An Efficient Circuit–Switched Broadcasting in Star Graph

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Abstract. In this paper, we propose an algorithm for broadcasting in star graph using circuit-switched, half-duplex, and link-bound communication. By using the algorithm, we show that the broadcasting in an n-dimensional star graph can be done in n-1 time steps. We also study the lower bound of time steps of the circuit-switched broadcasting in star graph, and we prove that the optimal broadcasting time steps in an n-dimensional star graph is $\lceil \log_n n! \rceil$. Finally, the computational results showed that the proposed algorithm gets nearly optimal solutions.

Keywords: Broadcasting, interconnection network, star graph, circuit-switched routing, link-bound.

1 Introduction

The star graph interconnection network, since being proposed in [1], [2], is receiving increasing attention in the literature. It has been considered as an attractive alternative to the popular hypercube as the network architecture for parallel processing. Part of the reason is its symmetric and recursive nature, and superior (lower) node degree and comparable diameter as opposed to the hypercube [14]. Many references can be found in studying the star graph regarding such as its topological properties [3], [4], [13], embedding capability [5], [6], fault-tolerant capability [7], [8], [9], and even the construction of incomplete stars [10].

Among the efforts of studying the star graph, one of the central issues is around the various versions of broadcasting problem, broadcasting refers to the process by which a data set is sent from one node to all other nodes. Results about broadcasting are summarized in papers by Hedtniemi et al. [11] and Fraigniaud et al. [12].

In this paper, we considered the problem of broadcasting in star graph using circuit-switched, half-duplex, and link-bound communication. We propose an Efficient Circuit-Switched Broadcasting (ECSB) algorithm for an n-dimensional star graph with n! nodes. By using this algorithm, we showed that the broadcasting for an n-dimensional star graph is done in n-1 time steps.

The rest of this paper is organized as follows. In Section 2, we describe our communication model. In section 3, we discuss lower bounds on the optimal circuit-switched

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broadcasting time steps. An efficient circuit-switched broadcasting algorithm is presented in Section 4. Finally, we give our concluding remarks in Section 5.

2 Communication Model

An n-dimensional star graph, also referred as n-star or S_n, is an undirected graph consisting of $n!$ nodes (or vertices) and $(n\text{-}1)n!/2$ edges. Each node is uniquely assigned a label $x_0x_1 \ldots x_{n\text{-}1}$, which is the concatenation of a permutation of n distinct symbols $\{x_0x_1 \ldots x_{n\text{-}1}\}$. Without loss of generality, let these n symbols be $\{0,1,\ldots,n\text{-}1\}$. Given any node label $x_0 \ldots x_i \ldots x_{n\text{-}1}$, let function g_i, $1 \le i \le n\text{-}1$, be such that $g_i(x_0 \ldots x_i \ldots x_{n\text{-}1})=x_i \ldots x_0 \ldots x_{n\text{-}1}$ (i.e., swap x_0 and x_i to keep the rest symbols unchanged). In S_n, for any node x, there is an edge joining x and node $g_i(x)$, and this edge is said to e along dimension i. It is known that S_n is node- and edge-symmetric and has a diameter of $D_n=\lfloor 3(n\text{-}1)/2 \rfloor$.

In circuit-switched model, a node x sends its message to a node y via a directed path. Between two neighbor nodes in star graph, there exists exactly one link which can be used for both directions (but only one direction at one time), i.e., half-duplex link, and the link-bound communication is assumed, i.e., a node can use all of its links at the same time.

Fig. 1 shows an example of the circuit-switched broadcasting in S_3 under our communication model. In Fig. 1(a), a source node 012 sends a message to nodes 210 and 201 during the first time step. Fig. 1(b) shows that the source node 012 and informed nodes, 210 and 201, send messages to the remaining three nodes during the next time step.

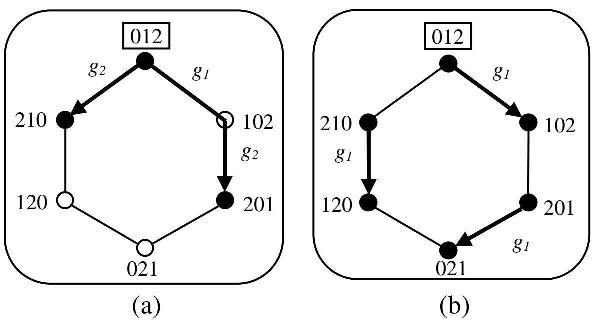


Fig. 1. An example of the circuit-switched broadcasting in S_3

3 Lower Bound of the Optimal Broadcasting Time Steps

In this section, we study the lower bound of time steps of the circuit-switched broadcasting in star graph.

Theorem 1. The optimal broadcasting time steps for an n-dimensional star graph with $n!$ nodes is $\lceil \log_n n! \rceil$.

Proof: The proof of Theorem 1 follows from the observation that each node can send the message to at most n-1 uninformed nodes at each time step in Fig. 1, because each node has n-1 degrees in S_n. For rapid broadcasting, source node and informed nodes must inform exactly n-1 other nodes at each time step except the last time step. Therefore, lower bound of the optimal broadcasting time steps is $\lceil \log_n n! \rceil$. □

4 An Efficient Circuit–Switched Broadcasting Algorithm

In this section, we present an efficient circuit-switched broadcasting algorithm in an n-dimensional star graph with broadcasting time steps n-1, $n \geq 3$. In order to facilitate our discussion, we introduce the following definitions.

Definition 1. The generator g_0 is defined by $vg_0 = v$, where v is a source node or a node of informed nodes in S_n.

Definition 2. We define a function *send*[v, $g_{x_0}g_{x_1}\ldots g_{x_c}$] to send the message from node v to the node located by $g_{x_0}g_{x_1}\ldots g_{x_c}$ function for $c \geq 0$, $x_i \geq 0$, where v is a source node or a node of informed nodes in S_n.

The proposed ECSB algorithm is shown in Fig. 2.

Algorithm. Efficient Circuit-Switched Broadcasting

Input: S_n and source node
Output: Broadcast to all the nodes
1: **begin**
2:   **for** i=n-1 to 2 **do**
3:     **for** j=i to 1 **pardo**
4:       *send*[v, $g_{j-1}g_i$];
5:     **end for**
6:   **end for**
7:   *send*[v,g_1];
8: **end**

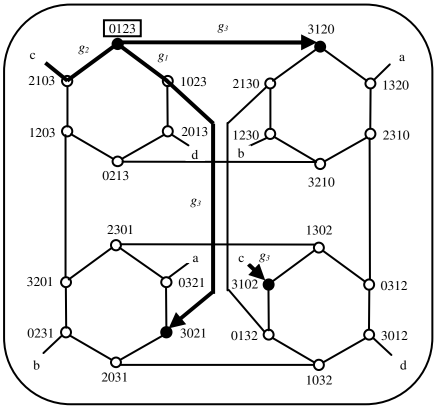
Fig. 2. The efficient circuit-switched broadcasting algorithm

Lemma 1. The efficient circuit-switched broadcasting algorithm for an n-dimensional star graph with $n!$ nodes can be done in n-1 time steps.

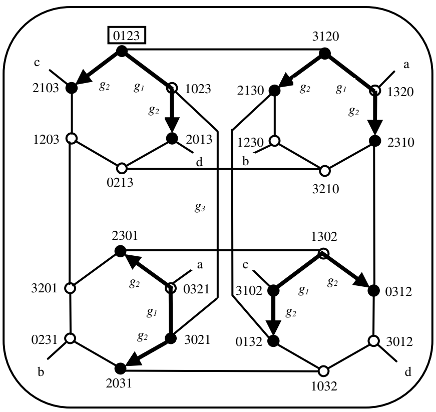
Proof: In the broadcasting algorithm, from step 2-6 execute n-2 time steps. Step 7 executes 1 time step. Hence, the algorithm executes n-1 time steps. □

Lemma 2. The algorithm can broadcasting from source node to all other nodes.

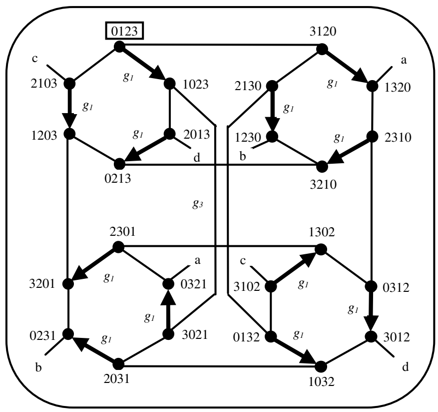
Proof: In efficient circuit-switched broadcasting algorithm, the steps 2-6 can send messages to other substar by each iteration, because v can send messages to other substar by $g_{j-1}g_i$ function. For example, v=0123 by g_2g_3 functions=3102, by g_1g_3 functions=3021, and by g_0g_3 functions=3120 in S_4. Step 7 can send messages to neighbor node by function g_1 in the last time step. Hence, the algorithm can broadcasting from the source node to all the other nodes. □



(a) 1st time step



(b) 2nd time step



(c) 3rd time step

Fig. 3. An example of the efficient circuit-switched broadcasting in S_4

Table 1. The comparison of the time steps of our proposed algorithm and lower bound time steps in n-dimensional star graph

Star size: n	Number of nodes: $n!$	Lower bound: $\lceil \log_n n! \rceil$	ECSB algorithm: n-1
3	6	2	2
4	24	3	3
5	120	3	4
6	720	4	5
7	5,040	5	6
8	40,320	6	7
9	362,880	6	8
10	3,628,800	7	9

Fig. 3 shows an example of the broadcasting in S_4. Figs. 3(a), 3(b), and 3(c) are the 1st time step, 2nd time step, and 3rd time step, respectively.

5 Conclusion

We considered the problem of broadcasting in *n*-dimensional star graph by using circuit-switched, half-duplex, and link-bound communication. The results showed that the broadcasting algorithm can be done in the star graph in nearly optimal time steps. The comparison of the time steps of our proposed algorithm and lower bound time steps in star graph is listed in Table 1.

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