PERFORMANCE ANALYSIS OF ADMISSION CONTROL ALGORITHMS FOR CDMA NETWORKS

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This paper proposes a modified mathematical formulation of revenue optimization problem in terms of CDMA admission control. The problem solution is based upon proposed algorithm as well as Lagrangean relaxation approach. Performance analysis on three algorithms with respect to voice activity factor (VAF) is considered, where 9 base stations, 50 existing mobile stations, and new mobile stations in Poisson arrival process (λ=100) on 500 test cases are given. Computational results illustrate that no matter which value of VAF is given; proposed algorithm always is with an outstanding performance on solution optimality, blocking rate, as well as revenue contribution.

1. Introduction

Generally, CDMA (code division multiple access) system provides no upper limit of available channels, but it is an interference-based channel assignment. For example, in the reverse-link, received signal-to-interference ratio (SIR) at the base station affects the connection quality. Interferences base station incurred must be lower than pre-defined acceptable interference threshold to ensuring communication quality of service (QoS) [4] [6] [7]. Thus, to preserve the whole system QoS, a number of interferences sources, including existing connections and other interferences propagated from cells, must be taken into account. Besides, mobility of new, handover, and outbound calls should be effectively managed [8]. To manage system capacity, call admission control (CAC) is a prevalent mechanism to allocating channel resources. The more users are admitted, the more revenue is contributed. However, less research discusses revenue optimization in terms of admission control [5] [9]. Even though previous research [9] has been illustrated the revenue analysis in terms of admission control, but no consideration of homing blocked mobile stations into another base station. It would be a poor performance. This paper not only modifies the formulation in [9], but also proposes a solution algorithm with dropping and homing mechanisms to enhance total system revenue. In order to proof the performance of proposed algorithm, analyses including solution optimality, blocking rate, revenue contribution are presented. The remainder of this paper is organized as follows. In Section 2, a modified mathematical formulation is expressed. Section 3 applies Lagrangean relaxation as a solution approach. Besides, the algorithm is also developed here. Section 4 illustrates the computational experiments and related analyses. Finally, Section 5 concludes this paper.

2. Admission Control Problem

In previous research [9], for simplicity of modeling admission control problem, only new mobile stations are considered. Besides, new mobile stations in previous model can either be homed to the controlling base station or blocked. However, to optimally contribute system revenue, the mechanism that will home blocked mobile stations to another base station is taken into account. Accordingly, we modify the previous formulation as problem (IP).

$$Z_{IP} = \max \sum_{t \in T''} a_t \sum_{j \in B} z_{jt} = \min(-\sum_{t \in T''} a_t \sum_{j \in B} z_{jt}) \qquad \text{(IP)}$$

subject to:

$$\left(\frac{E_b}{N_{total}}\right)_{req} \le \frac{\frac{S}{N_0}}{1 + \frac{1}{G}\alpha\frac{S}{N_0}\left(\sum_{t \in T'}\delta_{jt} + \sum_{t \in T''} z_{jt} - 1\right) + \frac{1}{G}\alpha\frac{S}{N_0}\sum_{\substack{j' \in B \\ j' \ne j}}\left(\sum_{t \in T'}\left(\frac{D_{j't}}{D_{jt}}\right)^{\tau}\delta_{j't} + \sum_{t \in T''}\left(\frac{D_{j't}}{D_{jt}}\right)^{\tau} z_{j't}\right)} \qquad \forall j \in B \quad (1)$$

$$\sum_{t \in T'} \delta_{jt} + \sum_{t \in T''} z_{jt} \le M_j \qquad \forall j \in B \quad (2)$$

$$D_{jt} z_{jt} \le R_j \mu_{jt} \qquad \forall j \in B, \forall t \in T'' \quad (3)$$

$$z_{jt} \le \mu_{jt} \qquad \forall j \in B, \forall t \in T'' \quad (4)$$

$$\sum_{j \in \{b', b_t\}} z_{jt} = 1 \qquad \forall t \in T'' \quad (5)$$

$$z_{jt} = 0 \text{ or } 1 \qquad \forall j \in B', \forall t \in T'' \quad (6)$$

$$\delta_{jt} = 0 \text{ or } 1 \qquad \forall j \in B, \forall t \in T' \quad (7)$$

The objective function is to minimize the total revenue loss in the process of admitting new mobile stations. A lot of constraints are the same in [9] except additional constraint (3) and (4) are added. Constraint (1) requires that every one mobile station is served with its homing base station in the required QoS. The left hand side of (1) is the threshold of acceptable SIR for each connection. The right hand side means the real SIR. The denominator of the right hand side is the total interference value, including white noise, the intra-cell interference as well as inter-cell interference. Constraint (2) is to ensure that the number of users who can be active at the same time in a base station would not exceed the base station's upper bound. Constraint (3) ensures that a base station can only serve the mobile stations inside its coverage of effective transmission power radius, where R_j is upper bound of power transmission

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Table 1. Description of Notations

Notation	Description
B	the set of candidate locations for base stations
b'	the artificial base station to carry the rejected call when admission control function decides to reject the call
B'	the set of $B \cup \{b'\}$
b_t	the controlling base station of mobile station t
T	the set of mobile stations
T'	the set of existing mobile stations
T''	the set of new mobile stations whose admittance into the cell is to be determined
G	the processing gain
S	the power that a base station received from a mobile station that is homed to the base station with perfect power control
E_b	the energy that BS received
N_{total}	total noise
N_0	the background noise
α	voice activity factor
τ	attenuation factor
D_{jt}	distance between base station j and mobile station t
M_j	upper bound on the number of users that can active at the same time in base station j
μ_{jt}	indicator function which is 1 if mobile station t can be served by base station j and 0 otherwise
a_t	the revenue from admitting mobile station $t \in T''$ into the system
R_j	upper bound of power transmission radius of base station j
δ_{jt}	indicator function which is 1 if mobile station t is homed to base station j and 0 otherwise
z_{jt}	decision variable which is 1 if mobile station t is served by base station j and 0 otherwise

radius of base station j. Constraint (4) guarantees that if a base station does not provide service to a mobile station, then the decision variable z_{jt} must be equal to 0. Constraint (5) ensures that each mobile station can be homed to only one physical base station or rejected. Constraint (6) and (7) guarantee the integer property of decision variables and indicator functions. Notations used to modeling the problem are described in Table 1.

3. Solution Procedure

3.1. *Lagrangean Relaxation Approach*

The solution approach applied to solving admission control problem is Lagrangean relaxation that is originally designed to solve large-scale linear as well as integer programming problems [1] [2]. Modified revenue optimization problem (IP) is transformed into the following Lagrangean relaxation problem (LR) where Constraints (1) (2) (3) are relaxed.

$$
\begin{aligned}
Z_D\left(v_j^1, v_j^2, v_{jt}^3\right) = \min - \sum_{t \in T''} a_t \sum_{j \in B} z_{jt} \\
+\sum_{j \in B} v_j^1 \left((\frac{E_b}{N_{total}})_{req} + (\frac{E_b}{N_{total}})_{req} \frac{1}{G} \alpha \frac{S}{N_0} \left(\left(\sum_{t \in T'} \delta_{jt} + \sum_{t \in T''} z_{jt} - 1 \right) + \sum_{\substack{j' \in B \\ j' \neq j}} \left(\sum_{t \in T'} \left(\frac{D_{j't}}{D_{jt}} \right)^{\tau} \delta_{j't} + \sum_{t \in T''} \left(\frac{D_{j't}}{D_{jt}} \right)^{\tau} z_{j't} \right) \right) - \frac{S}{N_0} \right) \\
+ \sum_{j \in B} v_j^2 \left(\sum_{t \in T'} \delta_{jt} + \sum_{t \in T''} z_{jt} - M_j \right) + \sum_{t \in T''} \sum_{j \in B} v_{jt}^3 \left(D_{jt} z_{jt} - R_j \mu_{jt} \right)
\end{aligned}
\quad \text{(LR)}
$$

subject to: (4) (5) (6) (7).

Lagrangean relaxation problem can further be decomposed into a lot of independent subproblems that could be optimally solved with respect to decision variables. To getting primal optimal solutions, we should iteratively adjust Lagrangean multipliers by subgradient method to optimally solve Lagrangean dual problem. Here, we express (LR) into subproblem 1 related to decision variables z_{jt}.

Subproblem 1: for z_{jt}

$$
\begin{aligned}
= \sum_{t \in T''} \left(\sum_{j \in B} \left(z_{jt} \left(-a_t + (\frac{E_b}{N_{total}})_{req} \frac{1}{G} \alpha \frac{S}{N_0} \left(v_j^1 + \sum_{\substack{j' \in B \\ j' \neq j}} v_{j'}^1 (\frac{D_{jt}}{D_{j't}})^{\tau} \right) + v_j^2 + v_{jt}^3 D_{jt} \right) - v_{jt}^3 R_j \mu_{jt} \right) \right) \\
+ \sum_{t \in T'} \left(\sum_{j \in B} \delta_{jt} \left((\frac{E_b}{N_{total}})_{req} \frac{1}{G} \alpha \frac{S}{N_0} \left(v_j^1 + \sum_{\substack{j' \in B \\ j' \neq j}} v_j^1 (\frac{D_{jt}}{D_{j't}})^{\tau} \right) + v_j^2 \right) \right) \\
+ \sum_{j \in B} \left(v_j^1 \left((\frac{E_b}{N_{total}})_{req} - \frac{S}{N_0} - (\frac{E_b}{N_{total}})_{req} \frac{1}{G} \alpha \frac{S}{N_0} \right) - v_j^2 M_j \right)
\end{aligned}
\quad \text{(SUB)}
$$

subject to: (4) (5) (6) (7).

In (SUB), indication function δ_{jt} and decision variable z_{jt} are used to track existing and new mobile stations, respectively. From which, δ_{jt} just indicates the homing status of existing mobile stations since existing ones would not be blocked at all. The second and the third term of (SUB) are constant, because of all variables are constant that can be calculated. Finally, the first term of (SUB) is what we intent to treat it. Let q_{jt} as follows, the first term of (SUB) can be decomposed into |T"| sub-problems for treatment of new mobile stations whether to be admitted or not in terms of revenue optimization. If q_{jt} is less than 0, assign z_{jt} to 1 or 0 otherwise.

$$
q_{jt} = -a_t + (\frac{E_b}{N_{total}})_{req} \frac{1}{G} \alpha \frac{S}{N_0} \left(v_j^1 + \sum_{\substack{j' \in B \\ j' \neq j}} v_j^1 (\frac{D_{jt}}{D_{j't}})^{\tau} \right) + v_j^2 + v_{jt}^3 D_{jt}
$$

According to the weak Lagrangean duality theorem [1], for any $(v_j^1, v_j^2, v_{jt}^3) \geq 0$, the objective value of $Z_D\left(v_j^1, v_j^2, v_{jt}^3 \right)$ is a lower bound of Z_{IP}. Based on problem (LR), the following dual problem (D) is constructed to calculate the tightest lower bound.

$$
Z_D = \max Z_D\left(v_j^1, v_j^2, v_{jt}^3 \right) \quad \text{(D)}
$$

subject to: $(v_j^1, v_j^2, v_{jt}^3) \geq 0$.

Then, subgradient method [3] is applied to solving the dual problem. Let the vector S is a subgradient of $Z_D\left(v_j^1, v_j^2, v_{jt}^3 \right)$ at (v_j^1, v_j^2, v_{jt}^3). In iteration k of subgradient optimization procedure, the multiplier vector π is updated by $\pi^{k+1} = \pi^k + t^k S^k$, where t^k is a step size determined by $t^k = \delta\left(Z_{IP}^* - Z_D(\pi^k)\right) / \|S^k\|^2$, where Z_{IP}^* is an upper bound on the primal objective function value after iteration k, and δ is a constant where $0 \le \delta \le 2$.

3.2. *Getting Primal Feasible Solutions*

Based on decision variables solved in Lagrangean dual problem (D), we must adjust them for getting primal feasible solutions of problem (IP). Actually, the primal feasible solution is an upper bound (UB) of (IP) while the solution of Lagrangean dual problem guarantees the lower bound (LB) of (IP). Iteratively, both solving Lagrangean dual problem and getting primal feasible solution, we get the LB and UB, respectively. The gap between UB and LB, expressed by (UB-LB)/LB*100%, illustrates the optimality of problem [1] [2]. In getting primal feasible solutions, there are two mechanisms, say call dropping and call homing, described as follows. 1) Call dropping mechanism (*CDM*): No surprisingly, some of new users would be dropped due to capacity and QoS constraints. However, selection of dropped users has something to do with the homing mechanism. If system properly chooses a new mobile station to be dropped, then it can be homed (added back) into another base station (system) later. For each mobile station, the more number that mobile station is covered by base stations, the more possibility that it can be homed into another base station in the homing stage. This implies total system revenue is enhanced. In this paper, we propose *CDM* with number it is covered by base stations (*CDM-NCBS*), from which the system will pick up the mobile station in the descending number that it is covered by base stations; 2) Call homing mechanism (*CHM*): the system tries to home each dropped new mobile station to another/candidate base station. Selection of candidate base station is also a key issue. Here we propose *CHM* with randomly selected candidate (*CHM-RSC*). Combining to *CDM-NCBS* and *CHM-RSC*, an algorithm, denoted algorithm AA, shown in the following is our solution for problem (IP).

[Algorithm AA]

Step 1. Check capacity constraint (2), for each base station $j \in B$, based upon decision variables z_{jt} solved in Lagrangean dual problem (D). Drop the new mobile station, i.e. set z_{jt} =0, with *CDM-NCBS*, if violates the constraint (2), or go to Step 2 otherwise.

Step 2. Make sure QoS constraint (1) for each base station *j* is satisfied. Drop the new mobile station, i.e. set z_{jt} =0, with *CDM-NCBS*, if violates the constraint (1), or go to Step 3 otherwise.

Step 3. Try re-adding back all dropped new mobile stations in Step 1 & 2 into system.

3-1) sequentially picks up a dropped new mobile station.

3-2) home to another base station based upon *CHM-RSC*, i.e. set z_{jt}=1 again, if this setting satisfies constraint (1) as well as capacity constraint (2) for each base station, or go to Step 4 otherwise.

Step 4. End algorithm AA.

4. Computational Experiments

4.1. *Other Primal Heuristics*

As mentioned in section 3.2, selection of candidate base station is a key issue. For the purpose of comparison with algorithm AA, other primal heuristic based upon another homing mechanism, denoted algorithm AB, is proposed that is *CHM* with rank of interference incurred at base station (*CHM-RII*). For *CHM-RII*, let SIR_j equals to the right side of constraint (1), based upon SIR_j in terms of ascending interference loading, candidate is selected. Algorithm AB is following up *CDM-NCBS* with *CHM-RII*, the system will re-add dropped mobile stations into candidate base station, i.e. set z_{jt} =1, if it is not violate constraints. Additional heuristic, denoted algorithm AC, applied in [9] is also implemented for performance analysis.

4.2. *Experiments Scenario*

A few of constants used in the formulation, including S/N_0, E_b/N_{total}, M_j, τ, G, and a_t are the same as Table II in [9]. Number of base stations (|B|), existing mobile stations (|T'|) are given to 9 and 50, respectively. Number of new mobile stations is generated in Poisson arrival process with λ=100. More generically, all locations of base stations, existing as well as new mobile stations are randomized, even thought a few of number of new mobile stations generated in Poisson process may be the same. For the purpose of statistic analysis, 500 test cases with Poisson arrival are generated. Three algorithms, say AA, AB, and AC, conjunction with voice activity factor (VAF) are analyzed.

4.3. *Performance Analysis*

4.3.1. Optimality of solution

Table 2 summaries a statistics of error gaps on best and average cases for three algorithms. Because of the problem is with a strong integrality property, no matter which algorithm applied, the more traffic is penetrated in the system (heavy VAF), the loose gaps the solutions are incurred. Even thought proposed algorithm AA is with an average gap up to 14.57%, it still is a best solution among three algorithms in all four VAF cases.

4.3.2. Blocking rate

Based on the solution the total call blocking rate is also analyzed, which is defined by ratio of blocked users (new mobile stations only) to total users (including existing and new mobile stations) in the system. Figure 1 illustrates the experiment results on blocking rate with respect to four VAF values. Inevitably, the bigger VAF is applied, the more blocking rate is incurred. Proposed algorithm AA is with an outstanding performance.

4.3.3. Revenue contribution

Table 3 summaries the aggregate revenue of problem (IP) on 500 test cases. Besides, improvement on algorithm AC is also shown. Proposed algorithm AA always contributes better revenue. This result is consistent with our assumption that CHM is an important issue for revenue optimization in terms of admission control policy. Another interesting finding is that the revenue improvement is monotonically increasing from 0.55% to 3.60% and from 0.49% to 3.13% for algorithm AA and AB, respectively, when VAF is increasing from 0.3 to 0.375. Especially, the more VAF is applied, the more improvement is calculated.

5. Conclusions

This paper analyzes the performance of three admission control algorithms for CDMA networks. First of all, a modified admission control problem is formulated as a mathematical revenue optimization problem and solution algorithm is proposed. The solution algorithm is based upon call dropping mechanism (*CDM*) and call homing mechanism (*CHM*). We propose an algorithm integrating *CDM* with number it is covered by base stations (*CDM-NCBS*) and *CHM* with randomly selected candidate (*CHM-RSC*) to handle mobile users. For proving the effectiveness of the proposed algorithm, other primal heuristic and previous algorithm in literature is implemented. Three algorithms for solving admission control problem are jointly considered. The analysis of experiments include optimality of solution, blocking rate, and total system revenue with respective to voice activity factor (VAF). Computational results illustrate that no matter which value of VAF is given, proposed algorithm AA always is with an outstanding performance on solution optimality, blocking rate, as well as revenue contribution.

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Table 2. Summary of error gaps percentage (%) for three algorithms based on 500 test cases of new mobile users with Poisson arrival process (λ=100) with respect to VAF.

VAF	0.3			0.325		
Algorithm	AA	AB	AC	AA	AB	AC
Best	0.00	0.00	0.00	0.00	0.00	0.00
Average	1.60	1.65	2.12	4.27	4.43	5.48
VAF	0.35			0.375		
Algorithm	AA	AB	AC	AA	AB	AC
Best	0.00	0.00	0.00	0.00	0.00	0.00
Average	8.50	8.78	10.52	14.57	14.98	17.59

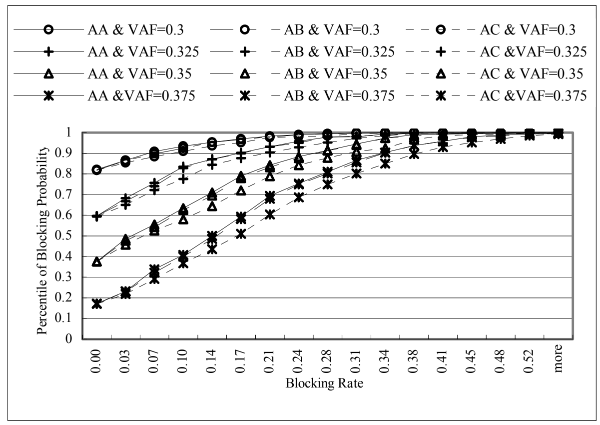

Figure 1. Percentile of blocking rate for three algorithms based on 500 test cases of new mobile users with Poisson arrival process (λ=100) with respect to VAF.

Table 3. Revenue aggregation and improvement on 500 test cases of new mobile users with Poisson arrival process (λ=100) with respect to VAF.

VAF	0.3		
Algorithm	AA	AB	AC
Revenue Aggregation	490960	490680	488270
Revenue Improvement on AC	0.55 %	0.49 %	N/A
VAF	0.325		
Algorithm	AA	AB	AC
Aggregate Revenue	476370	475560	470220
Improvement on AC	1.31 %	1.14 %	N/A
VAF	0.35		
Algorithm	AA	AB	AC
Aggregate Revenue	453580	452170	443640
Improvement on AC	2.24 %	1.92 %	N/A
VAF	0.375		
Algorithm	AA	AB	AC
Aggregate Revenue	421780	419830	407080
Improvement on AC	3.61 %	3.13 %	N/A