# Reliable Wireless Communication Network Design Considering Customized Multiple-Connectivity 

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#### Abstract

In this paper, we identify reliability issue for channelized wireless communication networks. Due to the time variance and unstable properties of wireless communications, customized multipleconnectivity wireless networks are necessary for many kinds of high-reliability communications. By introducing generic communication quality of service (QoS) assurance and concurrently sequential homing mechanisms, we can design a realistic and reliable wireless network.

We formulate this problem as a combinatorial optimization algorithm to design a generic wireless system, which is a multiple-sectorization, power controllable, customized multiple-connectivity, and communication QoS assurance network. We integrate long-term channel assignment and sequential homing mechanisms to ensure communication grade of service (GoS) and improve spectrum utilization. The objective function of this formulation is to minimize total cost of network system subject to configuration, capacity, k-connectivity, sequential homing, QoS and GoS constraints. The solution approach is Lagrangean relaxation. In the computational experiments, our proposed algorithm can achieve up to $35.51 \%$ improvement of the total cost of network design problems from heuristic algorithm.


## 1. Introduction

Due to the rapid growth of wireless applications, the reliability property is become a critical issue for any uninterrupted communication system. One promising technique to overcome spectrum unstable property is multiple-connectivity. By specifying location-based customized multiple-connectivity requirement, network designer must well deploy base stations (BSs) and arrange spectrum resource to ensure individual connectivity requirement.

Cellular systems are generally recognized as spectrum-efficient by increasing the frequency allocation, sectorizing the cells, and resizing the sectors [4]. In this paper, we adopt several resource
allocation mechanisms, such as channel assignment, power control, and BS configuration design, to optimize total wireless system costs. For modeling generic architecture of realistic networks, we allow each BS can be constructed by any number of sectors, whose radians and transmission powers can be adjusted as needed.

Efficient interference management aims at achieving acceptable carrier-to-interference ratio (CIR) in all active communication links and optimizing the system capacity. We accumulate co-channel interference (CCI), adjacent channel interference (ACI) and near channel interference ( NCI ) as total interference and consider the radio propagation characteristic to ensure communication QoS [2][7]. Furthermore, in order to ensure grade-of-service (GoS) and support real-time admission control, we develop a location-based sequential homing mechanism to provide multiple-connectivity requirement for each mobile terminal (MT) [5][6].

We formulate the wireless network design problem as a combinatorial optimization problem, where the objective function is to minimize total cost of system subject to configuration, capacity, k -connectivity, sequential homing, QoS and GoS constraints. To the best of our knowledge, the proposed algorithm is the first attempt to consider the problem with whole factors jointly and formulate it rigorously. This kind of problems is by nature highly complicated and NP-complete. Thus, we apply the Lagrange relaxation approach and the subgradient method to solve this problem.

The remainder of this paper is organized as follows. Section II provides the problem description and mathematical formulation. In Section III, we adopt Lagrangean relaxation as our solution approach. Section IV supports algorithm to get feasible solutions. Section V is our computation experiments. Finally, the conclusion of this paper is in Section VI.

## 2. Wireless Network Design Problem

## A. Problem Description

In this chapter, we develop a mathematical model to discuss an integrated wireless communication network design problem, consists of BS installation, sectorization, capacity allocation, channel assignment, power control, and sequential routing problems. We study how multi-configuration sectorization, generic channel interference, and terrain-based radio propagation, will influence the performance of cellular system. Furthermore, we consider the effects of multiple-connectivity and sequential routing properties to enhance reliability of cellular networks.

The system parameters consist of six parts: (1) BS information, (2) MT information, (3) system parameters, (4) resource properties, (5) cost functions and (6) propagation environments. We define the notations of given parameters and decision variables in Table 1 and 2 respectively.

Table 1. Notations for given parameters.

| Given Parameters |  |
| :---: | :--- |
| Notation | Descriptions |
| $A$ | The configuration set of sector number <br> $A \subset\left\{A_{0}, A_{1}, A_{2}, A_{3}, A_{4}, A_{5}\right\}$ |
| $T$ | The set of mobile terminals |
| $C$ | The set of BSs in the system |
| $M$ | The set of all kinds of sectorization and <br> deployment types |
| $S_{t}$ | The set of permutation for MT $t$ which is <br> integer value and $S_{t}=\left\{1,2, \ldots, K_{t}\right\}$ |
| $W$ | Maximum number of available channels |
| $G_{j a_{m}}$ | An arbitrarily large number for Sector <br> $a_{m}$ of BS $j$ |
| $K_{t}$ | Connectivity requirement of MT $t$ to <br> connect with $K_{t}$ candidate homes |
| $L_{t i a_{m}}$ | Path loss ratio of radio propagation from <br> Sector $\left(j, a_{m}\right)$ to MT $t$ |
| $\delta$ | Receiver sensitivity of each MT (in Watt) |
| $\gamma$ | Required CIR constraint <br> $\lambda_{t}$The mean traffic arrival rate of MT $t \in T$ <br> (in Erlang) |
| $\overline{g_{j}}$ | Upper bound of aggregate traffic for <br> Sector $a_{m}$ of BS $j$ |
| $\bar{n}_{j a_{m}}$ | Upper bound of channel number for Sector <br> $a_{m}$ of BS $j$ |
| $\bar{\beta}_{t}$ | Required grade of service (GoS) of MT $t$ |


| $\overline{p_{j}}$ | Upper bound of transmission power of <br> Sector $a_{m}$ of BS $j$ |
| :---: | :--- |
| $d\left(n_{j a_{m}}\right.$, <br> $\left.g_{j a_{m}}\right)$ | Blocking probability function for Sector <br> $\left(j, a_{m}\right)$ which is the Erlang-B formula |
| $\theta(\Delta i)$ | NFD ratio which is formed as a function of <br> the channel separation normalized to the <br> bit-rate |
| $\Delta_{m}$ | Cost of BS with configuration type $m$ <br> $\Delta_{C}\left(n_{j a_{m}}\right)$ |
| $\Delta_{F}$ | Capacity cost function of equipments to <br> assign $n_{j a_{m}} \quad$ number of channels |

Table 2. Notations descriptions for decision variables.

| Decision Variables |  |
| :---: | :---: |
| Notation | Descriptions |
| $c_{\text {jm }}$ | Decision variable of configuration type $m$ for BS $j$ |
| $n_{j a_{m}}$ | Number of channels assigned to Sector $a_{m}$ of BS $j$ |
| $p_{j a_{m}}$ | Effective isotropic radiated power (EIRP) of Sector ( $j, a_{m}$ ) (in Watt) |
| $g_{j a_{m}}$ | Aggregate flow on Sector ( $j, a_{m}$ ) ( in Erlangs) |
| $k_{t j a_{m}}$ | Decision variable which is 1 if MT $t$ can be served by Sector $\left(j, a_{m}\right)$ and 0 otherwise |
| $x_{t j a_{m} s}$ | Homing decision variable which is 1 if Sector $\left(j, a_{m}\right)$ is selected as the $s^{\text {th }}$ candidate path of MT $t$ and 0 otherwise |
| $y_{i j a_{m}}$ | Decision variable for channel assignment for Sector ( $j, a_{m}$ ) about Channel $i$ |
| $f_{i}$ | Licensed channel |
| $B_{t s}$ | Call blocking probability for the $s^{\text {th }}$ candidate homing policy for $t$, where $B_{t s} \in\left\{0,0.01,0.02, \ldots, \bar{B}_{t s}\right\}$ is a discrete set |
| $b_{t j a_{m}}$ | Blocking probability of Sector $a_{m}$ on BS $j$ which is referenced by MT $t$ |

## B. Problem Formulation

Objective function (IP):

$$
\begin{align*}
& Z_{I P}=\min \sum_{j \in C} \sum_{a_{m} \in A} \Delta_{C}\left(n_{j a_{m}}\right) \\
& +\sum_{j \in C} \sum_{m \in M} \Delta_{m} c_{j m}+\sum_{i \in F} \Delta_{F} f_{i} \tag{IP}
\end{align*}
$$

subject to:

$$
\begin{align*}
& \begin{array}{lr}
\prod_{s \in S} B_{t s} \leq \bar{\beta}_{t} & \forall t \in T \\
\sum_{j \in C} \sum_{a_{m} \in A} x_{t j a_{m} s} b_{t j a_{m}}=B_{t s} & \forall t \in T, s \in S_{t} \\
d\left(n_{j a_{m}}, g_{j a_{m}}\right)=b_{t j a_{m}} & \forall t \in T, j \in C, a_{m} \in A
\end{array} \\
& \sum_{t \in T} \lambda_{t} \sum_{s \in S_{t}}\left(x_{t j a_{m} s} \prod_{k=1}^{s-1} B_{t k}\right)=g_{j a_{m}} \quad \forall j \in C, a_{m} \in A \\
& \gamma \leq \frac{\frac{p_{j a_{m}}}{2 L_{i j a_{m}}}\left(y_{i j a_{m}}+k_{t j a_{m}}\right)+\left(2-y_{i j a_{m}}-k_{t j a_{m}}\right) G_{j a_{m}}}{\sum_{j^{\prime} \in C-\left\{j j j a^{\prime}{ }_{m} \in A\right.} \sum_{i{ }^{\prime}}\left(\frac{p_{j^{\prime} a_{m}^{\prime}}}{L_{t j^{\prime} a_{m}^{\prime}}} \sum_{i^{\prime} \in F} y_{i^{\prime} j^{\prime} a_{m}^{\prime}} \theta\left(\left|i-i^{\prime}\right|\right)\right)} \\
& \forall t \in T, i \in F, j \in C, a_{m} \in A \\
& k_{t j a_{m}} \delta \leq \frac{p_{j a_{m}}}{L_{t j a_{m}}} \\
& \sum_{j \in C} \sum_{a_{m} \in A} x_{t j a_{m} s}=1 \\
& \forall t \in T, j \in C, a_{m} \in A \\
& \sum_{s \in S_{t}} x_{t j a_{m} s} \leq k_{t j a_{m}} \\
& \forall t \in T, j \in C, a_{m} \in A \\
& \sum_{j \in C} \sum_{a_{m} \in A} k_{t j a_{m}} \geq K_{t} \\
& \forall t \in T \\
& \sum_{i \in F} y_{i j a_{m}}=n_{j a_{m}} \\
& \forall j \in C, a_{m} \in A \\
& \sum_{a_{m} \in A}\left(y_{i j a_{m}}+y_{(i+1) j a_{m}}\right) \leq 1 \\
& \forall i \in F, j \in C \\
& \sum_{i \in F} f_{i} \leq W \\
& y_{j i} \leq f_{i} \quad \forall i \in F, j \in C, a_{m} \in A \\
& y_{i j a_{m}} \leq c_{j m} \\
& \forall i \in F, j \in C, a_{m} \in A, m \in M \\
& p_{j a_{m}} \leq \bar{p}_{j a_{m}} \times \sum_{i \in F} y_{i j a_{m}} \quad \forall j \in C, a_{m} \in A \\
& \sum_{m \in M} c_{j m}=1 \\
& \forall j \in C \\
& c_{j m}=0 \text { or } 1 \\
& \forall j \in C, m \in M \\
& y_{i j a_{m}}=0 \text { or } 1 \\
& \forall i \in F, j \in C, a_{m} \in A \\
& x_{t a_{m} s}=0 \text { or } 1 \\
& \forall t \in T, j \in C, a_{m} \in A, s \in S_{t} \\
& k_{t j a_{m}}=0 \text { or } 1 \\
& \forall t \in T, j \in C, a_{m} \in A \\
& \begin{array}{l}
\forall i \in F \\
, a \in A
\end{array} \\
& f_{i}=0 \text { or } 1 \\
& y_{(|F|+1) j a_{m}}=0 \\
& \forall i \in F, j \in C, a_{m} \in A \\
& 0 \leq p_{j a_{m}} \leq \bar{p}_{j a_{m}}  \tag{23}\\
& \forall j \in C, a_{m} \in A \\
& 0 \leq n_{j a_{m}} \leq \bar{n}_{j a_{m}}  \tag{24}\\
& \forall j \in C, a_{m} \in A .
\end{align*}
$$

The objective function is to minimize the total cost of wireless communication networks, which sum up the costs of (1) fixed installation cost of BSs, (2) capacity equipment cost, and (3) the spectrum licensing fee. Constraint (1) is the acceptable upper
bound of call-blocking probability requirement. Constraints (2) and (3) calculate the call blocking probability of MT $t$ on the sequence $s$. Constraint (4) calculates the aggregate traffic for each sector under sequential routing effect. Constraint (5) ensures the CIR constraints. Constraint (6) enforces the receiver sensitivity constraint. Constraints (7) and (8) are sequential routing constraint. Constraint (9) enforces the k-connectivity constraint of MT $t$. Constraint (10) calculates the total capacity of channels for each sector. Constraint (11) enforce adjacent channel must not be assigned to the same BS. Constraints (12) and (13) ensure the number of assigned channels is less than the total available channels. Constraint (14) ensures channel can be assigned only if this sector is deployed on BS $j$. Constraint (15) ensures transmission power can larger than zero only if we have assigned some channels on this sector. Constraint (16) enforces that only one sectorization type can be selected for each BS. Constraints (17) to (21) enforce the integer property of the decision variables. Constraint (22) defines the value of boundary variables. Constraints (23) and (24) enforce the feasible regions of decision variables $p_{j a_{m}}$ and $n_{j a_{m}}$.

## 3. Solution Approach

By using the Lagrangean Relaxation method [1], we can transform the primal problem (IP) into the following Lagrangean relaxation problem (LR) where Constraints (3), (4), (5), (8), (9), (10), (11), and (13) are relaxed.

## A. Lagrangean Relaxation

For a vector of Lagrangean multipliers, a Lagrangean relaxation problem of (IP) is given by optimization problem (LR):

$$
\begin{aligned}
& Z_{L R}\left(\mu_{t j a_{m}}^{1}, \mu_{j a_{m}}^{2}, \mu_{t j a_{m}}^{3}, \mu_{t j a_{m}}^{4}, \mu_{t}^{5}, \mu_{j a_{m}}^{6}, \mu_{i j}^{7}, \mu_{i j a_{m}}^{8}\right)= \\
& \min \sum_{j \in C} \sum_{a_{m} \in A} \Delta_{C}\left(n_{j a_{m}}\right)+\sum_{j \in C} \sum_{m \in M} \Delta_{m} c_{j m}+\sum_{i \in F} \Delta_{F} f_{i} \\
& +\sum_{j \in C} \sum_{a_{m} \in A} \sum_{t \in T} \mu_{j j a_{m}}^{1}\left(d\left(n_{j a_{m}}, g_{j a_{m}}\right)-b_{t j a_{m}}\right) \\
& +\sum_{j \in C a_{m} \in A} \mu_{j a_{m}}^{2}\left(\sum_{t \in T} \lambda_{t} \sum_{s \in S_{t}}\left(x_{t j a_{m} s} \prod_{k=1}^{s-1} B_{t k}\right)-g_{j a_{m}}\right) \\
& +\sum_{j \in C C_{m} \in A} \sum_{i \in F} \sum_{t \in T} \mu_{t i j a_{m}}^{3}\left(\sum_{\left.j^{\prime} \in C-(j)\right\}} \sum_{a_{m} \in A}\left(\frac{p_{j^{\prime} a^{\prime}{ }_{m}}}{L_{t j^{\prime} a_{m}}} \sum_{i^{\prime} \in F} y_{i^{\prime} j^{\prime} a^{\prime}{ }^{\prime}} \theta\left(\left|i-i^{\prime}\right|\right)\right)\right. \\
& \left.-\frac{1}{\gamma}\left(\frac{p_{j a_{m}}}{2 L_{i j a_{m}}}-G_{j a_{m}}\right)\left(y_{i j a_{m}}+k_{i j a_{m}}\right)-\frac{2 G_{j a_{m}}}{\gamma}\right)
\end{aligned}
$$

$$
\begin{align*}
& +\sum_{j \in C} \sum_{a_{m} \in A} \sum_{t \in T} \mu_{t j a_{m}}^{4}\left(\sum_{s \in S_{t}} x_{t j a_{m} s}-k_{t j a_{m}}\right) \\
& +\sum_{t \in T} \mu_{t}^{5}\left(K_{t}-\sum_{j \in C} \sum_{a_{m} \in A} k_{t j a_{m}}\right) \\
& +\sum_{j \in C} \sum_{a_{m} \in A} \mu_{j a_{m}}^{6}\left(\sum_{i \in F} y_{i j a_{m}}-n_{j a_{m}}\right) \\
& +\sum_{j \in C i \in F} \sum_{i j} \mu_{i j}^{7}\left(\sum_{a_{m} \in A}\left(y_{i j a_{m}}+y_{(i+1) j a_{m}}\right)-1\right) \\
& +\sum_{j \in C} \sum_{a_{m} \in A} \sum_{i \in F} \mu_{i j a_{m}}^{8}\left(y_{i j a_{m}}-f_{i}\right) \tag{LR}
\end{align*}
$$

subject to: (1), (2), (6), (7), (12), (14), (15), (16), (17), (18), (19), (20), (21), (22), (23) and (24).

$$
\mu_{t j a_{m}}^{1}, \mu_{j a_{m}}^{2}, \mu_{t i j a_{m}}^{3}, \mu_{t j a_{m}}^{4}, \mu_{t}^{5}, \mu_{j a_{m}}^{6}, \mu_{i j}^{7}, \mu_{i j a_{m}}^{8} \quad \text { are }
$$

Lagrange multipliers and $\mu_{t i j a_{m}}^{3}, \mu_{t j a_{m}}^{4}, \mu_{t}^{5}, \mu_{i j}^{7}, \mu_{i j a_{m}}^{8} \geq 0$.
To solve (LR), we can decompose it into the following four independent solvable optimization sub-problems and develop several algorithms to determine configuration of each BS, transmission power of each sector, channel assignment plan of system, sequential homing policy of each MT, and average call blocking rate under multipleconnectivity constraints.

Subproblem (SUB1): (related with decision
variables $B_{t s}, b_{t j a_{m}}$, and $x_{t j a_{m} s}$ )

$$
\begin{align*}
& Z_{S U B 1}=\min -\sum_{j \in C} \sum_{a_{m} \in A} \sum_{t \in T} \mu_{t j a_{m}}^{1} b_{t j a_{m}} \\
& +\sum_{j \in C} \sum_{a_{m} \in A} \sum_{t \in T} \sum_{s \in S_{t}} x_{t j a_{m} s}\left(\mu_{j a_{m}}^{2} \lambda_{t} \prod_{k=1}^{s-1} B_{t k}+\mu_{t j a_{m}}^{4}\right) \tag{SUB1}
\end{align*}
$$

subject to: (1), (2), (7), (19), and

$$
\begin{array}{lc}
\sum_{s \in S} x_{t j a_{m} s} \leq 1 & \forall t \in T, j \in C, a_{m} \in A \\
\underline{B}_{t s} \leq B_{t s} \leq \bar{B}_{t s} & \forall t \in T, s \in S_{t}, B_{t s} \in K_{t s}  \tag{26}\\
0 \leq b_{t j a_{m}} \leq 1 & \forall t \in T, j \in C, a_{m} \in A .
\end{array}
$$

Because multiplier $\mu_{j a_{m}}^{2}$ is not required to be positive, this formulation is a signomial geometric programming problem. For dealing with this problem efficiency, we limit variable $B_{t s}$ to a discrete set $K_{t s}=\left\{\underline{B}_{t s}, \underline{B}_{t s}+0.01, \underline{B}_{t s}+0.02, \ldots, \bar{B}_{t s}-0.01, \bar{B}_{t s}\right\}$
by introducing an additional Constraint (26) where notations $\underline{B}_{t s}$ and $\bar{B}_{t s}$ are a sensible lower bound and upper bound. As the discrete property of $B_{t s}$, we can exhaustively search for all possible values of $B_{t s}$. Therefore, we can decompose this problem into $|T|$
independent sub-problems. Each subproblem solves the following problem (SUB1t),

$$
\begin{aligned}
& Z_{S U B 1 t}=\min -\sum_{j \in C} \sum_{a_{m} \in A} \mu_{t j a_{m}}^{1} b_{t j a_{m}} \\
& \quad+\sum_{j \in C} \sum_{a_{m} \in A} \sum_{s \in S_{t}} x_{t j a_{m} s}\left(\mu_{j a_{m}}^{2} \lambda_{t} \prod_{k=1}^{s-1} B_{t k}+\mu_{t j a_{m}}^{4}\right)
\end{aligned}
$$

subject to: (1), (2), (7), (19), (25), (26), and (27).
Decision variable $b_{t j a_{m}}$ can be determined by the following statements,

$$
b_{t j a_{m}}= \begin{cases}1, & \text { if } \sum_{s \in S_{t}} x_{t j a_{m} s}=0 \text { and } \mu_{t j a_{m}}^{1} \geq 0 \\ 0, & \text { if } \sum_{s \in S_{t}} x_{t j a_{m} s}=0 \text { and } \mu_{t j a_{m}}^{1}<0 \\ B_{t s}, & \text { if } \sum_{s \in S_{t}} x_{t j a_{m} s}=1\end{cases}
$$

where the assignment purpose is to minimize the objective value under a given combinatorial situation of $x_{t j a_{m} s}$ and $B_{t s}$. We define the notation $\operatorname{Coef}\left(x_{t j a_{m} s}\right)=\quad \mu_{t j a_{m}}^{1}\left(b_{t j a_{m}}-t e m p B_{t s}\right)$ $+\mu_{j a_{m}}^{2} \lambda_{t} \prod_{k=1}^{s-1} \operatorname{temp}^{t k}+\mu_{t j a_{m}}^{4}$ as the coefficient of. $x_{t j a_{m} s}$. In order to minimize this subproblem, we assign the $\left|S_{t}\right|$ number of smallest $\operatorname{Coef}\left(x_{t j a_{m} s}\right)$ of $x_{t j a_{m} s}$ to equal one. That is, we use $\left|S_{t}\right|$ number of $\mu_{j a_{m}}^{2} \lambda_{t} \prod_{k=1}^{s-1} B_{t k}+\mu_{t j a_{m}}^{4}-\mu_{t j a_{m}}^{1} B_{t s}$ to substitute the responded $\left\{\begin{aligned}-\mu_{t j a_{m}}^{1}, & \text { if } \mu_{t j a_{m}}^{1} \geq 0 \\ 0, & \text { if } \mu_{t j a_{m}}^{1}<0\end{aligned}\right.$ in order to minimize the objective value of this subproblem.

Subproblem (SUB2): (related with decision
variables $\mathrm{g}_{j a_{m}}$ and $n_{j a_{m}}$ )

$$
\begin{align*}
& Z_{S U B 2}=\min \sum_{j \in C} \sum_{a_{m} \in A}\left(\Delta_{C}\left(n_{j a_{m}}\right)-\mu_{j a_{m}}^{2} g_{j a_{m}}\right. \\
& -\mu_{j a_{m}}^{6} n_{j a_{m}}+\sum_{t \in T} \mu_{t j a_{m}}^{1} d\left(n_{j a_{m}}, g_{j a_{m}}\right) \tag{SUB2}
\end{align*}
$$

subject to: (24) and

$$
0 \leq g_{j a_{m}} \leq \bar{g}_{j a_{m}}
$$

$$
\forall j \in C, a_{m} \in A .(28
$$

We add a redundant Constraint (28) to improve dual solution quality. We decompose this problem into $|C| \times|A|$ independent sub-problems. Each subproblem solves the following problem (SUB2ja $a_{m}$ ),

$$
\begin{aligned}
& Z_{S U B 2 j a_{m}}=\min \Delta_{C}\left(n_{j a_{m}}\right)-\mu_{j a_{m}}^{2} g_{j a_{m}}-\mu_{j a_{m}}^{6} n_{j a_{m}} \\
& +\sum_{t \in T} \mu_{t j a_{m}}^{1} d\left(n_{j a_{m}}, g_{j a_{m}}\right)
\end{aligned}
$$

subject to: (24) and (28).
Because decision variable $n_{j a_{m}}$ is a positive and limited integer, we can exhaustive search $n_{j a_{m}}$ from zero to $\bar{n}_{j a_{m}}$. When give a certain value of $n_{j a_{m}}, d\left(n_{j a_{m}}, g_{j a_{m}}\right)$ is a convex function of decision variable $g_{j a_{m}}$. If multiple $\mu_{t j a_{m}}^{1} \geq 0$, problem $Z_{S U B 2 j a_{m}}$ becomes a convex function. To minimize objective value, the optimal $g_{j a_{n}}$ can be found by using line search technique (e.g. golden section method). Otherwise, if multiple $\mu_{t j a_{m}}^{1}<0$, problem $Z_{\text {SUB2 ja } a_{m}}$ becomes a concave function and the optimal solution will occurs either $g_{j a_{m}}=0$ or $g_{j a_{m}}=\bar{g}_{j a_{m}}$. The upper bound $\bar{g}_{j a_{m}}$ can be determined by function $d\left(\bar{n}_{j a_{m}}, g_{j a_{m}}\right)=\bar{b}_{t j a_{m}}$ where $\bar{b}_{t a_{m}}$ is an artificial probability threshold for MT $t$ being blocked by its candidate home $j a_{m}$.

Subproblem (SUB3): (related with decision
variables $c_{j m}, k_{t j a_{m}}, p_{j a_{m}}$, and $y_{i j a_{m}}$ )

$$
\begin{align*}
& Z_{S U B 3}=\min \sum_{j \in C} \sum_{m \in M} \Delta_{m} c_{j m} \\
& -\sum_{j \in C} \sum_{a_{m} \in A} \sum_{t \in T} k_{t j a_{m}}\left(\frac{1}{\gamma}\left(\frac{p_{j a_{m}}}{2 L_{t j a_{m}}}-G_{j a_{m}}\right) \sum_{i \in F} \mu_{t j a_{m}}^{3}\right. \\
& \left.\quad+\mu_{t j a_{m}}^{4}+\mu_{t}^{5}\right) \\
& +\sum_{j \in C} \sum_{a_{m} \in A} \sum_{i \in F} y_{i j a_{m}} \sum_{t \in T}\left(\frac{\mu_{t i j a_{m}}^{3} G_{j a_{m}}}{\gamma}-\frac{p_{j a_{m}}}{L_{t j a_{m}}} \frac{\mu_{t i j a_{m}}^{3}}{2 \gamma}\right. \\
& \left.\quad+\frac{p_{j a_{m}}}{L_{t j a_{m}}} \sum_{j^{\prime} \in C-\{j j\}} \sum_{a_{a_{m}} \in A} \sum_{i^{\prime} \in F} \mu_{t i^{\prime} j^{\prime} a^{\prime}{ }_{m}}^{3} \theta\left(\left|i^{\prime}-i\right|\right)\right) \\
& +\sum_{j \in C a_{m} \in A \in F} \sum_{i \in F} y_{i j a_{m}}\left(\mu_{j a_{m}}^{6}+\mu_{(i-1) j}^{7}+\mu_{i j}^{7}+\mu_{i j a_{m}}^{8}\right) \tag{SUB3}
\end{align*}
$$

subject to: (6), (14), (15), (16), (17), (18), (20), (23),
and

$$
\begin{array}{ll}
\sum_{i \in F} y_{i j a_{m}} \leq \bar{n}_{j a_{m}} & \forall j \in C, a_{m} \in A \\
\mu_{0 j}^{7}=0 & \forall j \in C, a_{m} \in A . \text { (30) } \\
\hline
\end{array}
$$

Without loss generality, we add an additional constraint (29) to improve quality of solutions. To
aggregate decision variable $y_{i j a_{m}}$, we reformulate this subproblem by removing Constraint (22) and introducing an additional constraint (30). Therefore, we decompose this problem into $|C|$ independent subproblems (SUB3j) and exhaustive search any kind of configuration $c_{i m}$ for each BS. After a temporary configuration temp $C_{j m}$ is determined, we can decompose the remaining problem into $|A|$ subproblems (SUB3ja $a_{m}$ ) as follows.

$$
\begin{aligned}
& Z_{S U B ~ 3 j a_{m}}=\min \\
& \quad-\sum_{t \in T} k_{t j a_{m}}\left(\mu_{t j a_{m}}^{4}+\mu_{t}^{5}+\frac{1}{\gamma}\left(\frac{p_{j a_{m}}}{2 L_{t j a_{m}}}-G_{j a_{m}}\right) \sum_{i \in F} \mu_{t i j a_{m}}^{3}\right) \\
& \quad+\sum_{i \in F} y_{i j a_{m}}\left(\mu_{j a_{m}}^{6}+\mu_{(i-1) j}^{7}+\mu_{i j}^{7}+\mu_{i j a_{m}}^{8}\right) \\
& \quad+\sum_{i \in F} y_{i j a_{m}} \sum_{t \in T}\left(\frac{\mu_{t i j a_{m}}^{3} G_{j a_{m}}}{\gamma}-\frac{p_{j a_{m}}}{L_{t j a_{m}}} \frac{\mu_{t i j a_{m}}^{3}}{2 \gamma}\right. \\
& \left.\quad+\frac{p_{j a_{m}}}{L_{t j a_{m}}} \sum_{j^{\prime} \in C-\left\{j j a^{\prime}{ }_{m} \in A\right.} \sum_{i^{\prime} \in F} \mu_{t i^{\prime} j^{\prime} a_{m}^{\prime}}^{3} \theta\left(\left|i^{\prime}-i\right|\right)\right)\left(\operatorname{SUB} 3 j a_{m}\right)
\end{aligned}
$$

subject to: (6), (14), (15), (18), (20), (29) and (30).
For each Sector, we can exhaustive search candidate transmission power $p_{j a_{m}}$ from zero to $\bar{p}_{j a_{m}}$. To determine the remaining decision variables $y_{i j a_{m}}$ and $k_{t j a_{m}}$, we denote the coefficients of $k_{t j a_{m}}$ and $\quad y_{i j a_{m}} \quad$ as $\operatorname{Coef}\left(k_{t j a_{m}}\right)$ and $\operatorname{Coef}\left(y_{i j a_{m}}\right)$ respectively. Their definitions are $\operatorname{Coef}\left(k_{t j a_{m}}\right)=\mu_{t j a_{m}}^{4}$ $+\mu_{t}^{5}+\frac{1}{\gamma}\left(\frac{p_{j a_{m}}}{2 L_{i j a_{m}}}-G_{j a_{m}}\right) \sum_{i \in F} \mu_{t i j a_{m}}^{3} \quad$ and $\quad \operatorname{Coef}\left(y_{i j a_{m}}\right)=$ $+\sum_{t \in T} \frac{p_{j a_{m}}}{L_{t j a_{m}}} \sum_{j^{\prime} \in C-\{j\}} \sum_{a_{m}{ }_{m} \in A i^{\prime} \in F} \sum_{t i^{\prime} j^{\prime} a_{m}}^{3} \theta\left(\left|i^{\prime}-i\right|\right) \quad-\sum_{t \in T} \frac{p_{j a_{m}}}{L_{t j a_{n}}} \frac{\mu_{t i j a_{m}}^{3}}{2 \gamma}$ $+\sum_{t \in T} \frac{\mu_{t i j a_{m}}^{3} G_{j a_{m}}}{\gamma}+\mu_{j a_{m}}^{6}+\mu_{(i-1) j}^{7}+\mu_{i j}^{7}+\mu_{i j a_{m}}^{8}$. Therefore, we can arrange the contribution of each decision variable and minimize Subproblem (SUB3ja ${ }_{m}$ ).

Subproblem (SUB4): (related with decision variables $f_{i}$ )

$$
\begin{equation*}
Z_{S U B 4}=\min \sum_{i \in F} f_{i}\left(\Delta_{F}-\sum_{j \in C a_{m} \in A} \mu_{i j a_{m}}^{8}\right) \tag{SUB4}
\end{equation*}
$$

subject to: (12), (21), and

$$
\begin{equation*}
\underline{F} \leq \sum_{i \in F} f_{i} \leq \bar{F} \tag{31}
\end{equation*}
$$

According to experience, we intend to find the lower bound $\underline{F}$ and upper bound $\bar{F}$ of $\sum_{i \in F} f_{i}$ to improve efficiency and quality of both dual and primal solutions for this subproblem. Therefore, we enhance the effect of Constraint (12) by introducing additional Constraint (31). Upper bound $\bar{F}$ can be the smaller one between the capacity upper bound summation of every BS or the total available channels in the system. However, it is difficult to find tighter lower bound $\underline{F}$ in this subproblem. We can arrange the channels in ascending order of $\operatorname{Coef}\left(f_{i}\right)=\Delta_{F}-\sum_{j \in C} \sum_{a_{m} \in A} \mu_{i j a_{m}}^{8}$ and make decision to minimize subproblem (SUB4).

## B. The Dual Problem and the Subgradient Method

According to the weak Lagrangean duality theorem, $Z_{D}$ is a lower bound on $Z_{I P}$ for any $\mu_{t i j a_{m}}^{3}, \mu_{t j a_{m}}^{4}, \mu_{t}^{5}, \mu_{i j}^{7}, \mu_{i j a_{m}}^{8} \geq 0$. The following dual problem (D) is then constructed to calculate the tightest lower bound.

```
Dual Problem (D):
    \(Z_{D}=\)
    \(\max Z_{L R}\left(\mu_{t j a_{m}}^{1}, \mu_{j a_{m}}^{2}, \mu_{t i j a_{m}}^{3}, \mu_{t j a_{m}}^{4}, \mu_{t}^{5}, \mu_{j a_{m}}^{6}, \mu_{i j}^{7}, \mu_{i j a_{m}}^{8}\right)\)
subject to:
    \(\mu_{t i j a_{m}}^{3}, \mu_{t j a_{m}}^{4}, \mu_{t}^{5}, \mu_{i j}^{7}, \mu_{i j a_{m}}^{8} \geq 0\)
```

In this dual problem, let a ( $|C| \times\{|A| \times[T|\times(|F|+2)+|F|+2]+|F|\}+|T| \quad$ )-tuple vector be a subgradient of problem $Z_{L R}$. In iteration $k$ of the subgradient method, the vector $\pi=\left(\mu_{t j a_{m}}^{1}, \mu_{j a_{m}}^{2}, \mu_{t i j a_{m}}^{3}, \mu_{t j a_{m}}^{4}, \mu_{t}^{5}, \mu_{j a_{m}}^{6}, \mu_{i j}^{7}, \mu_{i j a_{m}}^{8}\right)$ is updated by $\pi^{k+1}=\pi^{k}+t^{k} g^{k}$ [3]. The step size $t^{k}$ is determined by $t^{k}=\delta \frac{Z_{I P}^{h}-Z_{D}\left(\pi_{k}\right)}{\left\|g^{k}\right\|^{2}}$, where $Z_{I P}^{h}$ is the primal feasible objective function value from a heuristic solution (an upper bound on $Z_{I P}$ ).

## 4. Getting Primal Feasible Solutions

When we use Lagrangean relaxation and subgradient method as our methods to solve the problem, we not only get a theoretical lower bound of primal solution but also get some hints from solving
dual problem iteratively. Owing to the complexity of the primal problem, a divide-and-conquer strategy is proposed to get the primal feasible solution. We divide this integrated wireless communication network design problem into three parts. In each subproblem, we provide some heuristics to get primal feasible solution.

## A. Heuristic 1: BS Configuration Subproblem

In this part, we refer to the results of decision variables $c_{j m}, p_{j a_{m}}$ and $k_{t j a_{m}}$ that are calculated when solving subproblem (SUB3) as our initial values to determine the BS allocation, BS sectorization type, transmission power control, and candidate homing decisions. We also develop a drop-and-add procedure to find better feasible solution. The detail of the BS configuration heuristic is described in the following.

Step 1. Direct reference the results of $c_{j m}, p_{j a_{m}}$ and $k_{t j a_{m}}$ from LR dual problem as our initial configuration type.
Step 2. Considering connectivity constraint, if all of mobiles are feasible, go to Step 6. Otherwise, divide all of sectors into five groups and apply add-procedure to find feasible solution. If any candidate sector exists, go to Step 3 to add new home sector. Otherwise, if there is any enlarge-power sector, go to Step 4 to tune power level. Otherwise, go to Step 5 to deploy new BS.
Step 3. Without modify configuration and power level, we home all of the infeasible mobiles to its candidate sector in descending order of the value $\operatorname{Coef}\left(k_{t j a_{m}}\right)$ calculated in SUB3. Then, go to Step 2 for additional process.
Step 4. In order to determine the order of enlarge-power sectors, all of the infeasible mobiles elect for their favors. Then we enlarge the power level of the maximum-vote sector to reason level and then go to Step 2.
Step 5. The deployment decision is decided by vote of the entire infeasible mobiles. We deploy the most favor BS with the maximum-votes configuration and power level. Then, go to Step 2. This new deployment decision must minimize the interference to the existing deployed sectors and maximize the number of serviced mobiles. That is trade-off between configuration selection and power control.
Step 6. Applying drop-procedure to tune all configuration of deployed sector.

## B. Heuristic 2: Sequential Homing Subproblem

In this subproblem, we determine the decision variables $x_{t j a_{m} s}, B_{t s}, b_{t j a_{m}}$, and $\mathrm{g}_{j a_{m}}$ by referring to the result of $B_{t s}$ and the order of $\operatorname{Coef}\left(x_{t j a_{m s}}\right)$ calculated by (SUB1). According to the results of candidate homes $k_{t j a_{m}}$ calculated by Heuristic 1, we describe the detail of Heuristic 2 in the following.

Step 1. Direct use the sequential call-blocking probability to calculate $\operatorname{Coef}\left(x_{t j a_{m} s}\right)$.
Step 2. For each mobile, arrange its home sectors in ascending order of $\operatorname{Coef}\left(x_{t j a_{m} s}\right)$ and then assign the homing sequence $x_{t j a_{m} s}$ to this sector. Note that this assignment decision must satisfy the sequential homing constraint.
Step 3. For each sector, select the minimum associated sequential call-blocking probability as the call-blocking probability of this sector.
Step 4. Following the traffic aggregation constraint, we aggregate the associated traffic of sequential homing mobile to become the aggregate traffic of each sector.

## C. Heuristic 3: Channel Assignment Subproblem

In this subproblem, we determine the decision variables $y_{i j a_{m}}, f_{i}$, and $n_{j a_{m}}$ by referring to the result of $y_{i j a_{m}}$ and the order of $\operatorname{Coef}\left(y_{i j a_{m}}\right)$ calculated by (SUB3). Subject to CIR, adjacent channel, and call-blocking probability constraints, we use the LR-based channel order and most-capacity-requirement-first sector order to determine the channel assignment decision.

Step 1. For each sector, calculating the value of $\operatorname{Coef}\left(y_{i j a_{m}}\right)$ as our channel assignment decision. Arrange channel order in descending order of $\operatorname{Coef}\left(y_{i j a_{m}}\right)$.
Step 2. Calculate the minimum required channels for each sector to satisfy QoS constraint. Arrange sector order in descending order of required channels.
Step 3. For the first sector, which requires the greatest channel capacities, assign the first channel that has the smallest value of coefficient $\operatorname{Coef}\left(y_{i j a_{m}}\right)$ to this sector if this channel satisfies CIR and adjacent channel constraints. Minus the required capacity of this sector by
one
Step 4. If there has no channel can be assigned and there is any sector requires more capacity, we cannot find feasible solution. Otherwise, go to Step 2.
Step 5. Calculate the assigned capacity $n_{j a_{m}}$ for each sector and gather statistics for total used channel $f_{i}$ in order to calculate license fee.

## 5. Computational Experiments

Owing to the complexity of this problem, we cannot find tighter lower bound by solving dual problem. In order to prove that our LR-based heuristics are good enough, we also implement a primal algorithm to compare with our heuristics.

## A. Primal Algorithm

In previous chapter, we use some LR-based heuristics to determine (1) BS configuration subproblem, (2) sequential homing subproblem, and (3) channel assignment subproblem. Contrarily, we use an intuitive thought to determine them in this primal algorithm. We adopt election policy to determine best BS configuration and sequential homing decisions. For convenience, we denote this algorithm as $P A$ and describe its detail in the following.

Step 1. Initial transmission power of all sectors to maximum level.
Step 2. To minimize the coverage intersection with existing BSs, we reduce the power level if any mobile, which must not homed to this sector, locates in the coverage of this sector.
Step 3. We adopt election policy to determine the configuration of each BS. Each mobile votes its candidate home and favor configuration. Then, we deploy the maximum votes BS with the most popular configuration.
Step 4. We simply adopt the BS deployment order as the homing sequence of each mobile. For each sector, aggregate traffic and calculate the corresponding call-blocking probability.
Step 5. Arrange all existing sectors by most-capacity-requirement-first order. Confirm each channel's feasibility by checking the CIR and adjacent channel constraints for each sector. Then, assign required number of feasible channels to each sector.

## B. Lagrangean Relaxation Based Algorithm

When solving Lagrangean relaxation problem, we provide an iterative LR-based algorithm to get
primal feasible solutions. In this algorithm, we allow any kind of multiple-sectorization antennas can be deployed as our last solution of generic network design problems. In each iteration, we apply Heuristic 1, 2, and 3 to solve each subproblem and then find a feasible solution. For convenience, we denote it as $L R$.

## C. Experiment Results

In our computational experiments, we random generate several system scenarios with different (1) number of candidate BSs, (2) maximum connectivity requirements and (3) sectorization configurations. We random generate 20 BSs and 10 MTs as our test network. For comparison purpose, we group the experiment results and list in Table 3. To analyze the effect of sectorization, we explore different number of sectors in each BS from omni-direction to three sectors with the radian unit is 450 . We also explore the effect of multiple-connectivity on the total cost of cellular systems. In Table 3, we can find that as the connectivity requirement growth, the required number of deployed BSs also growth. As the allowed sector number growth, the required channel of cellular systems is smaller. The multiple-connectivity can improve communication reliability but will spend more deployment cost.

## 6. Conclusion

The proposed algorithm is the first attempt to consider the network design problem with whole factors jointly and formulate it rigorously. Due to the time variance and unstable properties of wireless communications, the proposed algorithm is helpful to design high-reliability wireless communication networks. In this paper, we identify reliability issue of channelized wireless communication networks by introducing customized multiple-connectivity effect.

The proposed algorithm not only designs a multipleconnectivity network but also guides to route MT among its candidate homes sequentially. We formulate a combinatorial optimization algorithm to deal with this problem. Because this problem is NP-complete, the solution approach we adopt is Lagrangean relaxation. In the computational experiments, we compared the proposed algorithm with the power dominant heuristic on test networks. The proposed algorithm can achieve up to $35.51 \%$ improvement of the total cost of network design problems.

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Table 3. Experiment results for analysis the effects of sectorization and connectivity

| $\|C\|$ | $\|T\|$ | $M$ | $K_{t}$ | $\lambda_{t}$ | Lower bound | Gap | $P A$ | $L R$ | Improve | \#BSs | \#Chs |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 10 | 1 | 1 | 3 | $1.6535 \mathrm{e}+007$ | $57.25 \%$ | $2.7600 \mathrm{e}+007$ | $2.6000 \mathrm{e}+007$ | $6.15 \%$ | 3 | 42 |
| 10 | 10 | 1 | 3 | 3 | $3.8268 \mathrm{e}+007$ | $43.20 \%$ | $6.3400 \mathrm{e}+007$ | $5.4800 \mathrm{e}+007$ | $15.69 \%$ | 9 | 36 |
| 10 | 10 | 3 | 1 | 3 | $1.6935 \mathrm{e}+007$ | $51.17 \%$ | $2.7600 \mathrm{e}+007$ | $2.5600 \mathrm{e}+007$ | $7.81 \%$ | 3 | 40 |
| 10 | 10 | 3 | 3 | 3 | $3.3600 \mathrm{e}+007$ | $41.67 \%$ | $6.2000 \mathrm{e}+007$ | $4.7600 \mathrm{e}+007$ | $30.25 \%$ | 7 | 48 |
| 20 | 10 | 1 | 1 | 3 | $1.5382 \mathrm{e}+007$ | $40.42 \%$ | $2.8400 \mathrm{e}+007$ | $2.1600 \mathrm{e}+007$ | $31.48 \%$ | 2 | 46 |
| 20 | 10 | 1 | 3 | 3 | $4.9083 \mathrm{e}+007$ | $1.87 \%$ | $5.6400 \mathrm{e}+007$ | $5.0000 \mathrm{e}+007$ | $12.80 \%$ | 8 | 38 |
| 20 | 10 | 3 | 1 | 3 | $1.5638 \mathrm{e}+007$ | $36.85 \%$ | $2.9000 \mathrm{e}+007$ | $2.1400 \mathrm{e}+007$ | $35.51 \%$ | 2 | 45 |
| 20 | 10 | 3 | 3 | 3 | $3.3769 \mathrm{e}+007$ | $47.47 \%$ | $4.9800 \mathrm{e}+007$ | $4.1600 \mathrm{e}+007$ | $16.47 \%$ | 6 | 41 |

