. And the third problem is how to configure these components properly. Most of all, these problems are related to each other. Obviously, the CATV network design problem is not easy to solve.

In order to solve this problem, we first constructed the mathematical formulation to describe the CATV design problem. We reformulate some equations to conform posynomial form. In such way, this problem can be solved by geometric programming method. In section 2.3, we demonstrated how to reformulate the equations by surrogate function method. We reformulated the objective function and several constraints. After that, we have constructed a set of equations that all conform posynomial form. In the next chapter, we will dualize the problem by geometric programming method, and show the solution procedure for the problem.

CHAPTER 3 SINGLE LAYER SOLUTION PROCEDURE AND COMPUTATIONAL EXPERIMENTS



3.1 Overview

In this chapter, we will discuss the solution procedures for CATV network designing problems. Since the network are treat as a plane (single layer) network in the solution procedure proposed in this chapter, the solution procedure are named as single layer solution procedure. In Chapter 2, we have constructed a mathematical model for CATV network designing problems. Besides, the original formulations are examined for the effectiveness and efficiency using mathematical programming methods. Some mathematical expressions are reformulated such that the geometric programming method can be used. The geometric programming method can be used to solve the CATV network designing problems; however, it could not guarantee an effective and efficient way to solve problems. There are some important factors that

influence the solution quality and computing time. In our computational experiments, we have found these factors and their relations. By our suggestion, the CATV network designing problems can be solved effectively and efficiently.

In the next section, we review the mathematical model introduced in the last chapter. By observing the structure of this model, we proposed two different types of steepest descent methods. Since the convex property of this mathematical model, we should solve this problem by any kind of steepest descent methods. However, in our computational experiments, we have found the steepest descent method used in general convex programming problems could not solve CATV network designing problems. Therefore, we tried another kind of steepest descent method, called modified steepest descent method, in this problem. The modified steepest method showed good performance on both solution quality and efficiency.

We have also do some experiments to improve the efficiency and effectiveness of the modified steepest method. The iterations of computations, or computing times, are influenced by the selection of several parameters. We examined the impact of these parameters. Finally we propose a mechanism for selection and determination of these parameters.

3.2 The single layer solution procedure

In this section, the single layer solution procedure for CATV network planning problems is described and explained. In figure 3-1, the single layer solution procedure for CATV network designing problems is showed.

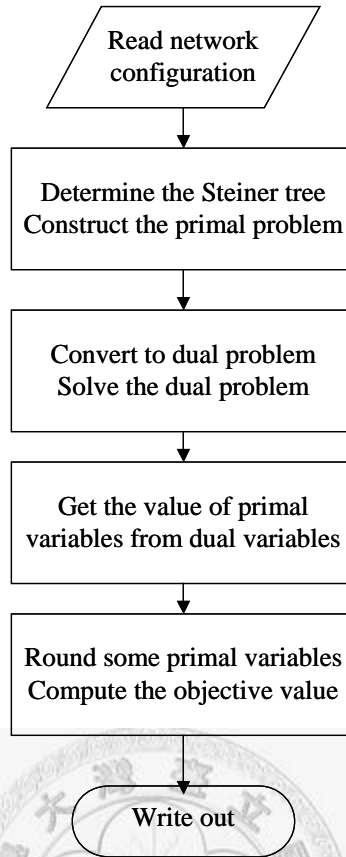


Figure 3-1: The single layer solution procedure

From the beginning, read information about the CATV network under planning. It includes the number and positions of users and position of head end. We also need the information of positions of possible link placement. With these data, we first construct the Steiner tree that connects the head end and all users. After that, we can construct the primal problem for geometric programming method.

By geometric programming method, the primal problem is converted into the dual problem. The dual problem will look like the following:

$$\max \quad v(\delta) = \left[\prod_{i=1}^n \left(\frac{c_i}{\delta_i} \right)^{\delta_i} \right] \prod_{k=1}^p \lambda_k(\delta)^{\lambda_k(\delta)} \quad (\text{IP4})$$

$$s.t. : \quad \delta_1 \geq 0, \delta_2 \geq 0, \dots, \delta_n \geq 0 \quad (4.1)$$

$$\sum_{j \in J[0]} \delta_j = 1 \quad (4.2)$$

$$\sum_{i=1}^n a_{ij} \delta_i = 0 \quad j = 1, 2, \dots, m \quad (4.3)$$

$$\begin{aligned}
\lambda(\delta) &= \sum_{i \in J[k]} \delta_i, k = 0, 1, \dots, p, \\
J[k] &= \{m_k, m_k + 1, m_k + 2, \dots, n_k\}, k = 0, 1, \dots, p, \\
m_0 &= 1, m_1 = n_0 + 1, m_2 = n_1 + 1, \dots, m_p = n_{p-1} + 1, n_p = n.
\end{aligned} \tag{4.4}$$

The next question is how to solve the dual problem. It is suggested to convert the above formulations into the following formulations [15,16,24]:

$$\min -\ln W + JX^2$$

where

$$W = \left[\prod_{i=1}^n \left(\frac{c_i}{\delta_i} \right)^{\delta_i} \right] \prod_{k=1}^p \lambda_k(\delta)^{\lambda_k(\delta)}$$

and

$$X^2 = \sum_{i=1}^n \left(\sum_{j \in J[0]} \delta_j - 1 \right)^2 + \left(\sum_{i=1}^n a_{ij} \delta_i \right)^2$$

Since the objective function is differentiable, it can be optimized by gradient-based method [14,18], like steepest descent method or Newton method. The item JX^2 is called penalty function, since it relaxed some constraints by adding them into the objective function with a very large value J .

By solving the dual problem by gradient-based method, we can get the optimal value of the dual problem. From the duality theorem [15], the constrained maximum value of the dual function is equal to the constrained minimum value of the primal function. Therefore, we can get solutions of primal problems from the solution of dual problems. Since some primal variable, like decision variable of whether to place an amplifier or not, must be integers, we still need to round those integer constrained primal variables into integers. The rounded variables must be checked, or they may violate some constraints after rounding.

In order to describe the solution procedure more clearly, we use an example as Figure 3-2 to explain our solution procedure.

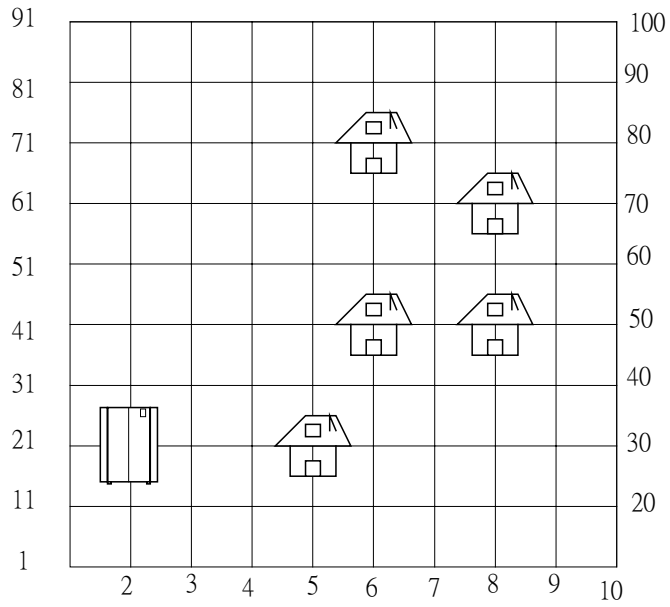


Figure 3-2 Location map of the example

Step I: Building a Steiner Tree Topology

Figure 3-3 shows the *tree* topology after Step I.

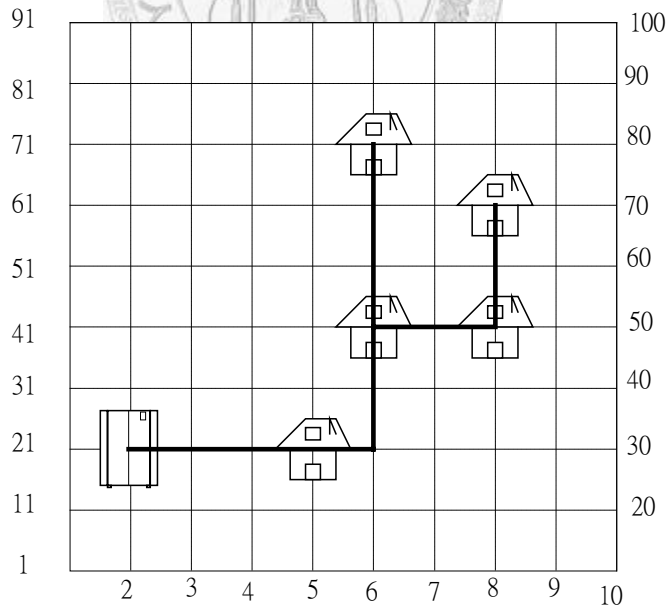


Figure 3-3: Tree Topology of Example 1

Step II: Determining the Locations of Amplifiers

Based on the tree *topology* above, we can formulate the relative posynomial

primal problem. By dualizing and solving this geometric problem, we obtain the numerical data, z_g terms, i.e., the decision variables of the amplifier positions. The topology positions of the amplifiers have a special appearance, which we call the "grouping". This indicates where to put the amplifiers, as in Figure 3-4.

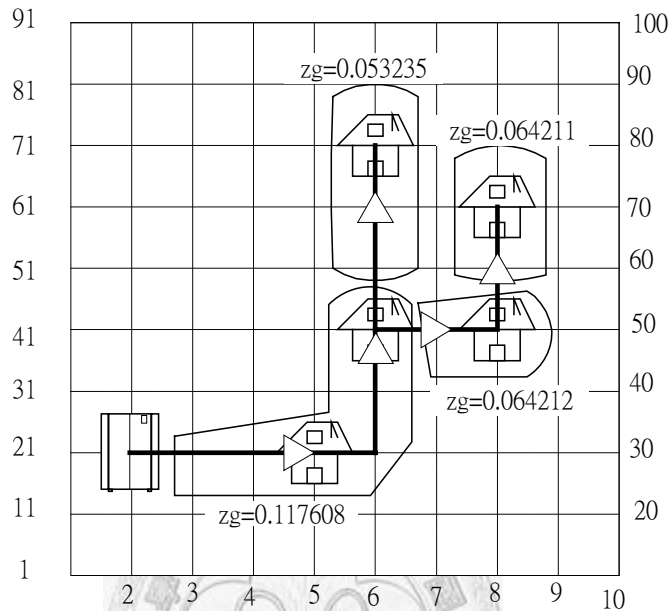


Figure 3-4: Locations to Put Amplifiers

Step III: Determining the Configuration of each Component

Having decided the locations of the amplifiers, we can reduce the problem size and formulate another posynomial primal problem. The primal decision variables are reduced to the configuration of the equipment. By dualizing and solving this geometric problem, we get the parameters of each amplifier, as shown in Table 3-1.

Table 3-1: Solution of the primal problem

Node	Attenuation	Gain	Noise Figure	X-MOD
25	0.448159	4.892901	24.496151	0.950541×10^7
46	0.448190	4.893536	24.486506	0.950336×10^7
47	0.256121	2.557893	24.483614	0.950649×10^7
58	0.256150	2.558247	24.509827	0.950693×10^7
66	0.234123	2.337965	24.520926	0.950686×10^7

Step IV: Adding Reverse Modules in Amplifiers

We add the reverse modules into the amplifiers to amplify the upstream signal and conform to the signal quality constraints. By dualizing and solving this geometric problem, we get the parameters of each amplifier in Table 3-2.

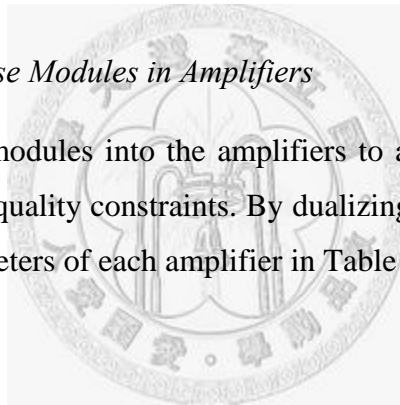


Table 3-2: Solution of primal problem

Reverse NF (dB)	8.808512
Input signal strength to reverse module (dB)	5.231051
Reverse CNR (in dB) of OD path from 25 to HE	51.927688
Reverse CNR (in dB) of OD path from 46 to HE	50.422538
Reverse CNR (in dB) of OD path from 48 to HE	49.542082
Reverse CNR (in dB) of OD path from 68 to HE	48.917388
Reverse CNR (in dB) of OD path from 76 to HE	49.542082



3.3 Computational experiments

In this section, we report on the experiments conducted to test the algorithm proposed above. The experiment parameters are described in Appendix B. The experimental platforms are IBM[®] personal computers with Microsoft[®] Windows XP Professional operational systems. These personal computers are equipped with a 1.86-GHz Pentium 4 CPU, 2 Giga-Bytes RAM, and one 80 Giga-Bytes Hard Disk.

The first experiment is the effectiveness of gradient-based methods. As shown in previous section, gradient-based methods are usually used to solve the dual problems. In [24], the steepest descent method was used to solve the dual problems. However, by comparing with other method, we found steepest descent method is not effective to solve the dual problems.

We have done experiments on some network examples to compare the results of steepest descent method. As shown in Table 3-3, the steepest descent method converged at some points that are not optimal. Since in steepest descent method, the program are terminated at the iteration with no more or too little improvement. In gradient-based method, the stopping point should be very close to the local minimum or maximum. However, it is not the case we have seen in CATV network designing problems.

By looking these formulations further, we have some findings that incur the ineffectiveness of the steepest descent method. First, the logarithmic part of the dual objective function is unstable near the optimal points [12]. The is the phenomenon we called “Zigzagging.” When we move to the points very close to the minimum or maximum points, the improvement is very small because we just move from one side of optimal points to the other. It is what happened when we solve by steepest descent method.



Table 3-3: Results of Steepest Descent Method

Net#	Steepest Descent Method		Optimal	
	Dual	Primal	Dual(converted)	Optimal
c00	-2.83746	17.07229	-7.979516	2920
c01	-2.27522	9.730104	-7.771648	2372
c02	-6.11063	450.6227	-8.268778	3900
c03	-1.69108	5.425345	-7.446525	1713
c04	-0.5784	1.783182	-8.155407	3482
c05	-3.86666	47.78255	-7.412583	1656
c06	-3.42424	30.69917	-7.351371	1558
c07	-3.94931	51.89957	-7.379032	1602
c08	-2.37665	10.76872	-7.159171	1285
c09	-2.33859	10.36658	-7.523604	1851

The penalty function used in dual objective function is another reason that steepest descent method is ineffective. Recall that multiplier J of penalty function should be a very large number to force the searching direction toward to some points that minimize the added constraints. When J equals to infinite real number, all the constraints added into the objective function must be satisfied, otherwise it will incur a very large penalty on object function. However, when J is very large, the searching route is directed to the point minimizing the penalty function first. Besides, the objective function would be very sensitive during searching.

For the above two reasons, we use the modified steepest descent method [31] proposed by B. T. Polyak to solve the dual problems. The major difference between the original and modified steepest descent methods is the mechanism to move to next searching point. In general steepest descent method, we move to the next searching point by the following equation:

$$X^{K+1} = X^K + r^K \nabla f(X^K)$$

The r^K is parameter called the optimal step size. The parameter r^K is determined such that X^{K+1} results in the largest improvement in f . On the other hand, in the modified steepest descent method, the parameter r^K is determined by a number series with several properties:

$$\lim_{k \rightarrow \infty} r^k = 0$$

$$\sum_{k=1}^{\infty} r^k = \infty,$$

The Polyak, B. T. had been proved that series of step size would always find the optimal solution as long as number of steps is large enough. For some extremal problems with gradient-based methods, this method always finds optimal solution. Since the ill-structure of CATV planning problems, the modified steepest descent method could solve by a different way.

The result generated by the modified steepest descent method are 51% to 92% better than the original steepest descent method, shown as Table 3-4. Based on our findings, we suggest that the modified steepest descent method are more suitable for the dual problems of CATV network planning problems.

Table 3-4: Comparison of original and modified steepest descent methods

Net#	Original	Modified	Difference	Improvement
c00	16212	2921	13292	82%
c01	14821	2372	12448	84%
c02	8008	3900	4108	51%
c03	11673	1714	9959	85%
c04	43524	3482	40042	92%
c05	5402	1657	3745	69%
c06	5770	1558	4212	73%
c07	5025	1602	3423	68%
c08	6332	1286	5046	80%
c09	10425	1851	8574	82%

In order to improve the efficiency and effectiveness of the modified steepest descent method, we have done more experiments on the settings of parameters in the modified steepest descent method. In the experiments, we found the initial step size play an important role to converge to optimal value. In modified steepest descent method, we use a harmonic series as the sequence of step size. However, for some large step sizes, the variables may go too far away and can never close to near optimal regions. The initial step size, therefore, must be carefully selected when we use the modified steepest descent method.

In our experiments, we found the feasible initial step size is impacted by several

factors. One is the size of Steiner trees, or the number of nodes on Steiner tree. From the structure of mathematical formulations, the more nodes needed to connect the head end and all users, the more primal variables are introduced. When we convert the primal problems into dual problems, the dual variables should decrease when number of primal variables increases. We tested fifty network examples for this factor. The relation of number of nodes and initial step size is shown in Figure 3-5.

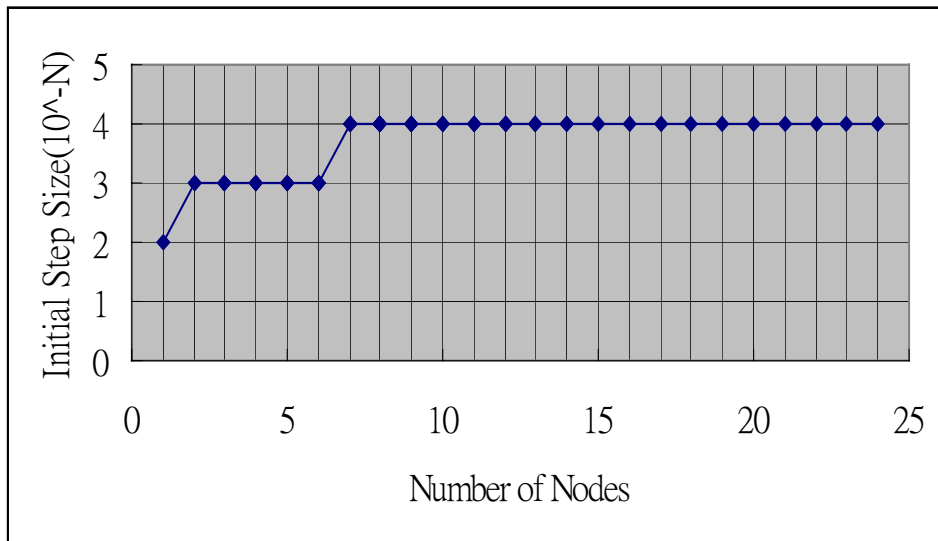


Figure 3-5: Initial step sizes vs. number of nodes

Another factor that influences the initial step size is the multiplier J of penalty function. Since the program converge to optimal value for many iterations, J can be set as small value in the beginning. The more we close to the optimal point, the larger J is set. However, a large J also needs a smaller step size. By testing different network examples and values of J, we have found the relation between these two parameters:

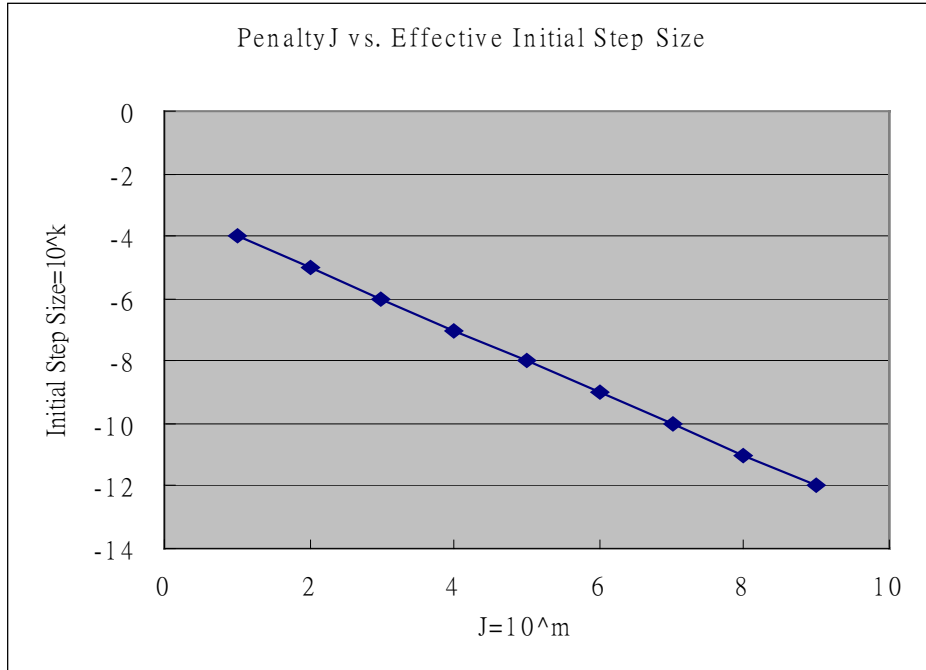


Figure 3-6: Initial step sizes vs. multiplier J of penalty function

From the results of experiments, we propose the adjustment procedure for initial step size and penalty multiplier J as follow:

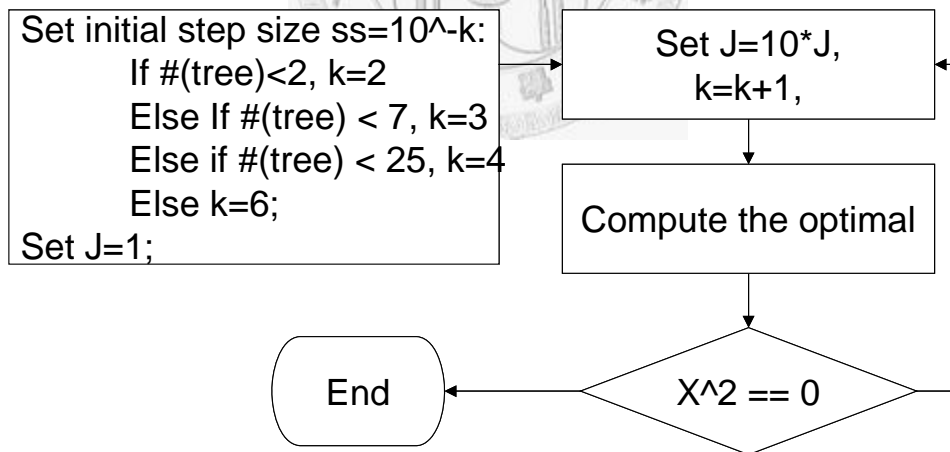


Figure 3-7: Adjustment procedure for initial step size and penalty coefficient

The adjustment procedure begins with determination of the initial step size by number of nodes on Steiner tree constructed. When the penalty multiplier J increases, the initial step size is decrease. The program would terminate when the optimal value is found and the part of penalty function is zero. For the examples we have done in

experiments, the number of iterations can be reduced by 43% to 231%. The modified steepest method is improved further by this adjustment procedure.

3.5 Concluding Remarks

In this chapter, the single layer solution procedure is described. Two kind of gradient-based methods are used and compared. In the modified steepest descent method, we have some findings on the parameter settings and adjustments. Finally we propose our adjustment procedure for initial step size and coefficient of penalty functions.

In the comparison of gradient-based methods, the result of experiments shows the effectiveness of modified and original steepest descent methods. The results of modified steepest descent method are 51% to 92% better than original steepest descent method.

Although the effectiveness of the modified steepest descent method, there are some factors that influence the efficiency of this method. First is the initial step size. Based on our analysis, improper selection of initial step size would make the method spend more time to find the optimal value. We have found the numbers of nodes in Steiner trees constructed has close relation with the value of initial step size. Generally, the more nodes in Steiner tree, the smaller value of initial step size is.

Another parameter that causes ineffectiveness is the multiplier J for penalty functions. For the original formulations, the multiplier J must be a very large value to force the constraints satisfied. However, the problems would be more sensitive with a larger J .

Combined with number of nodes and multiplier J , we proposed an adjustment procedure for initial step size in CATV network designing problems. With incremental adjustment these parameters, the modified steepest descent method can be more effectively used in solving CATV network design problems.

CHAPTER 4 MULTILAYER SOLUTION PROCEDURE AND COMPUTATIONAL EXPERIMENTS



4.1 Overview

In general, there are tens to hundreds thousands of subscribers in a CATV networks operated in real world. For CATV network operators, the problem size for network planning is too large to solve by the single layer solution procedure. With the scale of networks growing up, the computing time grows much faster than the scale of networks. Figure 4.1 shows the growing speeds when the network size grows up.

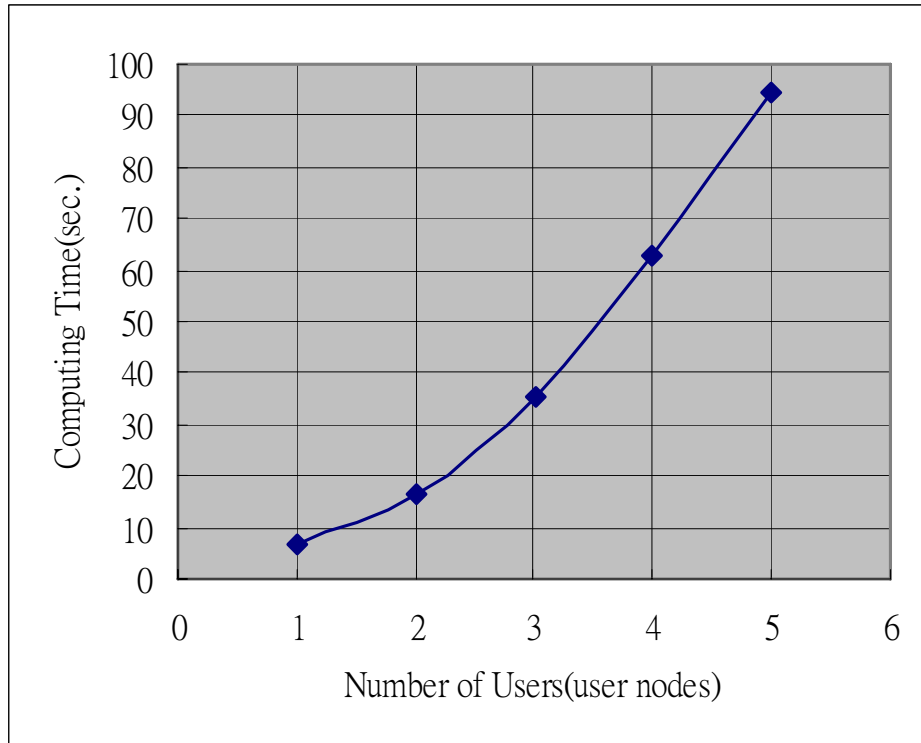


Figure 4-1: Number of Network Users versus Computing Time

The experiment parameters are described in Appendix B. The experiment platforms are IBM® personal computers with Microsoft® Windows XP Professional operational systems. These personal computers are equipped with a 1.86-GHz Pentium 4 CPU, 2 Giga-Bytes RAM, and one 80 Giga-Bytes Hard Disk.

In our experiments, we found the number of nodes in Steiner tree constructed is another factor related to the computing time. Here we report another finding that shows the relation between the number of nodes and computing time in Figure 4-2.

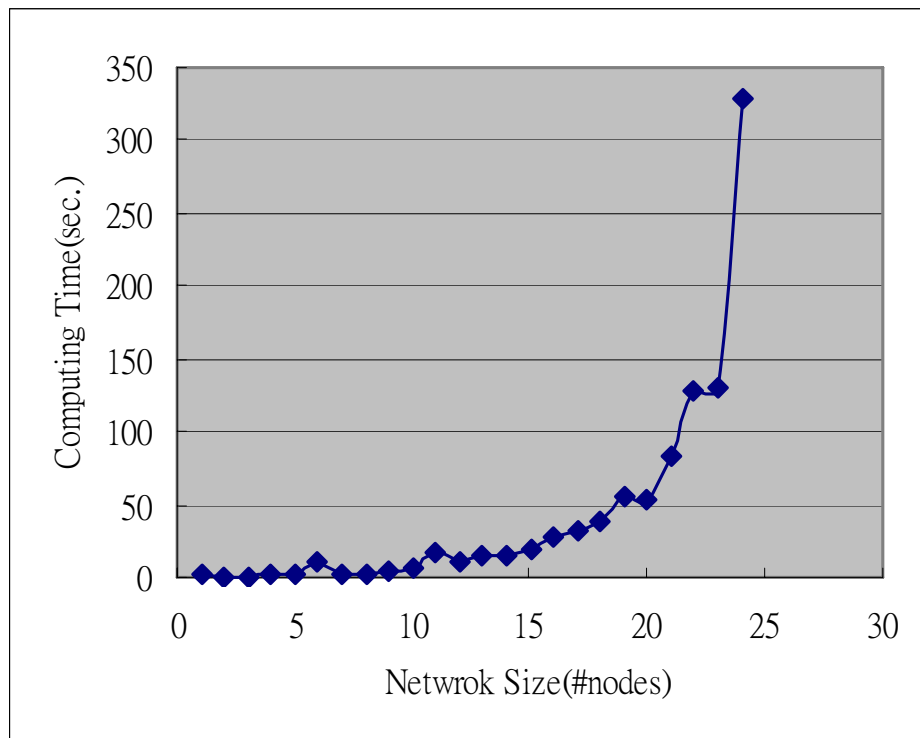


Figure 4-2: Network Size versus Computing Time

As the network size increases, the computing time needed for single layer solution procedure is growing up very quickly. Besides the computing time, the memory consumed during the computing process is also an important limitation. When the network size is more than about 40, the solution process would terminate sometimes because the memory is running out. The single layer solution procedure is unable to solve a network planning problem with more than 45 users on the platform we used in experiments. The network sizes for general CATV networks are ranged from hundreds to thousands of subscribers. Obviously, it is impractical to use single layer solution procedure to solve network planning problems for large scale CATV networks.

In this chapter, we propose the multilayer solution procedure for large CATV network planning problems. Since the problem size that single layer solution procedure can process is limited, we have to break down a large network into several smaller networks. The concept of “layering” can be used to describe the network planning procedure for large networks. Especially from the point of CATV operators, the entire networks can be seen as tree network with many branch networks. For example, there is a CATV network with a head end serving the branch networks in ten

cities in Figure 4-3.

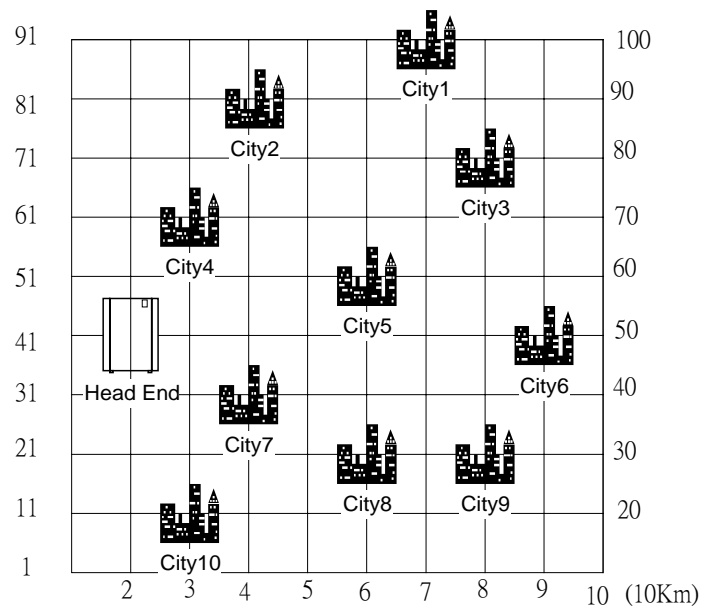


Figure 4-3: Location map of subscribers for a CATV network: Layer 1

And there are ten smaller branches in each city, serving the subscribers in each city, as in Figure 4-4.

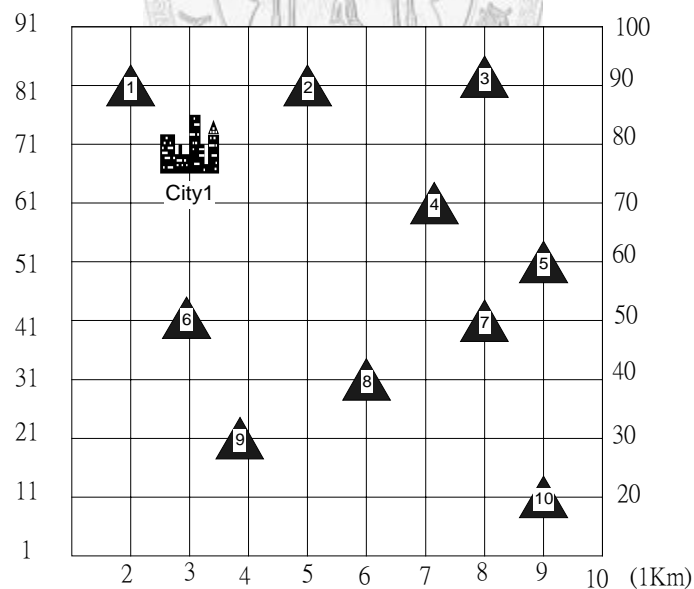


Figure 4-4: Location map of subscribers for a CATV network: Layer 2

In each city, the positions of the local branch box and subscribers are shown in Figure 4-5.

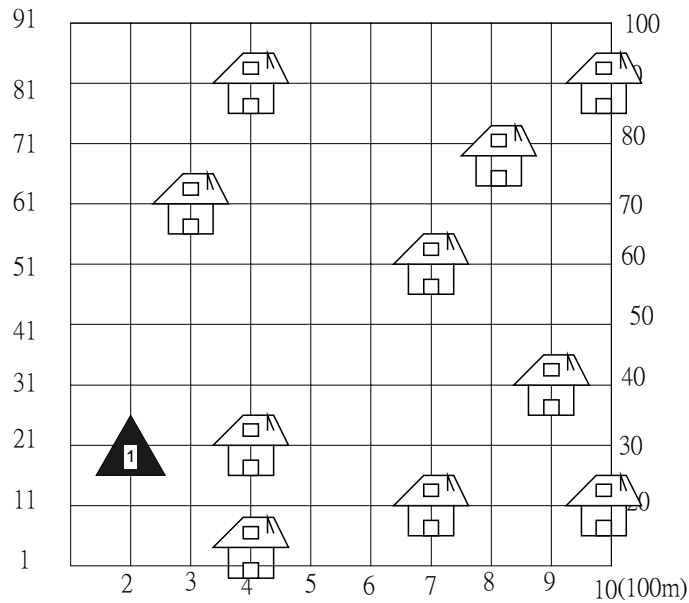


Figure 4-5: Location map of subscribers for a CATV network: Layer 3

Since the layers are inherently constructed along the tree structure, it is intuitively to solve large network by layering approaches. When the network size is small enough, it can be solved by single layer solution procedure. In this chapter, we propose multilayer solution procedure for CATV network planning problems. The clustering algorithm is introduced first. By grouping subscribers into several clusters, the planning problem of a large network can be decomposed into several smaller problems, which can be solved by single layer solution procedure. After that, we can construct an upper layer network that each cluster of subscribers is treated as a macro subscriber. Since the number of users in the upper layer network is small enough, we can solve the network planning problem for upper layer network by single layer solution procedure.

In the computational experiments, we have compared the result of single and multi layers solution procedure. The solution quality is evaluated by two criteria, network costs and computing time. As shown in experiment results, the costs of network constructed by multilayer solution procedure are greater than those by single layer solution procedure. However, the computing time can be dramatically decreased. For those large networks that could not be solved by single layer solution procedure in limited time, the multilayer solution procedure still provide another feasible

alternative.

4.2 Problem Descriptions

In multilayer solution procedure, the major problem is how to cluster the subscribers into different groups. In CATV industry, there are several ways to cluster subscribers. The first clustering strategy groups subscribers by number. In generally, CATV operators view the entire network as groups of subscribers. The number of subscribers in a group ranges from 500 to 2000, depends on the services running on. In traditional broadcasting CATV networks, two thousands subscribers are maximum number allowed in a group. In modern two-way, interactive CATV networks, only five hundreds subscribers can be serviced in a group. Therefore, clustering users until the maximum number reached is a strategy in CATV network planning problems. For example, there is a CATV network design problem with the location map of subscribers as Figure 4-6.

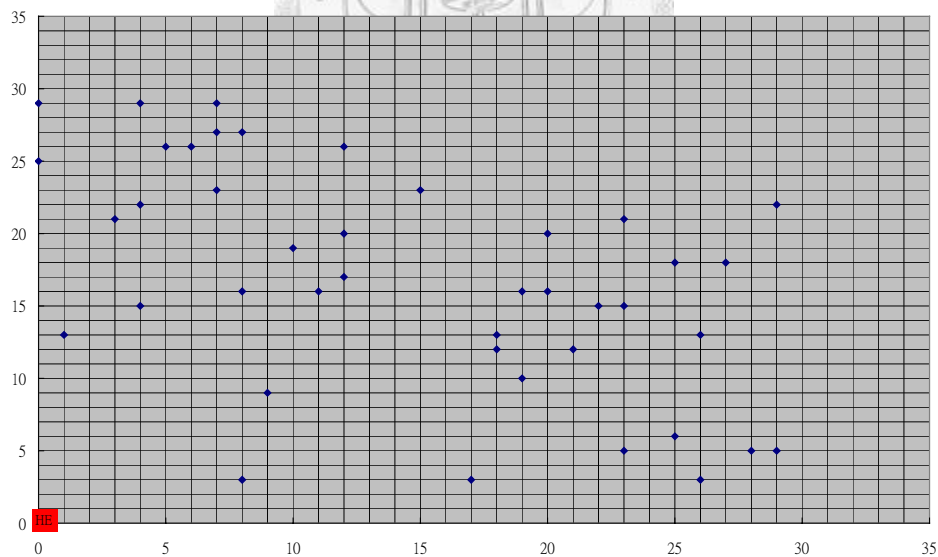


Figure 4-6: Location map of subscribers for a CATV network

If the maximum number of subscribers in a cluster is 5, we can group these subscribers as Figure 4-7.

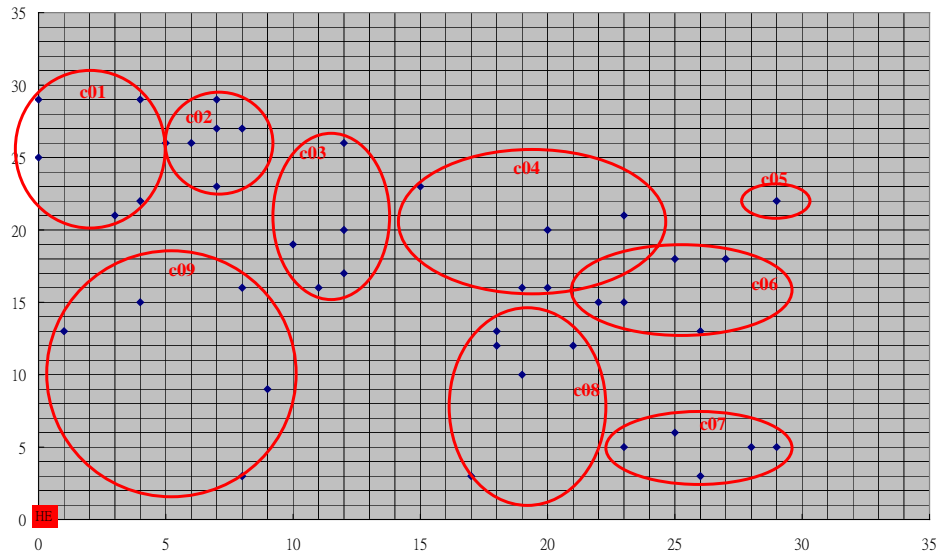


Figure 4-7: Clustering by number of subscribers

Another strategy to cluster subscribers is by positions. We can cut the entire network by several areas, which are based on geographical or administrative reasons. This strategy is generally used in telecommunication networks, for example, telephone networks. For the same example as above, the subscribers can be clustered as Figure 4-8.

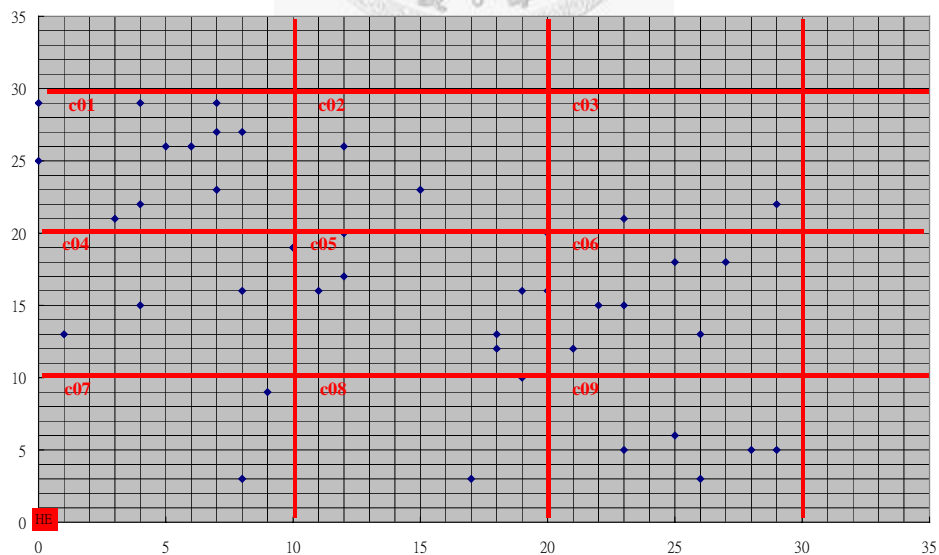


Figure 4-8: Clustering by subscribers' areas

For reducing cost of CATV networks, another strategy proposed to cluster subscribers is by diameters of clusters. The cost for each cluster is largely depended on the distance of lines. The longer the lines needed to connect all subscribers in a cluster, the higher the network cost is for this cluster. For example, if we limit the diameter of a cluster in six line segments, the subscribers will be clustered as Figure 4-9.

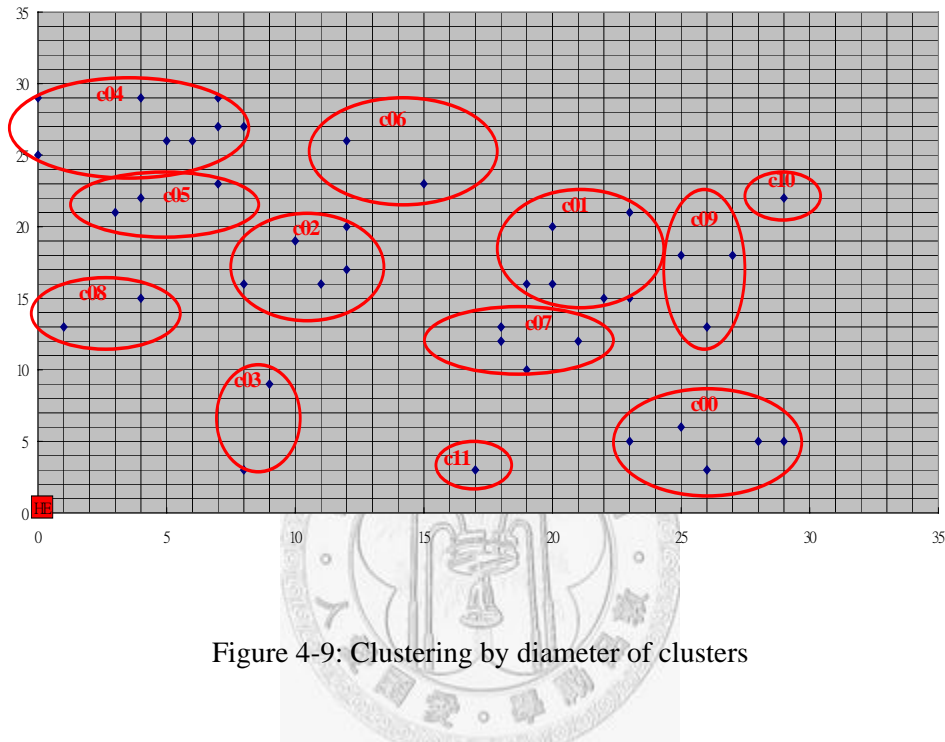


Figure 4-9: Clustering by diameter of clusters

Since the goal we are targeting is to minimize the cost of networks, we select the diameter-oriented strategy in our multilayer solution procedure. The purpose of clustering procedure should reduce the cost of each cluster. If only one subscriber permitted in a cluster, the solution procedure is the same as single layer solution procedure. However, it will make this problem hard to solve since the time to solve the problem would be too long.

The other objective is try to reduce the number of clusters in such a way the upper network is smaller enough to solve by single layer solution procedure. We can, of course, to develop more than two layers of hierarchy and solve them all. But it will increase the cost of entire networks when more layers are needed in solution procedures. Therefore our problem is how cluster all subscribers to minimize the total network cost, with diameter of each cluster is limited. Below is the description of this problem:

Given:

The number and positions of subscribers

To determine:

The member of each cluster

Objective:

To minimize the number of clusters

Subject to:

Maximum diameter allowed in a cluster

This clustering problem is a NP-Complete problem, and many heuristics are developed to solve this problem [30]. We have selected agglomerative clustering algorithm from these heuristics. Given a set of N instances to be clustered, and an N by N distance (or similarity) matrix, the agglomerative clustering algorithm clusters N instances into a cluster tree with all instances in a single cluster finally. The steps of agglomerative clustering algorithm are listed below:

Step 1: Start by assigning each item to its own cluster, so that if you have N items, you now have N clusters, each containing just one item.

Step 2: Let the distances (similarities) between the clusters equal the distances (similarities) between the items they contain.

Step 3: Find the closest (most similar) pair of clusters and merge them into a single cluster, so that now you have one less cluster.

Step 4: Compute distances (similarities) between the new clusters and each of the old clusters.

Step 5: Repeat steps 2 and 3 until all items are clustered into a single cluster of size n .

This algorithm is very close to what we need. However, we still need some modifications to fit our purpose well. First, the termination condition of programs should be different. The algorithm would terminate when all items are clustered into a single cluster. For multilayer solution procedure, the program should be terminated before the diameters of some clusters are larger than the maximum value allowed. Since we merge the closest pair of clusters in each iteration, whether the diameter of the new cluster after merged violates the constraint is the new termination condition. The step 3 should be modified as follow:

Step 3": Find the closest (most similar) pair of clusters, check the diameter D'' of merged cluster. If D'' is larger than maximum diameter allowed, write out the current members of each cluster and terminate the program. Otherwise, merge them into a single cluster, so that now you have one less cluster.

Another modification is the definition of distance between two clusters. There are many variations for definition of distance in clustering algorithms. For our purpose to confine the diameter of every cluster, the definition of distance should relate to diameter of clusters. In other words, the closet pair of clusters should be the less growth of diameter for all possible pairs. Round by round, the diameter of clusters would grow up and the program would terminate when the diameter of newly merged cluster is greater than maximum allowed. So the distance between two clusters is the longest distance between the most far away pair of members in two clusters. For example, the distance between clusters I and J is defined as D in Figure 4-10.

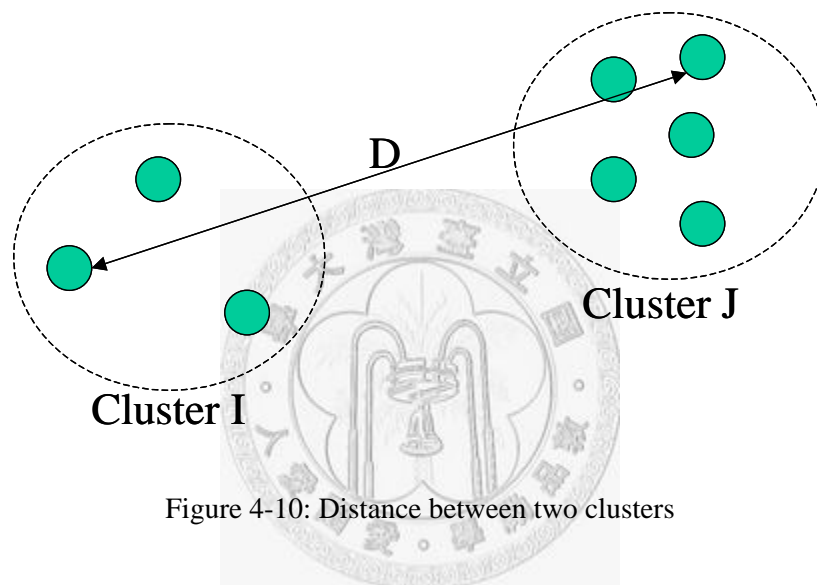


Figure 4-10: Distance between two clusters

With these two modifications from agglomerative clustering algorithm, our modified agglomerative clustering algorithm is described as follow:

Step 1: Start by assigning each item to its own cluster, so that if you have N items, you now have N clusters, each containing just one item.

Step 2: Let the distances (similarities) between two clusters equal the distance between the farthest pair of items contained in clusters.

Step 3: Find the closest (most similar) pair of clusters, check the diameter D'' of merged cluster. If D'' is larger than maximum diameter allowed, write out the current members of each cluster and terminate the program. Otherwise, merge them into a single cluster, so that now you have one less cluster.

Step 4: Compute distances (similarities) between the new clusters and each of the old clusters.

Step 5: Repeat steps 2 and 3 until all items are clustered into a single cluster of size n .

In the next section, we will compare multilayer and single layer solution procedures for two criteria. One is the computing time for two solution procedures. The other is the cost computed by these solution procedures. Besides the comparisons of different strategies, the selection algorithms of drop points for clusters are also discussed. Two selection algorithms are proposed and compared.

4.3 Computational Experiments

In this section, some comparisons about the multilayer solution procedure are presented. The parameters of experiments are listed in Appendix B. The experimental

platforms are IBM® personal computers with Microsoft® Windows XP Professional operational systems. These personal computers are equipped with a 1.86-GHz Pentium 4 CPU, 2 Giga-Bytes RAM, and one 80 Giga-Bytes Hard Disk.

The first experiment is to compare the time consumed in single layer and multilayer solution procedures. The result is shown in Figure 4-11.

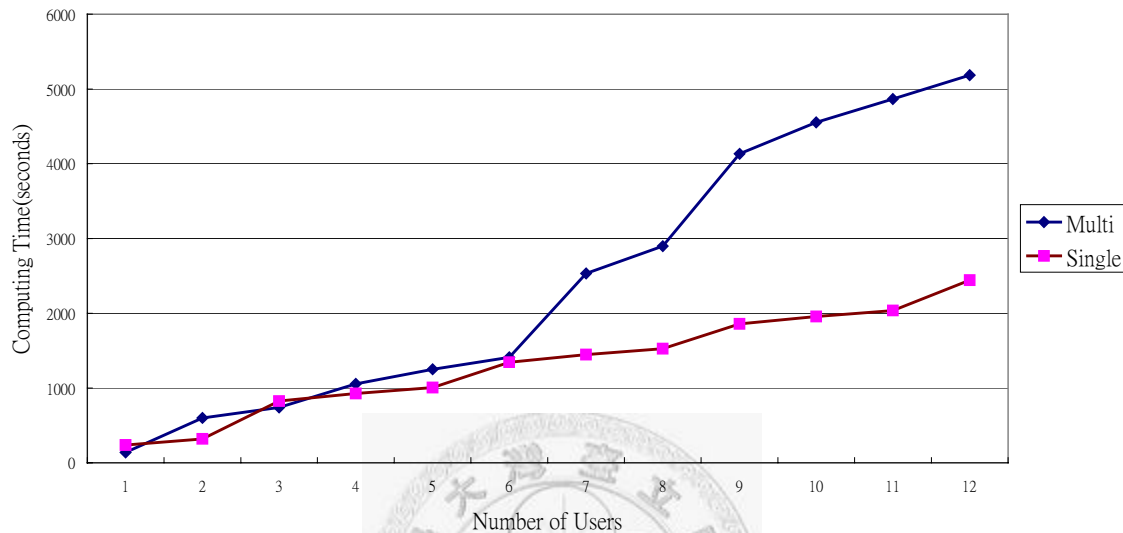


Figure 4-11: Comparison of computing time

The computing time for single layer solution procedure grows faster than multilayer solution procedure. In our experiments, the computing time for multilayer solution procedure ranges from 172% to 42% of single layer solution procedure. Especially with more users, the differences are more significant. As the number of users grows up, the computing time of single layer solution procedure would be no longer reasonable or possible for CATV network planners.

However, when we break down the entire network into several smaller networks, the optimality the single layer solution procedure is sacrificed. The question is: how much worse the multilayer solution procedure is? The other comparison is the costs between these two solution procedures, as Figure 4-12.

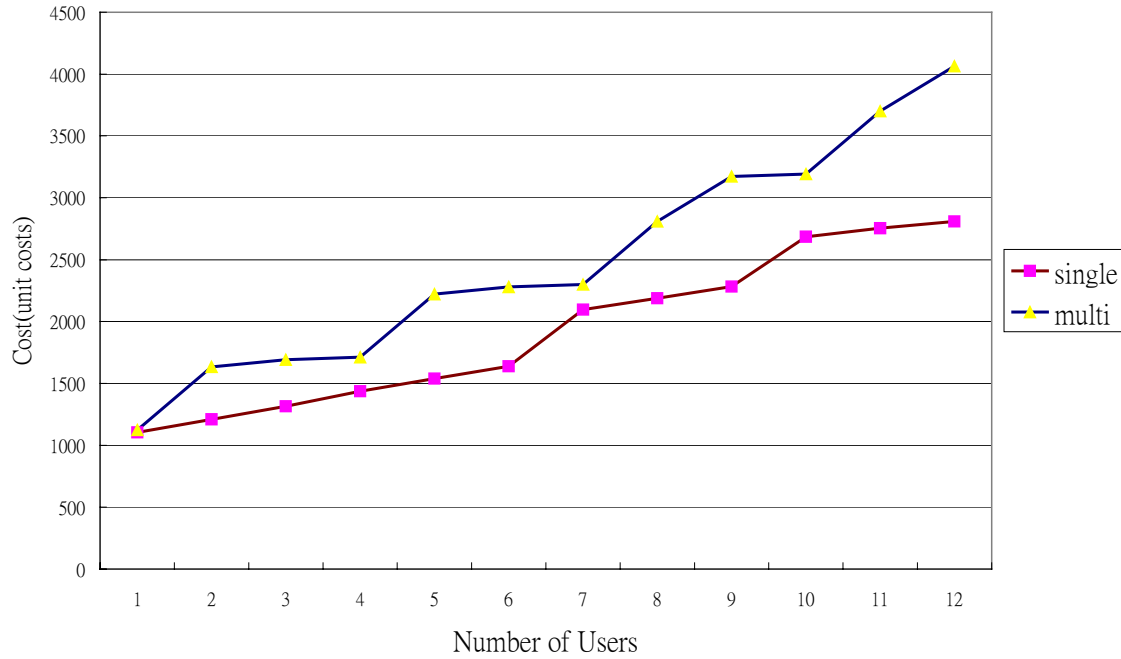


Figure 4-12: Comparison of computing time

Obviously the costs generated by multilayer solution procedure are larger than single layer solution procedure. The reason is straightforward: when we break into several clusters, the chances to globally optimized are lost. The cost differences are ranged from 2% to 45% for our experiments.

The next experiment is the scalability of multilayer solution procedure. We have solved problems with network users ranged from 1 to 300. The comparison of computing time is showed in Figure 4-13. The incremental speed of computing time in multilayer solution procedure is almost linear, but not exponential in single layer solution procedure.

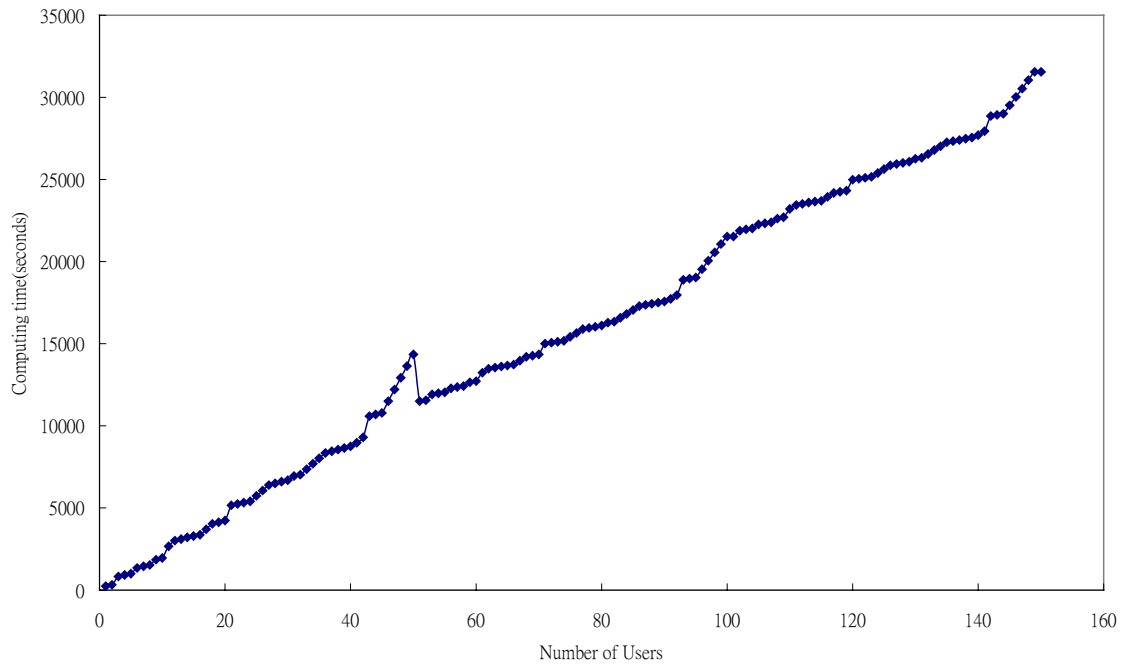


Figure 4-13 Growth rate of computing time

Although the costs are larger than single layer solution procedure, the computing times are still the major advantage for multilayer solution procedure. In general CATV networks, more than hundreds or thousands subscribers exists. Without multilayer solution procedure, it is impossible to use single layer solution procedure to design a CATV network.

Besides the comparison between the single layer and multilayer solution procedure, the drop point placement problem is also considered. After the clusters constructed, we need to place some network equipments to connect subscribers in cluster and head end. For example, there are two kinds of placements in Figure 4-14.

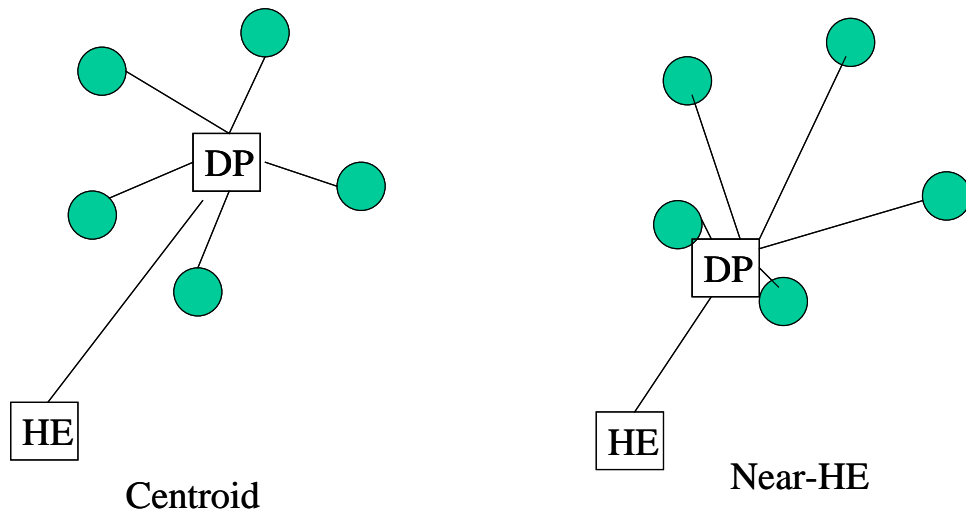


Figure 4-14: placements for drop points

Different placement policy changes the costs for networks intra-cluster and inter-cluster. In figure 4-14, the centroid policy, placing the drop points on the centroid points of clusters, should reduce the intra-cluster network costs. On the other hand, the near-HE policy, placing the drop points on the boundaries closest to the head end, may reduce the inter-cluster costs. If we can adjust the positions of drop points, we can improve the solution quality of multilayer solution procedure.

Here we did a sample test for placement policies. Considers the network clusters graph in Figure 4-15.

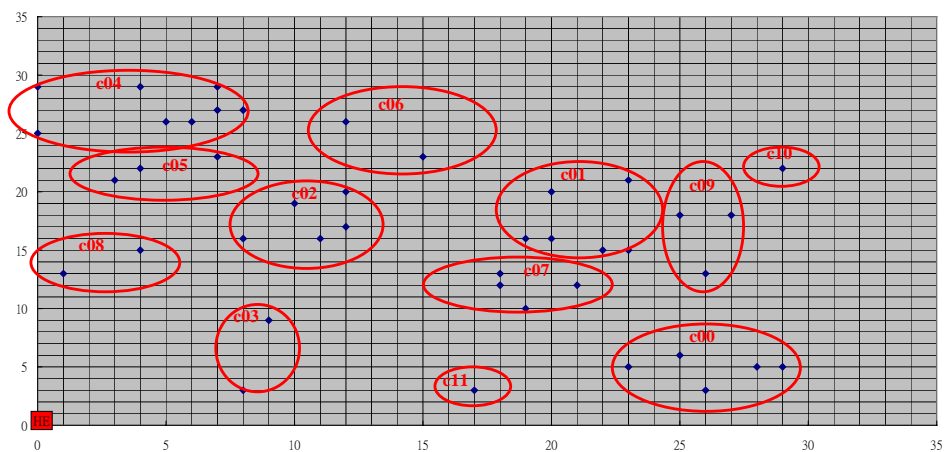


Figure 4-15: Network Clusters Graph

The network costs for different placement of drop points are generated by

multilayer solution procedure, as in Table 4-1.

Table 4-1: Computation of the cost for a 2-layer CATV network

Cluster	centroid	Near-HE
c00	2855	3173
c01	3300	3560
c02	3432	2396
c03	1450	1126
c04	3535	2300
c05	1443	1509
c06	1331	1324
c07	1690	1353
c08	1241	1245
c09	1662	1755
c10	707	707
c11	456	456
Layer 1	13554	17028
Total	36656	37934

The result shows some hints for us to reduce the cost for multilayer solution procedure. First, the cost of each cluster generated by centroid policy is usually lower than near-HE policy. It is not surprised since the centroid policy should reduce the length of wire in each cluster.

The second comparison between these two policies is the cost of upper layers. In our discussion before, the near-HE policy should reduce the cost for upper layers. But it is not in this example. In order to find the reason, we compare the Steiner trees constructed by two policies, as Figure 4-16.

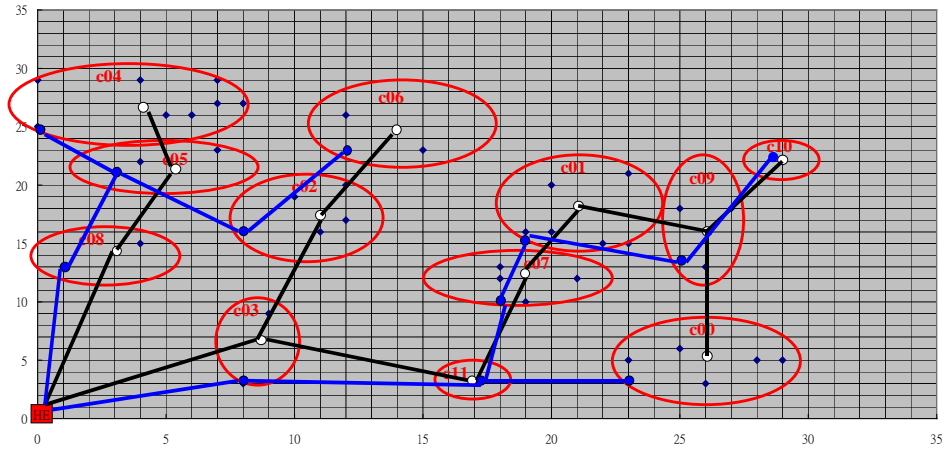


Figure 4-16: Layer 1 network topology for different placement of drop points

For some clusters, like clusters c03 and c08, the costs of upper layer are not changed to put the drop points near head end. However, the intra-cluster costs are increased. Therefore we could choose different policies in each cluster according to the differences of costs. The algorithm to adjust placement policies, named as Globally Adaptive Placement algorithm for drop points, is proposed as follow:

Globally Adaptive placement algorithm for drop points

Step 1: Clusters subscribers by modified agglomerative clustering algorithm.

Step 2: Compute the total cost of network by centroid placement policy.

Step 3: Considers every clusters, change different placement policy, and compare the total cost newly generated. If new placement policy can decrease the total cost, change the placement for this cluster.

Step 4: Repeat step 3 until all clusters are considered.

The comparison of network costs is listed in Table 4-2. It is clearly that the

globally adjustment procedure could reduce the cost of entire networks.

Table 4-2: Comparison of placement policies

Cluster	Centroid	NearHE	Globally AP
c00	2855	3173	2855
c01	3300	3560	3300
c02	3432	2396	2396
c03	1450	1126	1126
c04	3535	2300	2300
c05	1443	1509	1443
c06	1331	1324	1324
c07	1690	1353	1353
c08	1241	1245	1241
c09	1662	1755	1662
c10	707	707	707
c11	456	456	456
Layer1	13554	17028	12909
Total	36656	37934	33073

However, when we consider about the computing time for placement adjustment algorithms, it is possible to improve this algorithm from another perspective. Let us look the Steiner trees graph again.

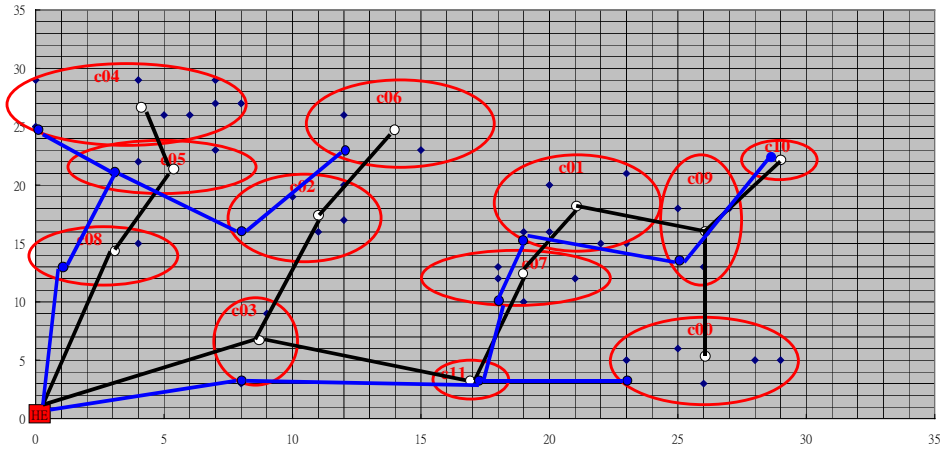


Figure 4-17: Layer 1 network topology for different placement of drop points

For those clusters on the leaf of trees, like clusters c04 and c06, different placements would change costs for inter- and intra- clusters. On the other hand, for those clusters are not leaf nodes in Steiner trees of upper layers, the network costs are changed very few when the placement policy changed. Therefore, it should be more time conserving to adjust placements only for those clusters on leaf nodes of Steiner trees of upper layers. We propose another adjustment algorithm described as follow.

Partially Adaptive placement algorithm for drop points

Step 1: Clusters subscribers by modified agglomerative clustering algorithm.

Step 2: Compute the total cost of network by centroid placement policy.

Step 3: Considers those clusters on the leaf nodes of Steiner tree in upper layer, change different placement policy, and compare the total cost newly generated. If new placement policy can decrease the total cost, change the placement for this cluster.

Step 4: Repeat step 3 until all clusters on the leaf nodes are considered.

The comparison of costs is showed in Table 4-3. In this example, the time for partially adaptive algorithm is 61.5% less than globally adaptive algorithm. On the other hand, the network cost for partially adaptive algorithm is 4.88% higher than globally. The choice between these two algorithms should depend on how much time allowed to compute. However, comparing with the fixed placement policies, no matter with the centroid or near-HE policy, the costs are improved.

Table 4-3: Comparison of placement policies

Cluster	Centroid	NearHE	Partial AP	Global AP
c00	2855	3173	2855	2855
c01	3300	3560	3300	3300
c02	3432	2396	3432	2396
c03	1450	1126	1450	1126
c04	3535	2300	2300	2300
c05	1443	1509	1443	1443
c06	1331	1324	1324	1324
c07	1690	1353	1690	1353
c08	1241	1245	1241	1241
c09	1662	1755	1662	1662
c10	707	707	707	707
c11	456	456	456	456
Layer1	13554	17028	13005	12909
Total	36656	37934	34865	33073

4.4 Concluding Remarks

In this chapter, we proposed multilayer solution procedure for CATV network planning problems. As analysis in the previous chapter, the computing time dramatically increases when the number of users grows up. Therefore, it is impractical

to solve large CATV network planning problems by single layer solution procedure.

In multilayer solution procedure, the major problem is how to cluster the subscribers into different groups. The clustering algorithm we adopted is modified from agglomerative clustering algorithm. Some modifications are added into the original agglomerative clustering algorithm. These modifications include the termination condition of clustering and the definition of distance of clusters. With modified agglomerative clustering algorithm, we could solve large CATV network planning problems.

The optimality of multilayer solution procedure is destroyed during the clustering procedure. In the computational experiments, we compared the solution quality between single layer and multilayer solution procedures. The differences on cost are ranged from 2% to 45% for our experiments. The solutions generated by multilayer solution procedure are worse than single layer unsurprisingly.

For the comparison of computing time, the multilayer solution procedure has the advantage over the single solution procedure. By using multilayer solution procedure, the computing time decreases dramatically. Especially when the number of subscribers grows up, the difference grows up quickly. For a general CATV network planning problem with hundreds or thousands subscribers, the multilayer solution procedure provide a possibility for using optimization based algorithms.

We have also compared the different placement for drop points in clusters. Different placement policies change the cost structure and the resultant total costs are different. In the section of computational experiments, we compare four placement policies for network costs and computing time. From the result of experiments, the globally adaptive placement algorithm provides better solution quality but needs more computing time. When the computing time is more critical, the partial adaptive placement algorithm provide a tradeoff between solution quality and computing time.

CHAPTER 5 CONCLUSION AND FUTURE WORK

5.1 Summary

In this dissertation, the CATV network design algorithms are proposed to minimize total installation costs, subject to the performance constraints. From the perspective of network management, a mathematical design algorithm should be helpful to network planners. Our algorithms are based on mathematical programming, relaxation techniques, and heuristics. Because of the nature of the problem, the nonlinear property is unavoidable and must therefore be dealt with.

In Chapter 2, we introduce a mathematical model to describe CATV networks. Based on some mathematical features of the model, some reformulations are necessary to solve the procedure. The surrogate functions are used to reformulate the objective function and some constraints. In Chapter 3, by applying some nonlinear programming techniques, the single layer solution procedure for CATV network planning problems is developed. Some computational experiments are described and explained. From the experiment results, the solution procedure we developed is better

than previous works. The comparison showed that our solution procedure is better in most of cases. The improvements on minimum costs are ranged from 51% to 92%. Based on the experiment results, we get some important finding in this problem, especially about the parameters settings in solution procedure. By the setting rules presented in this chapter, the solution quality, both the minimum cost and the scalability of the problem, can be further improved.

From the analysis of the solution procedure, however, we still could not deal with problems with too many nodes. Therefore, a multilayer solution procedure is proposed in Chapter 4. By layering a large network into several smaller networks, we can divide the problem and conquer every sub problems in reasonable time. After that, we can treat each network as a macro user in upper layer, and construct the network planning problem for upper layer. By summation the costs of upper layer and every sub layers, we can get the total cost of the entire network. By the multilayer solution procedure, we can solve CATV network planning problems with more nodes. We have compared with the single-layer solution procedure and show that only 40% of time is needed in multi-layer solution procedure. On the other side, the minimum costs solved by multilayer solution procedure are ranged from 2% to 45% larger than single-layer solution procedure. By balancing the computation time and solution quality, the multilayer solution procedure still provides a way to solve a larger network in limited time.

Besides the costs and computing time, we have developed algorithms for placement of drop points. In order to improve the costs of CATV networks, the placement of drop points in clusters is adjusted by proposed globally adaptive placement algorithm. Based on experiment results, the reduced costs ranges from 9% to 13%. With tradeoff between computing time and costs, we propose partially adaptive placement algorithm, which only adjust the leave nodes on upper layer networks. Compared with globally adjustment, the computing time is reduced to 61.5% and only 4.88% cost increased.

5.2 Future Work

Even though we have proposed a series of algorithms to deal with CATV networks planning problems, there are still many open issues to be further investigated. We point out three challenging issues to be tackled in the future.

1. The modified agglomerative clustering algorithm can be improved further by some heuristics. In modified agglomerative clustering algorithm, the subscribers are clustered step by step. It is possible that add-and-drop heuristic may improve the solution quality of clustering algorithm. We can drop some subscribers and add them into another cluster. In some cases, the possible improvement can get through this way. However, when and how this add-and-drop heuristic is applied would need more investigation.
2. Adjustment between the upper layer and lower layer of networks should be studied further. By multilayer solution procedure, the CATV networks are layed and solved separately. However, the parameter settings for upper layers not only change the solution of upper layers, but also change the solution of lower layers. By adjusting the parameter setting for upper and lower layers, the solution quality may improve further.
3. Apply the solution procedures for other kinds of application environments. Since the requirements changed, different applications on CATV networks should change the networks planning methods used in CATV industry. How to modify the mathematical model to accommodate new objectives and constraints for new services would be a challengeable work. Today, more and more new services are running on CATV networks. The new solution procedures for CATV networks could be developed based on our work.
4. In modern CATV networks, the hybrid fiber/coaxial technology is used. How to modified our mathematical model and solution procedures to solve problems with different technologies is another valuable work. Especially for other communication technologies with similar properties, like Passive Optical Networks, the modifications are needed.

APPENDIX A. LEGEND OF NOTATIONS

Notation	Description
L	The set of links
C_l	Installation cost of link l
Φ_l	Cost function of link l
V	The set of intermediate nodes
Φ_α	Cost function of active component (downstream amplifier)
$\Phi_{\bar{\alpha}}$	Cost function of active component (upstream amplifier)
Φ_β	Cost function of passive component
W	The set of user pairs
S_i	Input signal strength from the head end
P_w	The set of candidate paths for user pair w
H_{pc}	The number of class- C components on path p
λ_{pi}	Node v as the i^{th} equipment of path p
κ_{pi}	Link l as the i^{th} link of path p
N_i	Input noise strength from the head end
\bar{S}_i	Input signal strength from the user tap

\bar{N}_i	Input noise strength from the user tap
\bar{C}_{sys}	Upstream CNR performance requirement
$ W $	The cardinality of W
\bar{X}_{sys}	Upstream cross-modulation (X-MOD) performance requirement
L_{out}^v	The set of outgoing links for node v
L_{in}^v	The set of incoming links for node v
δ_{pl}	The indicator function, which is 1 if link l is on path p and 0 otherwise
F_v^L	Lower bound of the downstream noise figure for component on v
F_v^U	Upper bound of the downstream noise figure for component on v
\bar{F}_v^L	Lower bound of the upstream noise figure for component on v
\bar{F}_v^U	Upper bound of the upstream noise figure for component on v
F_l^L	Lower bound of the noise figure for link l
F_l^U	Upper bound of the noise figure for link l
M_v^L	Lower bound of the cross modulation intercept for component v
M_v^U	Upper bound of the cross modulation intercept for component v
O_v^L	Lower bound of the composite second order intercept for component v
O_v^U	Upper bound of the composite second order intercept for component v

B_v^L	Lower bound of the composite triple beat order intercept for component v
B_v^U	Upper bound of the composite triple beat order intercept for component v
\overline{G}_v^L	Lower bound of the downstream full gain for component v
\overline{G}_v^U	Upper bound of the downstream full gain for component v
$\overline{\overline{G}}_v^L$	Lower bound of the upstream full gain for component v
$\overline{\overline{G}}_v^U$	Upper bound of the upstream full gain for component v
\tilde{M}_v^L	Lower bound of the upstream cross modulation intercept for component v
\tilde{M}_v^U	Upper bound of the upstream cross modulation intercept for component v
K_e	The set of possible configurations for equipment e
K_c	The set of possible configurations for cable c
F_t^A	The set of available downstream noise figures for component t
\tilde{F}_t^A	The set of available upstream noise figures for component t
F_c^A	The set of available noise figures for link c
\overline{G}_t^A	The set of available downstream full gains for component t
$\overline{\overline{G}}_t^A$	The set of available upstream full gains for component t

\bar{M}_t^A	The set of available upstream cross modulations for component t
M_t^A	The set of available cross modulations for component t
O_t^A	The set of available composite second orders for component t
B_t^A	The set of available composite triple beats for component t
A_t^A	The set of available attenuation factors for component t
A_c^A	The set of available attenuation factors for cable c
ε	Threshold considered in the projection method
Decision Variables:	
y_l	Binary decision variable, which is 1 if link l is installed and 0 otherwise
z_v^α	Binary decision variable, which is 1 if component v is installed as a downstream active component and 0 otherwise
$z_v^{\bar{\alpha}}$	Binary decision variable, which is 1 if component v is installed as an upstream active component and 0 otherwise
z_v^β	Binary decision variable, which is 1 if component v is installed as a passive component and 0 otherwise
A_l	Attenuation factor of cable link l
F_l	Noise figure of cable link l
F_v	Downstream noise figure of equipment v

\overline{G}_v	Downstream full gain of equipment v
M_v	Downstream cross modulation intercept parameter of equipment v
B_v	Composite triple beat intercept parameter of equipment v
O_v	Composite second order intercept parameter of equipment v
\tilde{F}_v	Upstream noise figure of equipment v

$\overline{\overline{G}}_v$	Upstream full gain for component v
\tilde{M}_v	Upstream cross modulation intercept parameter of equipment v
x_p	Binary decision variable, which is 1 if path p is used and 0 otherwise
G_v	Downstream gain of equipment v
α_l	Cable splitting factor of link l
$\tilde{\overline{G}}_v$	Upstream gain of equipment v

APPENDIX B. EXPERIMENT PARAMETERS

The length of a cable segment	100 meters
The lower bound of the cable attenuation factor	0.01 or -20dB (100m)
The upper bound of the cable attenuation factor	0.5 or -3dB (100m)
The lower bound of amplifier full gain	100 or 20 dB
The upper bound of amplifier full gain	10000 or 40 dB
The lower bound of amplifier NF	6.3 or 8 dB
The lower bound of amplifier NF	31.6 or 15 dB
The lower bound of amplifier XMOD	10^7 or 70dB
The upper bound of amplifier XMOD	10^9 or 90 dB
Input signal strength	35 dBmv
The O-D pair CNR constraint	43 dB
The O-D pair XMOD constraint	-46 dB
The O-D pair signal strength constraint	10dbmv
The attenuation factor of a splitter or directional coupler	-1 dB
The attenuation factor of a user tap	-1 dB
The O-D pair upstream CNR constraint	20 dB

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