Supply Chain Network Design with Time and Capacity Requirements

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Abstract

An efficient supply chain network is crucial to the logistics of a modern business. Careful design of the supply chain network will lead to enormous cost saving and market opportunity. In the supply chain network design, how to determine the best location and capacity configurations of the Distribution Centers (DCs) and the distribution paths is a challenging issue. Intensive researches have tackled this problem. However, in this paper, for the first time, we consider the supply chain network design problem with time and capacity constraints.

The supply chain network considered here is composed of several potential locations for the DCs and predetermined locations and traffic requirements for the supplier (origin) and consumer nodes (destination). The origin can only make the shipment to the destination via DCs without the possibility of shipping to the destination directly. In this paper, only the traffic requirements (including the amount of goods to be shipped and the shipping time requirement) of each origin and destination pair (O-D pair) is known in advance. We try to determine the best locations for these DCs and their associated optimal capacity configurations, and to establish the optimal routing assignment for each O-D pair in order to meet the traffic requirements.

The objective of supply chain design problem is to minimize the total DCs construction cost (in terms of the capacity configurations for the DCs) and the transportation cost (in terms of the capacity configurations for the transporting vehicles), where the capacity and time requirements are explicitly enforced in the constraints. The capacity constraints include the capacity constraints for the DCs and the transporting vehicles. The capacity constraints for the DCs require that the amount of the distributed goods should not exceed the capacity of the DCs. The capacity constraints for the transporting vehicles require that the amount of transported goods should not exceed the capacity of the transporting vehicles. The transportation time constraints require that the end-to-end transportation time should not exceed the maximum allowable end-to-end delay for each O-D pair. In this paper, we assume the transporting time to be a function of the associated distance, and the time function does not need to be a linear function of distance, and even non-convex functions could be handled.

The problem is formulated as a non-linear and non-convex combinatorial optimization problem. The Largragean relaxation method is applied to solve this problem. From the computational results, by assessing the gap between heuristic upper bounds and the Lagrangean lower bounds and the computational time, we propose the high efficiency and effectiveness algorithms for the supply-chain network design problem.

Keywords: Location theory, Optimization, Largragean Relaxation, Supply Chain Network, Distribution Centers

1. Introduction

A number of researchers considered this facility layout and design problem since 1909[5]. And location theory is one of the most challenging problem faced by many logistics managers, which have been studied by many researchers[2, 6, 7]. Also many commercial software packages were developed to address this issue[3]. Most of these researches try to optimize the average/maximum travel time/cost with capacity constraints. And these solution techniques belong to the heuristics based or optimization based (convex or integer) approaches[5]. These problems require sophisticated design of the distributing and capacity management.

In this paper, for the first time, we try to determine the optimal capacity for the DCs and the transporting vehicles in order to meet the end-to-end delay for each O-D pair and the capacity constraints. The mathematical formulation with nonlinear and integer constraints is used to address this problem. And the algorithms based on the Lagragean Relaxation method are used to solve this problem. There are four parameters input to the formulation: traffic requirement for each O-D pair, the locations for the user(consumer and supplier) nodes and DC nodes, the time requirement for each O-D pair, the candidate cost and capacity configurations for the transporting vehicles and DC nodes. Four output parameters from this formulation: routing(distribution path) assignment, the cost and capacity assignments for transporting vehicles and DC nodes, the end-to-end delay for each O-D pair.

This paper is organized as follows. In Section 2, mathematical formulation of the Supply Chain network design with time and capacity requirements(SCTC) is proposed. In Section 3, the dual approach for the SCTC based on the Lagrangean relaxation is presented. In Section 4, the primal heuristics are developed to get the primal feasible solutions from the Lagrangean relaxation problem. In Section 5, the computational results are reported. In Section 6, the concluding remarks are presented.

2. Mathematical Formulation

The mathematical formulation of the SCTC is given in this section. Before demonstrating the mathematical formulation, the notation is shown as below.

L : the set of candidate links in the transportation network.

W : the set of origin-destination (O-D) pairs in the network.

λ_w : the traffic requirement for each O-D pair $w \in W$.

P_w : a given set of simple directed paths from the origin to the destination of O-D pair w.

δ_{pl} : the indicator function which is one if link l is on path p and zero otherwise.

ε_{pk} : the indicator function which is one if DC k is on path p and zero otherwise.

D_w : the maximum allowable end-to-end delay requirement for O-D pair w.

O : the set of candidate locations for DCs.

A_l : the set of candidate capacity configurations for link l.

R_w : the set of admissible capacity configurations for DC at location k.

$\varphi_l(C_l)$: the cost for sending transporting vehicles with capacity C_l.

$Q_k(J_k)$: the cost for constructing a DC at location k with capacity J_k.

$F_l(l)$: the transporting time which is a function of distance of link l.

And the decision variables are depicted as follows.

x_p: 1 when path $p \in P_w$ is used for shipping for O-D pair $w \in W$ and 0 otherwise.

C_l: the capacity assignment for link $l \in L$.

J_k: the capacity assignment for DC at location k.

From the above notation, it is easy to see that we model the transporting vehicles as the links in the transportation network, and the capacity of the transporting vehicles is the capacity of the associated links. In the following, (IP) gives the mathematical formulation for the SCTC problem. The objective function of the (IP) is to minimize the construction of the total DCs construction cost (in terms of the capacity configurations for the DCs) and the transportation cost (in terms of the capacity configurations for the transporting vehicles). The constraints for the SCTC problem could be divided into two main parts, the first part is the capacity constraints for the DCs and the transporting vehicles and the second part is the time constraints for each O-D pair.

$$\min \; Z_{IP} = \sum_{l \in L} \varphi_l(C_l) + \sum_{k \in O} Q_k(J_k, S_k) \qquad \text{(IP)}$$

subject to:

$$\sum_{l \in L} \sum_{p \in P_w} x_p \delta_{pl} F_l(l) \le D_w \qquad \forall w \in W \qquad (1.1)$$

$$\sum_{p \in P_w} x_p = 1 \qquad \forall w \in W \qquad (1.2)$$

$$x_p = 0 \text{ or } 1 \qquad \forall p \in P_w, w \in W \qquad (1.3)$$

$$\sum_{w \in W} \sum_{p \in P_w} x_p \varepsilon_{pk} \lambda_w \le J_k \qquad \forall k \in O \qquad (1.4)$$

$$\sum_{w \in W} \sum_{p \in P_w} x_p \delta_{pl} \lambda_w \le C_l \qquad \forall l \in L \qquad (1.5)$$

$$J_k \in R_k \qquad \forall k \in O \qquad (1.6)$$

$$C_l \in A_l \qquad \forall l \in L. \qquad (1.7)$$

Constraint (1.1) specifies the end to end transporting time requirement for each O-D pair. Constraints (1.2) and (1.3) denote the distribution path for each O-D pair should be distributed over one path and exactly only one path. Constraint (1.4) requires that the goods distributed by the DC should be within the capacity of that DC. Constraint (1.5) requires that the amount of the shipping goods should be within the capacity of the transporting vehicles. Constraint (1.6) locates the possible capacity configurations for each DC. Constraint (1.7) locates the possible capacity configurations for the transporting vehicles.

As could be seen from (IP), the integer property makes the problem more complicated. As a result, the Lagragean Relaxation method is applied to relax the complicated constraints into the objective function to obtain the dual problem which could be easily solvable. In the following section, the Lagrangean Relaxation method is demonstrated in detail.

3. Lagrangean Relaxation

$$\min Z_D(a, b, e) = \sum_{l \in L} \varphi_l(C_l) + \sum_{k \in O} Q_k(J_k) + \sum_{w \in W} a_w [\sum_{l \in L} \sum_{p \in P_w} x_p \delta_{pl} F_l(l) - D_w] +$$

$$\sum_{k\in O} b_k [\sum_{w\in W}\sum_{p\in P_w} x_p \varepsilon_{pk}\lambda_w - J_k] + \sum_{l\in L} e_l [\sum_{w\in W}\sum_{p\in P_w} x_p \delta_{pl}\lambda_w - C_l] \tag{LR}$$

subject to:

$$\sum_{p\in P_w} x_p = 1 \qquad \forall w \in W \tag{2.1}$$

$$x_p = 0 \text{ or } 1 \qquad \forall p \in P_w, w \in W \tag{2.2}$$

$$J_k \in R_k \qquad \forall k \in O \tag{2.3}$$

$$C_l \in A_l \qquad \forall l \in L. \tag{2.4}$$

Constraints (1.1), (1.4) and (1.5) of (IP) were dualized to obtain the (LR). After proper rearrangement, we can decompose (LR) into three independent subproblems.

Subproblem 2-1: for x_p:

$$\min \sum_{w\in W}\sum_{p\in P_w}[\sum_{l\in L}(a_w F_l(l) + e_l\lambda_w)x_p\delta_{pl} + \sum_{k\in O} b_k\lambda_w x_p \varepsilon_{pk}]$$

subject to: (2.1) and (2.2).

Subproblem 2-2: for J_k

$$\min \sum_{k\in O}[Q_k(J_k) - b_k J_k]$$

subject to: (2.3).

Subproblem 2-3 : for C_l:

$$\min \sum_{l\in L}(\varphi_l(C_l) - e_l C_l)$$

subject to: (2.4).

Subproblem 2-1 could be further decomposed into $|W|$ independent subproblems. In order to deal with the arc weight of the DC node, the node splitting technique is used[4]. Hence, the DC node is splitting into the incoming DC node and the outgoing DC node and the artificial link between the incoming and outgoing DC node. Figure 1 shows the illustrating diagram of the node splitting technique. Thus, the arc weight of this DC node is the arc weight of this artificial link. As a result, for each independent subproblem is a shortest path problem and could be optimally solved by the Dijkstra's algorithm since the coefficient of x_p are the linear combination of positive multipliers.

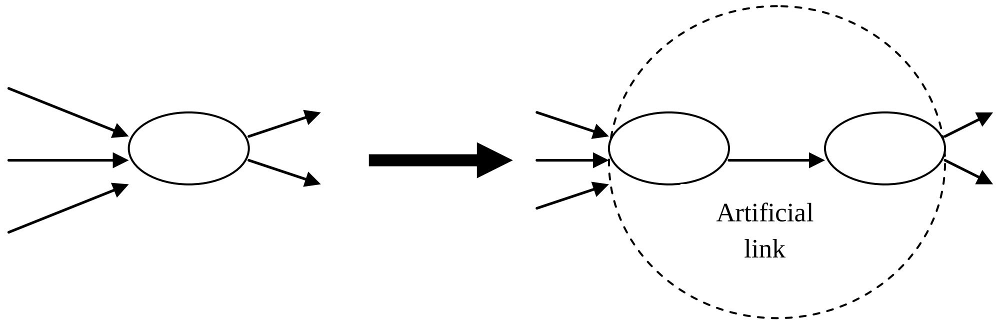

Figure 1. Node splitting technique

Subproblem 2-2 could be further decomposed into $|O|$ independent subproblems. For each independent subproblem, it could be optimally solved by the exhaustive search of all possible capacity configuration of the DC since the possible capacity configuration is finite.

Subproblem 2-3 also could be further decomposed into $|L|$ independent subproblems. For each independent subproblem, it could also be optimally solved by exhaustive search all possible finite capacity configuration of each link l.

From the arguments above that the algorithms developed for each subproblem could all be optimally solved, the weak Largragean Duality Theorem could be applied. That is, the lower bound from the dual Largragean formulation is a legitimate lower bound to the corresponding original problem[1].

4. Getting Primal Feasible Solution

In order to get the primal feasible solution, the solution to the (LR) problem is used. There are three possible ways to get the primal feasible solution from the solution to the (LR). The first is starting from the solutions to the subproblem 2-1. From the routing assignment (x_p) for each O-D pair, we could determine the aggregate flow for each link and DC node. Moreover, the least cost capacity configuration for each node and transporting vehicles could also be determined to satisfy the capacity constraints for the DC nodes and the links. If the transportation time requirement could also be satisfied by the link and node capacity determined above, the primal feasible solution is obtained.

The second is to start from the solutions to the subproblem 2-2. From the capacity of each DC node, it is not easy to construct the primal feasible solution, since the routing assignment is not known. Likewise, the third to start from the solutions to subproblem 2-3. Again, from the capacity of each transporting vehicles, it is not easy to construct the primal feasible solution due to same reason as subproblem 2-2.

In order to get a tighter upper bound, two primal heuristics from the solution to the subproblem 2-1 are used. The first heuristic is to consider the time constraints, when the time constraints is violated, identify the maximum end-to-end delay path, the arc weights along that path are increased, then the routing assignments are recalculated. On the other hand, the second heuristic is to consider the capacity constraints, when the solution is infeasible for capacity constraints, the arc weight for the overflow link is increased, then the routing assignments are recalculated. From the computational experiments, these two heuristics could provide better upper bound.

5. Computational Experiments

The computational experiments for the SCTC problem are performed. The algorithms developed in the above sections are coded in C++ and performed on a PC with INTEL^{TM} PⅢ-500 CPU. The tested network contain the ten user nodes(supplier and consumer nodes) and five potential DC nodes. The traffic requirement for each O-D pair is randomly generated. And the locations(x-axis and y-axis) for the user nodes and potential DC nodes are also randomly generated. Since the original node can only make the shipment to the destination via DCs without the possibility shipping to the destination directly, the total number of the potential links would only be 125(including five node-splitting links) instead of 215(including five node-splitting links). The computational time is about one to two minutes in this kind of the network size.

The time function $F_l(l)$ is assumed to be the Euclidean distance of the link. And the maximum allowable time requirement for each O-D pair is assumed to be a constant value, e.g. 50. The maximum number of iterations for the algorithms to solve (LR) is 1000, and the improvement counter is 30. The step size for the (LR) is initialized to be 2 and be half of its value when the objective value of the dual algorithm doesn't improve for 30 iterations.

We perform two sets of computational experiments. In the first set of computational experiments, the choice of the D_w value is fixed so as to examine the solution quality of the SCTC problem. Table 1 summarizes the results. The first column is the range of the randomly generated traffic requirements of each O-D pair. The second column reports the lower bound of the proposed dual Lagragean problem. The third column reports the upper bound of the proposed dual algorithm. The forth column reports the error gap between the lower bound and the upper bound. The fifth column reports the maximum end-to-end delay among all O-D pairs. The sixth column is the maximum allowable end-to-end delay (D_w). As can be seen in the forth column, the error gaps between the lower bound and the upper bound are almost the same for all different range of traffic requirements. And the error gaps are reasonably tight when the value of D_w is loose as compared to the maximum end-to-end delay among all O-D pairs. On the other hand, the ∞ symbol in the last row indicates that the primal feasible solution cannot be found.

Table 1 Comparison of solution quality obtained by various traffic requirements

Traffic requirements	Lower bound	Upper bound	Error Gap(%)	Maximum end-to-end delay	D_w
0~4	497.5	676	35.8	0.69	20
0~6	696.9	960	37.7	0.844	20
0~8	905.1	1237	36.6	0.886	20
0~10	1108.9	1504	35.6	0.819	20
0~11	1216.4	1634	34.3	0.937	20
0~12	1310.6	1767	34.8	0.844	20
0~13	1413.6	1900	34.4	0.843	20
0~14	1518.8	∞	∞	∞	20

Since the value for the maximum allowable end-to-end delay (D_w) for each O-D pair have a significant impact on the solution of the SCTC problem. In the second set of computational experiments, we try to examine the impact of the D_w value on the solution quality of SCTC problem. Table 2 summarizes this result. As could be seen from Table 2, the error gap is becoming slightly bigger under more fierce end-to-end delay requirements. From table 2, we could say that the solution quality is degrading gracefully under more and more fierce end-to-end delay requirements.

Table 2 Comparison of solution quality obtained by various D_w

Traffic requirements	Lower bound	Upper bound	Error Gap(%)	Maximum end-to-end delay	D_w
0~10	1108.9	1504	35.6	0.819	20
0~10	1108.9	1586	43.0	0.96	15
0~10	1108.9	1585	42.9	0.96	10
0~10	1108.9	1552	39.9	0.96	5
0~10	1108.9	1530	37.9	0.82	1
0~10	1108.9	1533	38.2	0.82	0.9
0~10	1108.9	1530	37.9	0.72	0.8
0~10	1108.9	1638	47.7	0.72	0.78

6. Concluding Remarks

In this paper, for the first time, we considered the problem of DC site selection and transporting vehicles assignment problem with maximum allowable end-to-end delay and capacity requirements. We formulate this problem as a nonlinear multi-commodity integral flow problem. The discrete (integer constraints) property makes the problem very difficult. We take an optimization-based approach by applying the Lagrangean relaxation technique in the algorithm development.

According to the first set of computational experiments, the error gaps are almost the same for all different range of traffic requirements. And the error gaps are reasonably tight when the value of D_w is loose as compared to the maximum end-to-end delay among all O-D pairs. On the other hand, from the second set of computational experiments, the solution quality is degrading gracefully under more and more fierce end-to-end delay requirements. Hence, the algorithms developed above provide a promising way of solving the complex supply chain network design problem.

References

[1] Ahuja R. K., T. L. Magnanti and J. B. Orlin, “Network Flows—Theory, Algorithms, and Applications”, Prentice Hall, ISBN0-13-027765-7, 1993.

[2] Ballou, Ronald H., “Business Logistics Management 3rd edition”, Prentice Hall, ISBN0-13-093410-0, 1992.

[3] Ballou, Ronald H. and J. M. Masters, “Facility Location Commercial Software Survey”, J. of Business Logistics, pp. 215-232, Vol. 20, No. 1, 1999.

[4] Lin Y. S. F., “Link Set Sizing for Networks Supporting SMDS”, IEEE/ACM Trans. On Networking, pp. 729-739, Vol. 1, No. 6, Dec. 1993.

[5] Margaret L. B. and S. S. Chiu, “Overview of Representative Location Research Problems”, Management Science, pp. 645-674, Vol. 35, No. 6, June 1989.

[6] Roberson J. F and W. C. Copacino, “The Logistics Handbook”, Free Press, ISBN 0-02-926595-9, 1994.

[7] Tsouros, C. and M. Satratzemi, “Optimal solution of a total time distribution problem”, International J. of Production Economics, pp. 438-478, Vol. 45, Issue 1-3, Aug. 1996.