Near Optimal Design of Lightpath Routing and Wavelength Assignment in Purely Optical WDM Networks

Hong-Hsu Yen[1] and Frank Yeong-Sung Lin
Dept. of Information Management
National Taiwan University
Taipei, Taiwan, R. O. C.
Tel: +886-2-23638423
Fax: +886-2-27584773
Email: d4725001@im.ntu.edu.tw yslin@im.ntu.edu.tw

Key words: Lightpath Routing, Wavelength Assignment, Lagrangean Relaxation, WDM Network, Optimization.

Abstract:
This paper proposes the optimal design of lightpath Routing and Wavelength Assignment (RWA) problem in Wavelength Division Multiplexing (WDM) networks without wavelength conversion. We formulate RWA as a mixed Integer Linear Programming (ILP) problem where the objective is to minimize the cost of wavelength assignment to the fiber links in the network. The Lagrangean relaxation technique and the optimization-based heuristics are used to solve this problem. Two sets of computational experiments are performed to test the algorithms for the maximum carried traffic and minimum wavelength requirements in three different network topologies (GTE, ARPA, OCT network). Based on solution quality to the computational experiments, the error gaps between the upper bound and the lower bound are close enough that the near optimal solutions could be obtained. On the other hand, we also show that the solution quality degrade gracefully under more and more heavy traffic network environment. By assessing solution quality and the computational time, we propose the efficient and effective optimization-based algorithms based on the Lagrangean relaxation method for the RWA problem in the WDM networks without wavelength conversion.

1. INTRODUCTION

WDM is a promising technique to utilize the enormous bandwidth of the optical fiber where the multiple wavelength-division multiplexed channels can be operated on a single fiber simultaneously [1]. A lightpath is an all-optical transmission path between two network nodes, implemented by the allocation of the same wavelength throughout the path [2]. However, how to route the wavelengths to a set of lightpaths is a challenging issue and

[1] Correspondence Author

proven to be a NP-Complete problem [6, 7, 9].

A number of researchers have addressed this issue. Chlamtac [2] and Liang [3] have introduced the semilightpath technique to find the routing path. Both of the two works try to find the routing path by shortest path algorithm based on the arc weight of the auxiliary graph. And the arc weight of the auxiliary graph is fixed in order to calculate the optimal routing path for the particular Origin-Destination (O-D) pair. However, the arc weight should change dynamically in different lightpath routing assignments for each O-D pair. Furthermore, they did not address the issue of how to avoid the wavelength assignment collision between all O-D pairs.

Kim [4] model the RWA in terms of the ILP formulation where the objective is to minimize the number of OXCs. And the heuristic algorithm based on branch & bound technique is proposed. However, no solution quality data between the solutions from the heuristic algorithm and the optimal solutions are reported from the computational experiments in order to clarify the effectiveness of this heuristic algorithm.

Chen [5] propose layer-graph model for solving the routing and wavelength assignment problem. Several heuristics based on shortest path algorithm and layer-graph approaches are proposed. However, no lower bound are reported in this work to verify the solution quality of the proposed heuristic algorithms.

Banerjee [6] formulated a Linear Programming (LP) formulation with the objective to minimize the number of wavelengths and solve by approximation algorithms in order to deal with large networks. However, in this work, only at most one lightpath is considered from a source to a destination.

In this paper, we try to optimize the cost of wavelength assignment on the fiber-optic links in the WDM networks such that to route the wavelengths to their destination without violating the wavelength continuity constraint. In addition, multiple lightpaths from a source to a destination is considered. The wavelength continuity constraint means that the same wavelength must be used on all the links along the selected path for the O-D pair [1, 5]. And the algorithms that we proposed, unlike previous researches, are based on the lower bound and upper bound approaches at the same time.

This paper is organized as follows. In Section 2, mathematical formulation of the RWA problem is proposed. In Section 3, the dual approach for the RWA problem based on the Lagrangean relaxation is presented. In Section 4, the getting primal feasible heuristic is developed to get the primal feasible solution from the solutions to the dual problem. In Section 5, the computational results are reported. In Section 6, the concluding remarks are presented.

2. RWA PROBLEM FORMULATION

The WDM network is modeled as the graph $G(W, L)$ where W is the set of O-D pairs and L is the set of fiber links. Here we assume that each node has the switching capability to route the wavelength to proper links. We show the definition of the following notation.

L	the set of candidate fiber links in the WDM network
W	the set of O-D pairs in the WDM network
J	the set of admissible wavelengths in the WDM networks
λ_w	the traffic requirement (in number of wavelengths) for each O-D pair $w \in W$
P_w	a given set of simple directed paths from the origin to the destination of O-D pair $w \in W$
δ_{pl}	the indicator function which is one if link $l \in L$ is on path $p \in P_w$ and zero otherwise
$\varphi_{lj}(C_{lj})$	the cost for installing wavelength $j \in J$ on link $l \in L$

And the decision variables are depicted as follows.

x_{pj}	1 when path $p \in P_w$ with wavelength $j \in J$ is used to transmit the traffic for O-D pair $w \in W$ and 0 otherwise
C_{lj}	1 when wavelength $j \in J$ is installed on link $l \in L$ and 0 otherwise

The RWA problem is formulated as a mixed ILP optimization problem, as shown below.

$$Z_{IP} = \min \sum_{l \in L} \sum_{j \in J} \varphi_{lj}(C_{lj}) \qquad \text{(IP)}$$

subject to:

$$\sum_{p \in P_w} \sum_{j \in J} x_{pj} = \lambda_w \qquad \forall w \in W \quad (1.1)$$

$$x_{pj} = 0 \text{ or } 1 \qquad \forall p \in P_w, w \in W, j \in J \quad (1.2)$$

$$\sum_{w \in W} \sum_{p \in P_w} x_{pj} \delta_{pl} \le C_{lj} \qquad \forall l \in L, j \in J \quad (1.3)$$

$$C_{lj} = 0 \text{ or } 1 \qquad \forall l \in L, j \in J . \quad (1.4)$$

The objective function of (IP) is to minimize the total cost of wavelength assignments in the WDM networks. Constraints (1.1) and (1.2) require that the wavelength requirements for each O-D pair should be routed to its destination. Here distinct routing paths for the O-D pair with different wavelength requirements are allowed only if each wavelength is routed on one path. Hence, the wavelength continuity constraint is explicitly enforced in these two constraints.

Constraint (1.3) enforces that any wavelength should be installed on the link before assigned by the O-D pairs for routing on this link. Constraint (1.4) requires that the wavelength assignment on each link is a zero/one integer constraint, which means each wavelength could only be installed on each link for one time or none. From Constraints (1.3) and (1.4), each wavelength could only be assigned by no more than one O-D pair on every link is strictly enforced.

3. LAGRANGEAN RELAXATION FOR (IP)

In order to solve the above formulation successfully, we relax (1.3) to obtain the following (LR) for (IP).

$$Z_D(a) = \min \sum_{l\in L}\sum_{j\in J}\varphi_{lj}(C_{lj}) + \sum_{l\in L}\sum_{j\in J} a_{lj}\Big(\sum_{w\in W}\sum_{p\in P_w} x_{pj}\delta_{pl} - C_{lj}\Big) \qquad \text{(LR)}$$

subject to:

$$\sum_{p\in P_w}\sum_{j\in J} x_{pj} = \lambda_w \qquad \forall w \in W \qquad (2.1)$$

$$x_{pj} = 0 \text{ or } 1 \qquad \forall p \in P_w, w \in W, j \in J \qquad (2.2)$$

$$C_{lj} = 0 \text{ or } 1 \qquad \forall l \in L, j \in J. \qquad (2.3)$$

We can decompose (LR) into two independent subproblems.

Subproblem 1: for x_{pj}

$$\min \sum_{w\in W}\sum_{l\in L}\sum_{j\in J}\sum_{p\in P_w} a_{lj} x_{pj}\delta_{pl}$$

subject to (2.1) and (2.2).

Subproblem 2: for C_{lj}

$$\min \sum_{l\in L}\sum_{j\in J}(\varphi_{lj}(C_{lj}) - a_{lj}C_{lj})$$

subject to (2.3).

Subproblem 1 could be further decomposed into $|W|$ independent subproblems. For each independent subproblem, it looks like a shortest path problem but the wavelength assignment makes this subproblem slightly more complicated. We propose the Wavelength-Routing Algorithm (WR) to solve this subproblem.

Wavelength-Routing Algorithm(WR)

Step 1: For each O-D pair $w \in W$, first finding the shortest path with respect to each wavelength. Since the multiplier is positive, so the Dijkstra's Shortest Path Algorithm could be applied.

Step 2: There are total $|J|$ shortest paths for this O-D pair. Then, the optimal solutions for O-D pair $w \in W$ are λ_w number of shortest paths with the lowest costs.

Subproblem 2 could also be further decomposed into $|L||J|$ independent subproblems. For each independent subproblem, which could be solved by the Wavelength-Assignment Algorithm (WA).

Wavelength-Assignment Algorithm(WA)

Step 1: For each wavelength $j \in J$ on each link $l \in L,$ calculate the value of $\varphi_{lj}(C_{lj}) - a_{lj}$.

Step 2: If this value is greater than zero, assign C_{lj} to zero else assign C_{lj} to one.

According to the algorithms proposed above, we could successfully solve the Lagrangean relaxation problem optimally. By using the weak Lagrangean duality theorem (for any given set of non-negative multipliers, the optimal objective function value of the corresponding Lagrangean relaxation problem is a lower bound on the optimal objective function value of the primal problem), $Z_D(a)$ is a lower bound on Z_{IP}. We construct the following dual problem to calculate the tightest lower bound and solve the dual problem by using the subgradient method.

$$Z_D = \max Z_D(a) \qquad \text{(D)}$$

subject to: $a \geq 0$.

Let the vector S be a subgradient of $Z_D(a)$ at (a). In iteration x of the subgradient optimization procedure, the multiplier vector $m^x=(a^x)$ is updated by $m^{x+1}= m^x+ \alpha^x S^x$, where $S^x(a) =(\sum_{w\in W}\sum_{p\in P_w} x_{pj}\delta_{pl} - C_{lj}$).

The step size α^x is determined by $\delta \dfrac{Z_{IP}{}^h - Z_D(m^x)}{\left\| S^x \right\|^2}$, where $Z_{IP}{}^h$ is an primal objective function value at iteration k (an upper bound on optimal primal objective function value), and δ is a constant ($0 \le \delta \le 2$).

4. GETTING PRIMAL FEASIBLE SOLUTIONS

To obtain the primal feasible solutions to the RWA problem, solutions to the Lagrangean relaxation problems (LR) is considered. From the wavelength-routing assignment x_{pj} to determine the wavelength assignment for each link C_{lj} in order to satisfy the Constraint (1.3). The algorithm to get the primal feasible solution is proposed as follows.

Joint Wavelength-Routing and Wavelength-Assignment Algorithm(WR-WA)

Step 1: For each O-D pair, say w_b, the wavelength routing assignment x_{pj} from the solutions to the dual problem is used. If the wavelength j along the routing path x_{pj} have not been used, assign the associated C_{lj} along this routing path to be one and go to Step 4. If wavelength assignment violation occurs at any link, go to Step 2.

Step 2: Find all the wavelengths on the links where $C_{lj} = 1$ and add the associated arc weight a_{lj} with a very large number, say *G1*. Then identify the O-D pair, say w_c, that use this conflict wavelength link. Go to Step 3.

Step 3: There are two ways to resolve conflict. First to release the associated C_{lj} along the routing path selected by w_c, and let the w_b find the minimum shortest path and the associated a_{lj} is modified accordingly. Then, finding the minimum shortest path of w_c. Compute the total cost, say *t1*, by adding the cost of the shortest paths for w_b and w_c. Second is to locate the minimum shortest path of w_b with the shortest path of w_c remain unchanged. The total cost of w_b and w_c is also computed, say *t2*. If the lower cost of *t1* and *t2* is greater than *G1*, then it is an infeasible primal solution and stop the whole algorithm else assign the associated C_{lj} to be one and go to Step 4.

Step 4: Repeat the whole process until all O-D pairs are executed. And the primal feasible solutions are obtained. Stop the whole algorithm.

5. COMPUTATIONAL EXPERIMENTS

The computational experiments for the RWA algorithms developed in section 3 and 4 are coded in C and performed at PC with INTEL^{TM} PIII-500 CPU. We tested the algorithm for 3 network topologies--ARPA, GTE, OCT with 21, 12 and 26 nodes. The network topologies are shown in Fig. 1, 2 and 3.

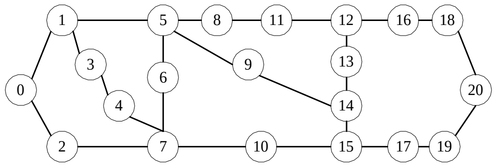

Figure 1. 21-node 52-link ARPA Network

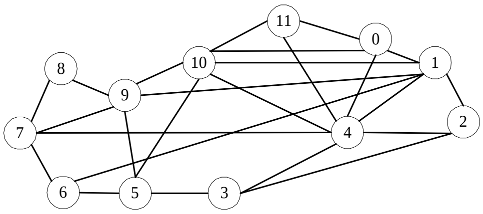

Figure 2. 12-node 50-link GTE Network

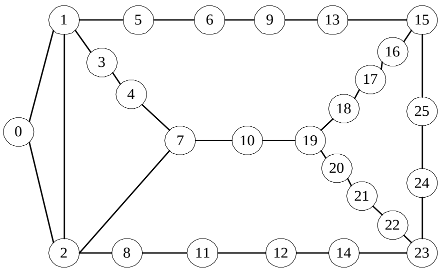

Figure 3. 26-node 60-link OCT Network

The maximum number of iterations for the proposed dual Lagrangean algorithm are 1500, and the improvement counter is 30. The step size for the dual Lagrangean algorithm is initialized to be 2 and be halved of its value when the objective value of the dual algorithm does not improve for 30 iterations.

Two sets of computational experiments are performed. The computational time for these two sets of computational experiments are all within fifteen minutes. In these computational experiments, the cost of installing each wavelength on each link is randomly generated from one to ten. In the first set of computational experiment, we try to explore the threshold of the traffic that the networks could afford in the different network topologies under the given total number of available wavelengths. This is to maximize the carried traffic for the RWA problem, which is similar to the objectives in a number of related researches [5, 8]. In this experiment, the total number of wavelengths available for each link is 40. Table 1 summarizes the results.

Here the error gap is defined as the {(upper bound – lower bound)/lower bound} * 100%. Since the traffic requirement is randomly generated, in the second column of Table 1, it means the range of traffic requirements (in wavelengths) for each O-D pair.

As could be seen from Table 1, the error gap is tighter (all below 3%) in lightly loaded (below 0~6) GTE network environment and become loser in highly loaded environment. And when the range of traffic requirement is above 12, no feasible solution could be found. On the other hand, in the ARPA and OCT network topologies, it is not easy to find the feasible solution, since the degree of the nodes is small as compared to the GTE network. Table 1 shows that no feasible solution could be found when the range of traffic requirements is above 3 in both of these two network topologies. In other words, no primal feasible solution could be obtained even in the lightly loaded environment when the degree of the nodes is small. Hence, the degree of the nodes affects the solution quality tremendously.

In the second set of computational experiment, we try to explore the threshold of the number of available wavelengths under fixed traffic requirements. In this experiment, the GTE network topology is tested, and the traffic requirements (in wavelengths) are randomly generated from zero to four for each O-D pair. Table 2 summarizes the results.

As could be seen from Table 2, the solution quality is getting loser as the number of available wavelengths for each link is getting smaller. And for the number of wavelengths to be no greater than 13, no feasible solution could be found. It is interesting to see that as the number of wavelengths is approaching the threshold, the solution quality is becoming unstable. That is, when the number of available wavelengths is 15, no feasible solution could be obtained, but feasible solution could be found when the number of available wavelengths is 14. In addition, the error gap is only 7% at the threshold of the number of wavelengths. Some researchers try to minimize the number of available wavelengths [6], in the similar way, we have located the minimum number of wavelengths needed to support a given set of lightpaths on a GTE topology. In this set of computational experiments, the minimum number of wavelengths is 14. By assessing the error gap (7%) at this number of wavelengths, we also provide the effective algorithms to find the minimum number of wavelengths.

6. CONCLUDING REMARKS

In this paper, we successfully solve the RWA problem in which wavelength conversion is not considered. We formulate RWA problem as a mixed ILP problem and solved by Lagrangean relaxation method. We introduce two algorithms to solve each independent dual subproblem successfully, and we also propose an optimization-based heuristic to get the primal feasible solution based on the solutions to the dual problem.

Two sets of computational experiments are performed. In the first set of computational experiments, the threshold of the traffic requirements is explored in different network topologies under a given number of wavelengths. The solution quality is good (error gap are below 3 percent) in lightly loaded network and reasonably good (error gap are below 13 percent) in more highly loaded network. On the other hand, we also show that the degree of the nodes is an important factor for the algorithms to find the feasible solutions. That is, in low degree network topology, it is difficult to find feasible solution even in the lightly loaded traffic environments.

In the second set of computational experiments, we try to locate the minimum number of total wavelengths which is the objective function in the past research. The approach to locate the number of total wavelengths is by iterative decreasing the number of total wavelengths when the feasible solution could be obtained. At the time that no primal feasible solution could be found, the minimum number of wavelengths is located. Based on the computational experiments, the solution quality is still good at the minimum number of wavelengths.

As recalled from the computational time and the solution quality of the computational experiments, we propose the efficient and effective algorithms to solve the RWA problem. The further research is to solve the RWA with wavelength conversion problem.

References:

[1] B. Ramamurthy and B. Mukherjee, “Wavelength Conversion in WDM Networking,” *IEEE JSAC*, Vol., 16, No. 7, pp. 1061-1073, Sep. 1998.

[2] I. Chlamtac, A. Faragó and T. Zhang, “Lightpath (Wavelength) Routing in Large WDM Networks,” *IEEE JSAC*, Vol., 14, No. 5, pp. 909-913, June 1996.

[3] W. Liang, G. Havas and X. Shen, “Improved Lightpath (Wavelength) Routing in Large WDM Networks,” *18th International Conf. On Distributed Computing Systems*, pp. 516-523, 1998.

[4] B. Kim, J.-H. Eom, J. Park and C.-J. Chae, “A Lightpath Routing Reconfiguration Algorithm for WDM Optical Networks,” *5th Asia-Pacific Conf. On Communication and 4th Optoelectronic and Communication Conf.*, Vol. 1, pp.11-14, 1999.

[5] C. Chen and S. Banerjee, “A new model for optimal routing and wavelength assignment in wavelength division multiplexed optical networks,” *Proc. of INFOCOM*, pp. 164-171, 1996.

[6] D. Banerjee and B. Mukherjee, “A Practical Approach for Routing and Wavelength Assignment in Large Wavelength-Routed Optical Networks,” *IEEE JSAC*, Vol., 14, No. 5, pp. 903-908, June 1996.

[7] P. Raghavan and E. Upfal, “Efficient Routing in All-Optical Networks,” *Proc. 26th ACM STOC*, pp. 384-395, 1994.

[8] N. Wauters and P. Demeester, “Design of the Optimal Path Layer in Multiwavelength Cross-Connected Networks,” *IEEE JSAC*, Vol. 14, No. 5, pp. 881-892, June 1996.

[9] I. Chlamtac, A. Ganz and G. Karmi, “Lightpath Communications: An Approach to High

Bandwidth Optical WAN's," *IEEE Trans. on Communications*, Vol. 40, No. 7, pp. 1171-1182, July 1992.

TABLE 1 – COMPUTATIONAL RESULTS (THRESHOLD OF THE TRAFFIC REQUIREMENTS UNDER DIFFERENT NETWORK TOPOLOGY)

Network topology	Traffic requirements	Lower bound	Upper bound	Error gap(%)
GTE	0~2	1.454951e+002	1.490000e+002	2.408975
	0~3	3.620000e+002	3.710000e+002	2.486190
	0~4	6.159618e+002	6.270000e+002	1.792022
	0~5	9.386195e+002	9.630000e+002	2.597483
	0~6	1.278489e+003	1.313000e+003	2.699331
	0~7	1.660505e+003	1.768000e+003	6.473647
	0~8	2.198478e+003	2.338000e+003	6.346305
	0~9	2.827843e+003	2.947000e+003	4.213689
	0~10	3.379600e+003	3.736000e+003	10.545640
	0~11	4.285055e+003	4.814000e+003	12.343940
	0~12	5.387118e+003	X#	X
ARPA	0~2	2.329307e+003	2.412000e+003	3.550106
	0~3	6.698546e+003	X	X
OCT	0~2	5.146004e+003	5.471000e+003	6.315504
	0~3	1.205400e+004	X	X

means no feasible solution could be found.

TABLE 2 – COMPUTATIONAL RESULTS (THRESHOLD OF THE NUMBER OF AVAILABLE WAVELENGTHS)

Network topology	# of available wavelengths	Lower bound	Upper bound	Error gap(%)
GTE	20	1.491610e+003	1.529000e+003	2.506690
	19	1.426938e+003	1.456000e+003	2.036654
	18	1.401527e+003	1.441000e+003	2.816450
	17	1.389967e+003	1.411000e+003	1.513170
	16	1.700296e+003	1.776000e+003	4.452425
	15	2.052708e+003	X	X
	14	1.883020e+003	2.024000e+003	7.486924
	13	2.204916e+003	X	X