# Modeling and Performance Evaluation of Survivable CDMA Networks

Kuo-Chung Chu* and Frank Yeong-Sung Lin
*d5725003@im.ntu.edu.tw
Department of Information Management
National Taiwan University
Taipei, Taiwan 106, ROC

## Abstract

In this paper, we consider the network survivability in CDMA networks, from which some of base stations may be broken. A survivable network is modeled as mathematical optimization problem to enhance quality of service (QoS) in terms of BS recovery. The objective is to minimize the total blocking traffic of the network after partial broken base stations are fixed. The solution approach is Lagrangean relaxation in conjunction with an algorithm development. We analyze the performance on call blocking rate, solution gap, service rate, as well as CPU time with respect to base station recovery ratio (BSRR) and threshold of call blocking probability for each base station. We evaluate the algorithm for 3 combinations of base stations/mobile users– 9/500, 16/1000, 25 /1500. The experiment results illustrate, 1) call blocking rate is much less affected by call blocking probability in light loading than heavy loading; 2) All gaps in 9/500 combination are less than 0.12%, it is calculated with near-optimal solution; 3) BS recovery is much more important in light loading than in heavy loading; 4) time consumed in 25/1500 case is up to 30 minutes.

**Keywords:** Base Station Recovery, Lagrangean Relaxation, Mathematical Modeling, Network Survivability, Performance Evaluation, Quality of Service.

## 1. Introduction

Wireless communications have been highly improved and broadly applied in human life. CDMA seems to be the most popular standard of third generation (3G) broadband wireless communications networks [1][2] For the services of 3G mobile environment, there are a variety of requirements, e.g. multimedia data services, higher data rate, mobility as well as quality of service (QoS). To providing service quality, most of previous researches discuss the issue of multimedia traffics in terms of call admission control [3][4]. Generally, network systems manage the available resources and allocate them in an optimal way among the system users. For example, in the reverse link, signal to interference ratio (SIR) received at the base station impacts the QoS of the connection. To maintain the whole system QoS, it needs to consider the interferences on neighboring cells when admitting a new call in a cell. Besides, survivable service is another important issue in wireless communications networks [5]. To provisioning reliable and survivable wireless and mobile services, network providers must consider ways to decrease the number of network failures and to cope with failures when they do occur. Actually, reliability, availability, and survivability have long been important research areas for wire-line networks, such as the public switched telephone networks (PSTN) and asynchronous transfer mode (ATM) networks.

A typical infrastructure of cellular network consists of a number of components, including base stations (BS), base station controllers (BSC), mobile switching centers (MSC), home location registers (HLR) and visiting location registers (VLR), signaling system 7 (SS7), high-capacity trunks, etc [6][7]. A failure in a MSC, a HLR/VLR, an MSC-PSTN link, an SS7, or a PSTN trunk affects nearly all customers under a mobile switching center—perhaps hundreds of thousands of people. Failure in other components would be less severe but still significant. A base station of wireless communications networks is a core station to assign channel for each mobile user [8]. Probably, the natural disasters, e.g. floods, earthquakes, or some human factors, often cause the BS broken. Thus, partial users may be out of service such that overall QoS must be degraded. In terms of operating, if we can properly recover some of broken BS, it will enhance QoS and provide survivable service. Accordingly, BS recovery is one of the most important approaches to minimize total system call blocking rate. To the best of our knowledge, although intensive research on comparison of FH-CDMA and DS-CDMA for wireless survivable networks has been conducted [9], relatively little work has been attempted to attack the overall call blocking problem in conjunction with BS recovery.

In this paper, we focus on BS recovery to modeling survivable networks as a combinatorial optimization problem in terms of call blocking control. The objective is to minimize the overall system blocked traffic after allocating some of restricted resources to recover broken base stations. The remainder of this paper is organized as follows. In Section 2, a mathematical problem formulation of survivable networks is proposed. Section 3 presents a solution approach to the problem based on

* Corresponding author, e-mail: d5725003@im.ntu.edu.tw, who is also a lecturer of the Department of Information Management, Jin-Wen Institute of Technology, Taipei, Taiwan.

Lagrangean relaxation. In Section 4, heuristic is developed to calculate good primal feasible solutions. Section 5 illustrates the computational experiments. Finally, Section 6 concludes this paper.

## 2. Problem Formulation

We suppose that there are $|B|$ base stations in the system, a number of base stations, say $|F|$, are failed due to an emergency. To provisioning service quality in terms of minimizing total blocked traffic, some of them, say $|F'|$, can be fixed by allocating restricted resources and timing considerations. Accordingly, the system that available base stations cooperate with fixed base stations could provide survivable services in overall minimal traffic blocking. Here, we ignore the consideration of existing connections. Besides, a number of assumptions including perfect power control, reverse link perfectly separated from the forward link, fading and forward link are not considered. In terms of new call blocking analysis, some complicated scenarios like new, re-homing, outbound, handover calls are not dealt with. Notations used to modeling the problem are listed in the Table I.

For each base station, Erlangs-B formula is applied to modeling call blocking process. The survivable problem is formulated as a following mathematical optimization problem (IP) that the objective is to minimize the total call blocking rate of overall system. Even though the call blocking probability, $B_j$ of Erlangs-B formula, is nondifferentiable with respect to $c_j$ and nonconvex with respect to $g_j$, but the call blocking rate, i.e. $g_jB_j$ , is a convex function of $g_j$ rather than $B_j$ alone [10].

$$Z_{IP} = Min \sum_{j \in B''} g_j B_j(g_j, c_j) + g' \quad \text{(IP)}$$

subject to:

$$(\frac{E_b}{N_{total}})_{req} \le \frac{\frac{S}{N_0}}{(1+\frac{1}{G}\alpha\frac{S}{N_0}(c_j-1)+\frac{1}{G}\alpha\frac{S}{N_0}\sum_{\substack{j' \in B \\ j' \neq j}}\left(\frac{\frac{r_{j'}}{2}}{Max(D_{jj'}-\frac{r_{j'}}{2},\varpi)}\right)^{\tau} c_{j'})} \quad \forall j \in B'' \quad (1)$$

$$D_{jt} z_{jt} \le r_j \delta_{jt} \quad \forall j \in B'', \forall t \in T \quad (2)$$

$$\sum_{t \in T} \lambda_t z_{jt} = g_j \quad \forall j \in B'' \quad (3)$$

$$\sum_{j \in B''} z_{jt} \le 1 \quad \forall t \in T \quad (4)$$

$$c_j \le M_j \quad \forall j \in B'' \quad (5)$$

$$c_j \in Z^+ \quad \forall j \in B'' \quad (6)$$

$$0 \le r_j \le R_j \quad \forall r_j \in Y_j, j \in B'' \quad (7)$$

$$B_j(g_j, c_j) \le \beta_j \quad \forall j \in B'' \quad (8)$$

Table 1 Description of notation

| Notation | Description |
|---|---|
| $B$ | The set of base stations |
| $F$ | The set of broken base stations, F⊂ B |
| $F'$ | The set of fixed base stations, F'⊂ F |
| $B'$ | The set of available base stations, B'=B-F |
| $B''$ | The set of workable base stations, B"=B'∪F' |
| $T$ | The set of mobile stations |
| $B_j$ | Call blocking probability in base station j, j∈B |
| $M_j$ | Upper bound on the number of users that can active at the same time at base station j, j∈B |
| $R_j$ | Upper bound on the transmission power radius of base station j, j∈B |
| $\beta_j$ | Threshold of call blocking probability foe each base station j, j∈B |
| $c_j$ | The number of users who can be active at the same time in the base station j, j∈B |
| $r_j$ | Transmission power radius of base station j, j∈B |
| $Yj$ | The set of transmission radius of base station j |
| $g_j$ | Aggregate flow on base station j, j∈B" |
| $g'$ | Aggregate flow of mobile stations not served by B", where $g'=\sum_{t\in T}\lambda_t \sum_{j\in B''}(1- z_{jt})$ |
| $S$ | The power that a base station received from a mobile station that is homed to the base station with perfect power control |
| $E_b$ | The energy that BS received |
| $N_{Total}$ | Total noise |
| $\varpi$ | A small number |
| $G$ | The processing gain |
| $N_0$ | The background noise |
| $\alpha$ | Voice activity factor |
| $\tau$ | Attenuation factor |
| $\lambda_t$ | The traffic requirement of mobile station t (in Erlangs), t∈T |
| $D_{jt}$ | Distance between base station *j* and mobile terminal *t* |
| $D_{jj}$ | Distance between base station j and j' |
| $\delta_{jt}$ | Coverage indicator function which is 1 if mobile station t can be served by base station j and 0 otherwise, j∈B |
| $z_{jt}$ | Granting decision variable which is 1 if mobile station t is serviced by base station j and 0 otherwise, j∈B |

$$z_{jt} = 0 \text{ or } 1 \quad \forall j \in B'', \forall t \in T \quad (9)$$

The objective function is to minimize the total blocking rate, the sum of average blocked and lost traffics, which is subject to a number of constrains. First of all, Constraint (1) ensures that each traffic demand is served with base station in the required QoS. For more generic, we do not consider the multi-user detection in this model. Constraint (2) requires that a mobile station would be in the service (coverage) area of a base station before being served by that base station. Constraint (3) checks aggregate flow of each base station, j∈B", which is based upon all granting mobile stations. Constraint (4) guarantees that each mobile station can be homed to no more than one base station, where $z_{jt}$ =0 in case of mobile station is not in the service area of a base station j∈B". Constraint (5) and (6) are to ensure that the number of users who can be active at the same time in a base station be no greater than upper bound $M_j$ and nonnegative integer, respectively. Constraint (7) is to ensure that transmission power radius of each base station j∈B" be between 0 and $R_j$. Constraints (8) requires that any base station can serve its slave mobile station under certain (system defined) call blocking probability. Constraint (9) is to enforce the integer property of the decision variables.

## 3. Solution Approach

### 3.1 Overall Procedure

To simplify the treatment of solution procedure, it is convenient to introduce an overall procedure as follows,

Step 1.Specify the number of base station |*F'*| that will be fixed, from which the specific number of combinations to fix base station can be generated, say *C*(|*F*|, |*F'*|).

Step 2.Sequentially fix base stations elected from each combination identified in Step 1.

Step 3.Solve problem (*IP*) that is based upon all workable base stations (∀j∈B") by Lagrangean relaxation approach (in Section III-B) to getting optimal value of the problem in each combination; Compare each optimal value to keeping overall optimal value among combinations; If end of combination, go to step 2, or step 4 otherwise.

Step 4. End solution procedure.

### 3.2 Lagrangean Relaxation

The approach to solving the problem (IP) is Lagrangean relaxation [11], which including the procedures that relax complicating constraints, multiple the relaxed constraints by corresponding Lagrangean multipliers, and add them to the primal objective function. Based on above procedures, we transform the primal optimization problem (IP) into the following Lagrangean relaxation problem (LR) where Constraints (1)(2)(3) are relaxed. Furthermore, LR can be decomposed into two independent subproblems.

$$Z_D(\mu_j^1,\mu_{jt}^2,\mu_j^3)=\min\sum_{j\in B''} g_jB_j(g_j,c_j)+\sum_{t\in T}\lambda_t(1-\sum_{j\in B''} z_{jt})+$$
$$\sum_{j\in B}\mu_j^1\left( (\frac{E_b}{N_{total}})_{req}+(\frac{E_b}{N_{total}})_{req}\frac{1}{G}\alpha\frac{S}{N_0}\right.$$
$$\left.\left((c_j-1)+\sum_{\substack{j'\in B''\\ j'\neq j}}(\frac{\frac{r_{j'}}{2}}{Max(D_{jj'}-\frac{r_{j'}}{2},\varpi)})^\tau c_{j'}\right)-\frac{S}{N_0}\right)$$
$$+\sum_{j\in B''}\sum_{t\in T}\mu_{jt}^2(D_{jt}z_{jt}-r_j\delta_{jt})+\sum_{j\in B''}\mu_j^3(\sum_{t\in T}\lambda_t z_{jt}-g_j) \qquad \text{(LR)}$$

subject to: (4)-(9).

Here, we decompose (LR) into two independent subproblem 1 and subproblem 2 related to decision variables $c_j$, $r_j$, $g_j$, and $z_{jt}$, respectively.

**Subproblem 1**: for $c_j$, $r_j$, and $g_j$

$$Z_{SUB1}=\min\sum_{j\in B''} g_jB_j(g_j,c_j)+$$
$$\sum_{j\in B''}\mu_j^1\left( (\frac{E_b}{N_{total}})_{req}+(\frac{E_b}{N_{total}})_{req}\frac{1}{G}\alpha\frac{S}{N_0}\right.$$
$$\left.\left((c_j-1)+\sum_{\substack{j'\in B''\\ j'\neq j}}\left(\frac{\frac{r_{j'}}{2}}{Max(D_{jj'}-\frac{r_{j'}}{2},\varpi)}\right)^\tau c_{j'}\right)-\frac{S}{N_0}\right)$$
$$-\sum_{j\in B''}\sum_{t\in T}\mu_{jt}^2 r_j\delta_{jt}-\sum_{j\in B''}\mu_j^3 g_j$$
$$=\min\sum_{j\in B''}\left( g_jB_j(g_j,c_j)+(\frac{E_b}{N_{total}})_{req}\frac{1}{G}\alpha\frac{S}{N_0}c_j\right.$$
$$\left(\mu_j^1+\sum_{\substack{j'\in B''\\ j'\neq j}}\mu_{j'}^1(\frac{\frac{r_j}{2}}{Max(D_{jj'}-\frac{r_j}{2},\varpi)})^\tau\right)-\sum_{t\in T}\mu_{jt}^2 r_j\delta_{jt}$$
$$\left.-\mu_j^3 g_j+\mu_j^1((\frac{E_b}{N_{total}})_{req}-\frac{S}{N_0}-(\frac{E_b}{N_{total}})_{req}\frac{1}{G}\alpha\frac{S}{N_0})\right) \qquad \text{(SUB1)}$$

subject to: (5)-(8).

Problem (SUB1) can be decomposed this into |B| subproblems since the value of $r_j$ and $c_j$ is discrete and limited. To getting optimal solution, we just exhaustively search for all possible combinations of $c_j$, $r_j$, and $g_j$.

**Subproblem 2**: for $z_{jt}$

$$Z_{SUB2}=$$
$$\min\sum_{j\in B''}\sum_{t\in T}v_{jt}^2D_{jt}z_{jt}+\sum_{j\in B''}v_j^3\sum_{t\in T}\lambda_t z_{jt}+\sum_{t\in T}\lambda_t(1-\sum_{j\in B''}Z_{jt})$$
$$=\min\sum_{j\in B''}\left(\sum_{t\in T}(v_{jt}^2D_{jt}+\left(v_j^3-1\right)\lambda_t)\right)z_{jt}+\sum_{t\in T}\lambda_t \qquad \text{(SUB2)}$$

subject to: (4) and (9).

In (SUB2), the second term $\sum_{t\in T}\lambda_t=|T|\times\lambda_t$ is a constant, total aggregate traffic in the system, which can be dropped and added back to the optimal value since it will not affect the optimal solution of (SUB2). Then (SUB2) can be decomposed into |*B"*|×|*T*| independent subproblems for each $l_{jt}=(v_{jt}^2D_{jt}+\left(v_j^3-1\right)\lambda_t)$ where *j*∈*B"* and *t*∈*T*. To getting minimal value of problem (SUB2), we assign either $z_{jt}$=1 for all $l_{jt}\le 0$, or $z_{jt}$=1 for the minimal value of $l_{jt}$ among all |*B"*|×|*T*| subproblems.

### 3.3 Lagrangean Relaxation Algorithm

According to the weak Lagrangean duality theorem [12], for any $(\mu_j^1, \mu_{jt}^2) \geq 0$ and $\mu_j^3$, the objective value of $Z_D(\mu_j^1, \mu_{jt}^2, \mu_j^3)$ is a lower bound of $Z_{IP}$. Based on Problem (LR), the following a dual problem (D) is constructed to calculate the tightest lower bound.

$$Z_D = \max Z_D(\mu_j^1, \mu_{jt}^2, \mu_j^3) \quad \text{(D)}$$

subject to: $(\mu_j^1, \mu_{jt}^2) \geq 0$ and $\mu_j^3$

Then, subgradient method [9] is applied to solving the dual problem. Let the vector S is a subgradient of $Z_D(\mu_j^1, \mu_{jt}^2, \mu_j^3)$ at $(\mu_j^1, \mu_{jt}^2, \mu_j^3)$. In iteration k of subgradient optimization procedure, the multiplier vector $\pi$ is updated by $\pi^{k+1} = \pi^k + t^k S^k$, in which $t^k$ is a step size determined by $t^k = \delta\left(Z_{IP}^* - Z_D(\pi^k)\right) / \|S^k\|^2$, where $Z_{IP}^*$ is an upper bound on the primal objective function value after iteration k, and $\delta$ is a constant where $0 \leq \delta \leq 2$.

## 4. Getting Primal Feasible Solutions

As mentioned in Section 3, the solution approach applied to solving the problem of network survivability and mobile stations rearrangement is Lagrangean relaxation and subgradient method. The procedure not only guarantees theoretical lower bound of primal feasible solution, but also provides useful hints to getting better primal feasible solution in the process of iteratively solving dual problem. Generally speaking, the better primal feasible solution, say upper bound of the problem (*IP*), is taken by solving Lagrangean relaxation problem (*LR*), for which the decision variables solved in the dual problem (*D*) are applied, in case of either the decision variables are also feasible in the primal problem (*IP*), or adjustment on the decision variables is treated to getting primal feasible solution. The following is a heuristic, denoted Algorithm *A*, for getting primal feasible solution in this paper.

**[Algorithm *A*]**

Step 1. QoS constraint (1) related to inter/intra cell interferences is checked on each base station *j*, $\forall j \in B''$. Adjust the transmission power radius $r_j$ down if the QoS constraint is still violated, or go to Step 2 otherwise.

Step2. Compute the aggregate traffic flow $\sum_{t \in T} \lambda_t z_{jt} = g_j$ of each base station j, based on the transmission power radius rj determined in Step 1.

Table 2 Given parameter for experiments

| Notation | Value |
|---|---|
| $S/N_0$ | 7 db |
| $Eb/N_{total}$ | 6 db |
| $M_j$ | 120 |
| $\tau$ | 4 |
| $G$ | 156.25 |
| $a_t$ | 10 |
| $\alpha$ | 0.75 |

Step3. Check the call blocking constraint $B_j(g_j, c_j) \leq \beta_j$. Assign the available channel $c_j$ up to meet the call blocking requirements $\beta_j$, if the call blocking constraint is still violated, or go to Step 4 otherwise.

Step 4. Adjust the transmission power radius $r_j$ down to the extent for just far enough to covering all mobile stations in each base station j.

Step 5. Calculate the whole blocked traffic in the system that is based upon the decision variables, including $c_j$, $r_j$, $g_j$, and $z_{jt}$, solved in previous steps.

Step 6. End algorithm.

## 5. Computational Experiments

### 5.1 Experiment Environment

For experiment purpose, constants used in the problem (IP) are listed in Table 2. The proposed algorithm for the survivable network problem developed in Sections 3 and 4 is coded in C and run on a PC with INTEL™ P4-1.6GHZ CPU and 256 MB RAM. We evaluate the algorithm for 3 base stations (BS)/ mobile users (MU) combinations – 9/500, 16/1000, 25 /1500. Locations of (BS) as well as (MU) are generated in uniform distribution. Number of broken base stations ($|F|$) is given to four. BS recovery ratio (BSRR) is assigned to 0.25, 0.5, 0.75, and 1.0, for which $|F'|$=1, 2, 3, and 4 BS is fixed, respectively. Besides, different predefined thresholds of call blocking probability ($\beta_j$) are also applied to see what extent of total calling rate can be reduced in terms of different BSRR. The maximum number of iterations for the proposed dual Lagrangean algorithm is 1000, and the improvement counter is 25. The parameter $\delta$ adopted in the subgradient method is initialized to be 2 and halved when the dual objective function value does not improve for 25 iterations.

#### 5.2 Performance Evaluation

Table 3, 4, 5 illustrate the experiment results for each BS/MU of 9/500, 16/1000, 25/1500, respectively. Five results of problem (IP) solved by Lagrangean relaxation approach, i.e. upper bound (UB), lower bound (LB) , error gap, service rate, and CPU time, are illustrated. The error gap is expressed by (UB-LB)/ LB*100%. Accordingly, another stop condition of iteration process, if error gap is less than 0.001%, is also set. If so, this implies the proposed algorithm almost optimally solve the problem. Service rate is defined as the ratio of mobile users which is admitted into the system to total mobile users after optimal solutions are made. Time consumed in solving the problem is listed in CPU.

*1) Call blocking rate:* The analysis results of call blocking rate are shown in Figure 1. No matter which BS/MU is combined, for each BSRR, the call blocking rate is a montonically increasing function of $\beta_j$. However, the combination of 9/500 is more gradual than the other two. This implies call blocking rate is much less affected by $\beta_j$ in light loading than heavy loading.

*2) Gap:* For getting optimal solution, the more heavy loading is incurred, the more time consumed is required. Since the maximum number of iterations is 1000, Figure 2 illustrates that computation of 25/1500 is with loosest gaps in range from 8.52% to 13.84%. All gaps in 9/500 combination are less than 0.12%, it is calculated with near-optimal solution.

*3) Service rate*: Based on Figure 3, the service rate is also a monotonically increasing function of $\beta$j. An interesting finding is that the more heavy loading is incurred, the less service rate is varied. For each BSRR, service rates are varied from 0.624 to 0.91, from 0.785 to 0.937, and from 0.844 to 0.948 in 9/500, 16/1000, and 25/1500 combination, respectively. In other words, BS recovery is much more important in light loading than in heavy loading.

*4) CPU time:* The statistics of CPU time consumed in all experiments are depicted in Figure 4. Obviously, BSRR=0.25 and BSRR=0.75 consumed four times the CPU of the BSRR=1.0, while BSRR=0.5 consumed six times of it. The reason of time consumed varied is that the optimal solution is calculated in combination of C(|F|, |F'|). For 25/1500 case, the time consumed is up to 30 minutes (1848 sec.) to decide which 2 of 4 broken base staions should be fixed. This is a acceptable in terms of planning and operation monitoring considerations, but not real tme control.

## 6. Conclusions

For CDMA networks, this paper focuses on a probable and occasional scenario of base stations crash. We propose a mathematical model to deal with the scenario. Proposed model in conjunction with developed algorithm recovers some of broken base stations to guarantee sustainable services as well as to assure quality of service. Be more generic, performance evaluation considering existing users as well as mobility will be taken into account in the future works.

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Table 3 Experiment Results of 9/500 with Respect to BSRR & βj

| BSSR | $\beta_j$ | UB | LB | Gap* | ServiceRate | CPU+ |
|---|---|---|---|---|---|---|
| 0.25 | 0.01 | 15.90983 | 15.90903 | 0.01 | 0.624 | 59 |
| 0.25 | 0.02 | 16.06501 | 16.06292 | 0.01 | 0.624 | 59 |
| 0.25 | 0.03 | 16.12252 | 16.11785 | 0.03 | 0.624 | 58 |
| 0.25 | 0.04 | 16.3259 | 16.31399 | 0.07 | 0.624 | 58 |
| 0.25 | 0.05 | 16.48268 | 16.46374 | 0.12 | 0.624 | 57 |
| 0.5 | 0.01 | 10.64054 | 10.64001 | 0.01 | 0.72 | 88 |
| 0.5 | 0.02 | 10.83417 | 10.83277 | 0.01 | 0.72 | 87 |
| 0.5 | 0.03 | 10.88018 | 10.87714 | 0.03 | 0.72 | 87 |
| 0.5 | 0.04 | 11.10629 | 11.09885 | 0.07 | 0.72 | 87 |
| 0.5 | 0.05 | 11.26306 | 11.25125 | 0.11 | 0.72 | 86 |
| 0.75 | 0.01 | 5.386978 | 5.386709 | 0.01 | 0.816 | 59 |
| 0.75 | 0.02 | 5.591834 | 5.591275 | 0.01 | 0.816 | 58 |
| 0.75 | 0.03 | 5.66057 | 5.659495 | 0.02 | 0.816 | 58 |
| 0.75 | 0.04 | 5.934523 | 5.932387 | 0.04 | 0.816 | 58 |
| 0.75 | 0.05 | 6.110325 | 6.106783 | 0.06 | 0.816 | 58 |
| 1.0 | 0.01 | 0.249247 | 0.249245 | 0.00 | 0.91 | 12 |
| 1.0 | 0.02 | 0.496858 | 0.496853 | 0.00 | 0.91 | 12 |
| 1.0 | 0.03 | 0.602978 | 0.602978 | 0.00 | 0.91 | 12 |
| 1.0 | 0.04 | 0.876931 | 0.876922 | 0.00 | 0.91 | 12 |
| 1.0 | 0.05 | 1.100581 | 1.10057 | 0.00 | 0.91 | 11 |

Table 4 Experiment Results of 16/1000 with Respect to BSRR & βj

| BSSR | $\beta_j$ | UB | LB | Gap* | Service Rate | CPU+ |
|---|---|---|---|---|---|---|
| 0.25 | 0.01 | 17.36336 | 17.21684 | 0.80 | 0.785 | 345 |
| 0.25 | 0.02 | 18.02989 | 17.8512 | 1.00 | 0.785 | 345 |
| 0.25 | 0.03 | 18.50718 | 18.28213 | 1.23 | 0.785 | 344 |
| 0.25 | 0.04 | 19.21121 | 18.98003 | 1.22 | 0.785 | 344 |
| 0.25 | 0.05 | 19.99411 | 19.72739 | 1.35 | 0.785 | 344 |
| 0.5 | 0.01 | 11.58889 | 11.49326 | 0.83 | 0.838 | 517 |
| 0.5 | 0.02 | 12.31528 | 12.20156 | 0.93 | 0.838 | 517 |
| 0.5 | 0.03 | 12.79257 | 12.65001 | 1.13 | 0.838 | 516 |
| 0.5 | 0.04 | 13.60332 | 13.45784 | 1.08 | 0.838 | 517 |
| 0.5 | 0.05 | 14.38622 | 14.22393 | 1.14 | 0.838 | 516 |
| 0.75 | 0.01 | 6.125002 | 6.077356 | 0.78 | 0.888 | 345 |
| 0.75 | 0.02 | 6.894594 | 6.842319 | 0.76 | 0.888 | 346 |
| 0.75 | 0.03 | 7.45353 | 7.380829 | 0.99 | 0.888 | 346 |
| 0.75 | 0.04 | 8.264281 | 8.199668 | 0.79 | 0.888 | 346 |
| 0.75 | 0.05 | 9.047181 | 8.965949 | 0.91 | 0.888 | 346 |
| 1.0 | 0.01 | 0.765985 | 0.762387 | 0.47 | 0.937 | 88 |
| 1.0 | 0.02 | 1.57397 | 1.565937 | 0.51 | 0.937 | 87 |
| 1.0 | 0.03 | 2.20696 | 2.194015 | 0.59 | 0.937 | 87 |
| 1.0 | 0.04 | 3.017711 | 3.001055 | 0.56 | 0.937 | 87 |
| 1.0 | 0.05 | 3.80061 | 3.77584 | 0.66 | 0.937 | 87 |

Table 5 Experiment Results of 25/1500 with Respect to BSRR & βj

| BSSR | $\beta_j$ | UB | LB | Gap* | Service Rate | CPU+ |
|---|---|---|---|---|---|---|
| 0.25 | 0.01 | 18.11724 | 16.44271 | 10.18 | 0.844 | 1229 |
| 0.25 | 0.02 | 19.14714 | 17.17124 | 11.51 | 0.844 | 1226 |
| 0.25 | 0.03 | 20.3903 | 18.10475 | 12.62 | 0.844 | 1226 |
| 0.25 | 0.04 | 21.3712 | 18.97807 | 12.61 | 0.844 | 1226 |
| 0.25 | 0.05 | 22.28902 | 19.57873 | 13.84 | 0.844 | 1227 |
| 0.5 | 0.01 | 11.99584 | 10.91632 | 9.89 | 0.881 | 1841 |
| 0.5 | 0.02 | 13.06911 | 11.78045 | 10.94 | 0.881 | 1841 |
| 0.5 | 0.03 | 14.39214 | 12.83625 | 12.12 | 0.881 | 1841 |
| 0.5 | 0.04 | 15.37304 | 13.71601 | 12.08 | 0.881 | 1843 |
| 0.5 | 0.05 | 16.42581 | 14.56473 | 12.78 | 0.881 | 1848 |
| 0.75 | 0.01 | 6.221366 | 5.692113 | 9.30 | 0.917 | 1229 |
| 0.75 | 0.02 | 7.3545 | 6.66978 | 10.27 | 0.917 | 1230 |
| 0.75 | 0.03 | 8.677526 | 7.79715 | 11.29 | 0.917 | 1230 |
| 0.75 | 0.04 | 9.765154 | 8.776325 | 11.27 | 0.917 | 1231 |
| 0.75 | 0.05 | 10.81792 | 9.644996 | 12.16 | 0.917 | 1232 |
| 1.0 | 0.01 | 1.0738 | 0.989522 | 8.52 | 0.948 | 310 |
| 1.0 | 0.02 | 2.236798 | 2.0408 | 9.60 | 0.948 | 310 |
| 1.0 | 0.03 | 3.619934 | 3.273026 | 10.60 | 0.948 | 310 |
| 1.0 | 0.04 | 4.707562 | 4.25677 | 10.59 | 0.948 | 309 |
| 1.0 | 0.05 | 5.870181 | 5.259548 | 11.61 | 0.948 | 309 |

* The exact gap is less than 0.001%. For simplicity of table expression, only two decimal places are presented.

\+ CPU time consumed in seconds.

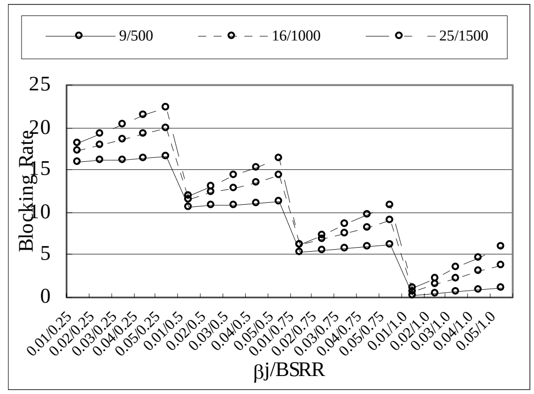


Figure 1 Call blocking rate as a function of $\beta_j$/BSRR combination with respect to BS/MU.

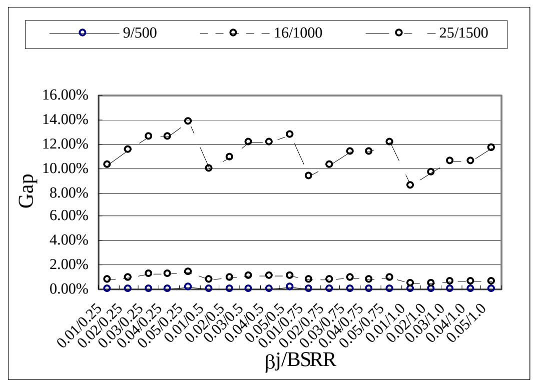


Figure 2 Gap as a function of $\beta_j$/BSRR combination with respect to BS/MU.

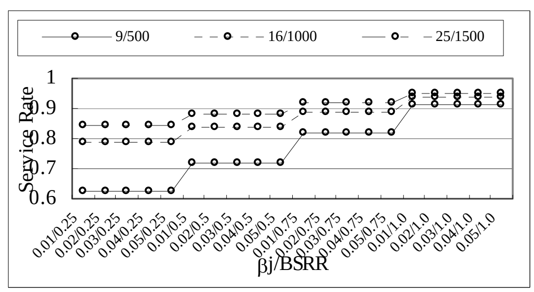


Figure 3 Service rate as a function of $\beta_j$/BSRR combination with respect to BS/MU.

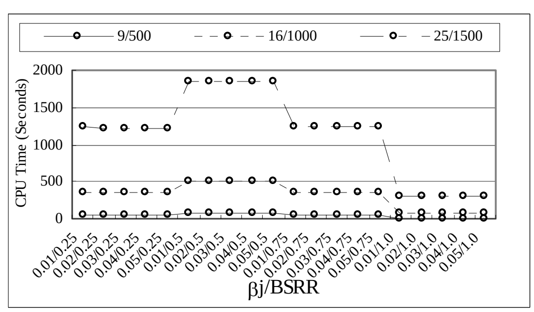


Figure 4 CPU time consumed a function of $\beta_j$/BSRR combination with respect to BS/MU.