A Capacitated Minimum-Cost Multicast Routing Algorithm for Multirate Multimedia Distribution

Hsu-Chen Cheng and Frank Yeong-Sung Lin
Department of Information Management
National Taiwan University
50, Lane 144, Keelung Rd., Sec.4, Taipei, Taiwan, R.O.C.
Tel: +886-2-2511-4301, Fax: +886-2-2521-1912
E-mail: hccheng@ieee.org

Abstract: In this paper, we attempt to solve the problem of min-cost multicast routing for multirate multimedia distribution in a Capacitated Network. More specifically, for (i) a given network topology (ii) a given link capacity (iii) the destinations of a multicast group and (iv) the bandwidth requirement of each destination, we attempt to find a feasible routing solution to minimize the cost of a multicast tree for multirate multimedia distribution. This problem has been proved to be NP-hard. First, we model this problem as an optimization problem, which is a linear programming problem. Then, we propose a simple heuristic algorithm and an optimization based heuristic to solve this problem. Computational experiments have been performed on regular networks, random networks, and scale-free networks.

1. Introduction

Multimedia application environments are characterized by large bandwidth variations due to the heterogeneous access technologies of networks and different receivers' quality requirements. In video multicasting, the heterogeneity of the networks and destinations makes it difficult to achieve bandwidth efficiency and service flexibility. There are many challenging issues that need to be addressed in designing architectures and mechanisms for multicast data transmission [1].

Taking advantage of recent advances in video encoding and transmission technologies, either by a progress coder [2] or video gateway [3][4], different destinations can request a different bandwidth requirement from the source, after which the source only needs to transmit signals that are sufficient for the highest bandwidth destination into a single multicast tree. This concept is called single-application multiple-stream (SAMS). A multi-layered or multirate encoder encodes video data into more than one video stream, including one base layer stream and several enhancement layer streams. The base layer contains the most important portions of the video stream for achieving the minimum quality level. The enhancement layers contain the other portions of the video stream for refining the quality of the base layer stream.

Reference [14] gives an intact survey of the multirate video multicast. Reference [15] [16] discuss the flow control issues in multirate multicast networks. The objective is to achieve the fairness transmission rates that maximize the total receiver utility. Reference [5] discusses the cost issue of multirate video distribution in multicast networks and proposes a heuristic to solve this problem, namely: the modified T-M heuristic (M-T-M Heuristic). Its goal is to construct a minimum cost tree from the source to every destination without considering the link capacity constraints. However, the reference provides only experimental evidence for its performance and does not consider the link capacity constraints. Reference [6] extends this concept to present heuristics with provable performance guarantees for the Steiner tree problem and proves that this problem is NP-hard, even in the special case of broadcasting. From the results, the cost of the multicast tree generated by M-T-M heuristics was no more than 4.214 times the cost of an optimal multicast tree. However, no simulation results are reported to justify the approaches in [6]. The solution approaches described above are heuristic-based and could be further optimized. Consequently, for multimedia distribution on capacitated multicast networks, we intend to find the multicast trees that have a minimal total incurred cost for multi-layered video distribution.

The minimum cost multicast tree problem, which is the Steiner tree problem, is known to be NP-complete. The Steiner tree problem is different to the minimum spanning tree problem in that it permits us to construct, or select, intermediate connection points to reduce the cost of the tree. For the conventional Steiner tree problem, the link costs in the network are fixed. However, for the minimum cost multirate video multicast tree, the link costs are dependent on the set of receivers sharing the link. This is a variant of the Steiner tree problem. The heterogeneity of the networks and destinations makes it difficult to design an efficient and flexible mechanism for servicing all multicast group users. Without considering the link capacity constraints, the multirate multicast routing problem with several commodities can be treated as several single-commodity problems. We would solve each single-commodity problem separately using the techniques such as M-T-M heuristic. With regard to the link capacity constraints, however, different groups may share the common links, so the individual single commodity problems are not independent.

In this paper, we intend to deal with the link-constrained multirate multicast optimization problem. We formally model this issue as an optimization problem. We apply the Lagrangean relaxation method and the subgradient method to solve the problems [7] [8]. Properly integrating the M-T-M heuristics and the results of Lagrangean dual problems may be useful to improve the solution quality. In addition, the Lagrangean relaxation method not only gets a good feasible solution, but also provides the lower bound of the solution, which helps to verify the solution quality. We name this method Lagrangean-multiplier-based Heuristics.

The rest of this paper is organized as follows. In Section 2, we describe the detail of the simple heuristic we proposed, which is composed of the M-T-M heuristic and the adjustment procedures to ensure the link capacity constraint. In Section 3, we formally define the problem being studied, and propose a mathematical formulation of min-cost optimization is proposed. Section 4 applies Lagrangean relaxation as a solution approach to the problem. Section 5, illustrates the computational experiments. Finally, in Section 6, we present our conclusions and the direction of future research.

2. A Simple Heuristic of Link Constrained Multirate Multimedia Multicasting

Reference [9] proposes an approximate algorithm named Takahashi-Matsuyami (T-M) heuristic to deal with the Steiner tree problem, which is a min-cost multicast tree problem. The T-M heuristic uses the idea of minimum depth tree algorithm (MDT) to construct the tree. To begin with, the source node is added to the tree permanently. At each iteration of MDT a node is temporarily added to the tree until the added node is a receiver of the multicast group. Once the iterated tree reaches one of the receivers of the multicast group, it removes all unnecessary temporary links and nodes added earlier and mark the remaining nodes as permanently connected to the tree. The depth of the permanently connected nodes is then set to zero and the iterations continue until all receivers are permanently added to the tree. In [5], the author gives examples of the performance of the T-M heuristic and shows that in some cases the T-M heuristic does not achieve the optimum tree.

Reference [5] modified the T-M heuristic to deal with the min-cost multicast tree problem in multi-layered video distribution. For multirate video distribution, which is different from the conventional Steiner tree problem, each receiver can request a different quality of video. This means that each link's flow of the multicast tree is different and is dependent on the maximum rate of the receiver sharing the link. The author proposes a modified version of the T-M heuristic (M-T-M heuristic) to approximate the minimum cost multicast tree problem for multi-layered video distribution.

The M-T-M heuristic separates the receivers into subsets according to the receiving rate. First, the M-T-M heuristic constructs the multicast tree for the subset with the highest rate by using the T-M heuristic. Using this initial tree, the T-M heuristic is then applied to the subsets according to the order of receiving rate from high to low. For further details of the M-T-M heuristic, please refer to reference [5].

Under the link capacity constraint, the routing decision generated by the heuristic described above may cause the overflow of the links. We propose an adjustment procedure (AP), which is used to adjust the multicast tree resulting from the M-T-M heuristic in order to find a feasible solution and comply with the link capacity.

The adjustment procedure is used to adjust the initial multicast tree constructed by the M-T-M heuristic. Nevertheless, redundantly checking actions may cause a serious decline in performance, even if the total cost is reduced. Therefore, we consider the most useful occurrence to reduce the total cost and control the used resources in an acceptable range. The details of the procedure are:

Adjustment Procedure (AP)

1) Compute the aggregate flow of each link.
2) Sort the links by the difference between aggregate flow of each link and the link capacity in descending order.
3) Choose the first link. If the difference value of the link is positive, go to Step 4, otherwise Step 6.
4) Choose the maximal loaded group on that link to drop and re-add it to the tree. Consider the following possible adding measures and set the best one to be the final tree. Either adds the dropping node to the source node, or to other nodes having the same hop count, or to the nodes having a hop count larger or smaller by one.
5) If a feasible solution is found, go to Step2, otherwise Step 6.
6) Stop.

The performance evaluation of the simple heuristic will be shown in section 5.

3. Problem Formulation

3.1. Problem Description

The network is modeled as a graph where the switches are depicted as nodes and the links are depicted as arcs. A user group is an application requesting transmission in this network, which has one source and one or more destinations. Given the network topology, the capacity of the links and the bandwidth requirement of every destination of a user group, we want to jointly determine the following decision variables: (1) the routing assignment (a tree for multicasting or a path for unicasting) of each user group; and (2) the maximum allowable traffic rate of each multicast user group through each link.

By formulating the problem as a mathematical programming problem, we intend to solve the issue optimally by obtaining a network that will enable us to achieve our goal, i.e. one that ensures the network operator will spend the minimum cost on constructing the multicast tree. The notations used to model the problem are listed in Table 1.

Table 1: Description of Notations

Given Parameters	
Notation	**Description**
a_l	Transmission cost associated with link l
α_{gd}	Traffic requirement of destination d of multicast group g
G	The set of all multicast groups
V	The set of nodes in the network
L	The set of links in the network
D_g	The set of destinations of multicast group g
h_g	The minimum number of hops to the farthest destination node in multicast group g
C_l	The capacity of link l
I_v	The incoming links to node v
r_g	The multicast root of multicast group g
I_{r_g}	The incoming links to node r_g

P_{gd}	The set of paths destination d of multicast group g may use
δ_{pl}	The indicator function which is 1 if link l is on path p and 0 otherwise
Decision Variables	
Notation	**Descriptions**
x_{gpd}	1 if path p is selected for group g destined for destination d and 0 otherwise
y_{gl}	1 if link l is on the subtree adopted by multicast group g and 0 otherwise
m_{gl}	The maximum traffic requirement of the destinations in multicast group g that are connected to the source through link l

3.2. Mathematical Formulation

According to the description in previous section, the min-cost problem is formulated as a combinatorial optimization problem in which the objective function is to minimize the link cost of the multicast tree. Of course a number of constraints must be satisfied.

Objective function:

$$Z_{IP} = \min \sum_{g\in G}\sum_{l\in L} a_l \, m_{gl} \qquad \text{(IP)}$$

subject to:

$$\sum_{p\in P_{gd}} x_{gpd}\alpha_{gd}\delta_{pl} \le m_{gl} \qquad \forall g\in G, d\in D_g, l\in L \qquad (1)$$

$$\sum_{g\in G} m_{gl} < C_l \qquad \forall l\in L \qquad (2)$$

$$m_{gl} \in [0, \max_{d\in D_g} \alpha_{gd}] \qquad \forall l\in L, g\in G \qquad (3)$$

$$y_{gl} = 0 \text{ or } 1 \qquad \forall l\in L, g\in G \qquad (4)$$

$$\sum_{l\in L} y_{gl} \ge \max\{h_g, |D_g|\} \qquad \forall g\in G \qquad (5)$$

$$\sum_{d\in D_g}\sum_{p\in P_{gd}} x_{gpd}\delta_{pl} \le |D_g| y_{gl} \qquad \forall g\in G, l\in L \qquad (6)$$

$$\sum_{l\in I_v} y_{gl} \le 1 \qquad \forall g\in G, v\in V-\{r_g\} \qquad (7)$$

$$\sum_{l\in I_{r_g}} y_{gl} = 0 \qquad \forall g\in G \qquad (8)$$

$$\sum_{p\in P_{gd}} x_{gpd} = 1 \qquad \forall d\in D_g, g\in G \qquad (9)$$

$$x_{gpd} = 0 \text{ or } 1 \qquad \forall d\in D_g, g\in G, p\in P_{gd} \qquad (10)$$

The objective function of (IP) is to minimize the total transmission cost of servicing the maximum bandwidth requirement destination through a specific link for all multicast groups G, where G is the set of user groups requesting connection. The maximum bandwidth requirement on a link in the specific group m_{gl} can be viewed so that the source would be required to transmit in a way that matches the most constrained destination.

Constraint (1) and (2) are referred to as the capacity constraints, which require that the aggregate flow on each link l does not exceed its link capacity C_l. In constraint (1), a variable m_{gl} is introduced, where the variable m_{gl} can be interpreted as the "estimate" of the aggregate flow. Since the objective function is strictly an increasing function with m_{gl} and (IP) is a minimization problem, each m_{gl} will equal the aggregate flow in an optimal solution. Constraint (3) is a redundant constraint which provides upper and lower bounds on the maximum traffic requirement for multicast group g on link l. Constraints (4) and (5) require that the number of links on the multicast tree adopted by the multicast group g be at least the maximum of h_g and the cardinality of D_g. The h_g and the cardinality of D_g are the legitimate lower bounds of the number of links on the multicast tree adopted by the multicast group g. Constraint (6) is referred to as the tree constraint, which requires that the union of the selected paths for the destinations of user group g forms a tree. Constraints (7) and (8) are both redundant constraints. Constraint (7) requires that the number of selected incoming links y_{gl} to node is 1 or 0, while constraint (8) requires that there are no selected incoming links y_{gl} to the node that is the root of multicast group g. As a result, the links we select can form a tree. Finally, constraints (9) and (10) require that only one path is selected for each multicast source/destination pair.

4. Solution Approach

4.1 Lagrangean Relaxation

Lagrangean method has become one of the best tools for optimization problems such as integer programming, linear programming combinatorial optimization, and non-linear programming [7] [8]. The Lagrangean relaxation method permits us to remove constraints and place them in the objective function with associated Lagrangean multipliers instead. The optimal value of the relaxed problem is always a lower bound (for minimization problems) on the objective function value of the problem. By adjusting the multiplier of Lagrangean relaxation, we can obtain the upper and lower bounds of this problem. The Lagrangean multiplier problem can be solved in a variety of ways. The subgradient optimization technique is possibly the most popular technique for solving the Lagrangean multiplier problem [7] [10].

By using the Lagrangean Relaxation method, we can transform the primal problem (IP) into the following Lagrangean Relaxation problem (LR) where Constraints (1) and (6) are relaxed. For a vector of non-negative Lagrangean multipliers, a Lagrangean Relaxation problem of (IP) is given by

Optimization problem (LR):

$$\begin{aligned} Z_D(\beta,\theta) = \min & \sum_{g\in G}\sum_{l\in L} a_l m_{gl} \\ & + \sum_{g\in G}\sum_{d\in D_g}\sum_{l\in L}\sum_{p\in P_{gd}} \beta_{gdl} x_{gpd}\alpha_{gd}\delta_{pl} \\ & - \sum_{g\in G}\sum_{d\in D_g}\sum_{l\in L} \beta_{gdl} m_{gl} \\ & + \sum_{g\in G}\sum_{l\in L}\sum_{d\in D_g}\sum_{p\in P_{gd}} \theta_{gl} x_{gpd}\delta_{pl} \\ & - \sum_{g\in G}\sum_{l\in L} \theta_{gl} |D_g| y_{gl} \end{aligned} \qquad \text{(LR)}$$

subject to: (2) (3) (4) (5) (7) (8) (9) (10).

Where β_{gdl}, θ_{gl} are Lagrangean multipliers and β_{gdl}, $\theta_{gl} \ge 0$. To solve (LR), we can decompose (LR) into the following three independent and easily solvable optimization subproblems.

Subproblem 1: (related to decision variable x_{gpd})

$$Z_{Sub1}(\beta,\theta) = \min \sum_{g\in G}\sum_{d\in D_g}\sum_{p\in P_{gd}}[\sum_{l\in L}\delta_{pl}(\beta_{gdl}\alpha_{gd}+\theta_{gl})]x_{gpd}$$

subject to: (9) (10).

Subproblem 1 can be further decomposed into |G||D$_g$| independent shortest path problems with nonnegative arc weights. Each shortest path problem can be easily solved by Dijkstra algorithm.

Subproblem 2: (related to decision variable y_{gl})

$$Z_{Sub2}(\theta) = \min \sum_{g\in G}\sum_{l\in L}(-\theta_{gl}|D_g|)y_{gl}$$

subject to: (4) (5) (7) (8).

The algorithm to solve Subproblem 2 is:

Step 1 Compute max{h_g, $|D_g|$} for multicast group g.

Step 2 Compute the number of positive coefficients $\theta_{gl}|D_g|$ for all links in the multicast group g.

Step 3 If the number of positive coefficients is greater than max{h_g, $|D_g|$} for multicast group g, then assign the corresponding positive coefficient of y_{gl} to 1 and 0 otherwise.

Step 4 If the number of positive coefficients is no greater than max{h_g, $|D_g|$} for multicast group g, assign the corresponding positive coefficient of y_{gl} to 1. Then, assign [max{h_g, $|D_g|$} – the number of positive coefficients of y_{gl}] numbers of the smallest negative coefficient of y_{gl} to 1 and 0 otherwise.

Subproblem 3: (related to decision variable m_{gl})

$$Z_{Sub3}(\beta) = \min \sum_{g\in G}\sum_{l\in L}(a_l - \sum_{d\in D_g}\beta_{gdl})m_{gl}$$

subject to: (2) (3).

We decompose Subproblem 3 into |L| independent problems. For each link $l \in L$:

$$Z_{Sub3.1}(\beta) = \min \sum_{g\in G}(a_l - \sum_{d\in D_g}\beta_{gdl})m_{gl}$$

subject to: (2) (3).

The algorithm to solve Subproblem 3.1 is:

Step 1 Compute $a_l - \sum_{d\in D_g}\beta_{gdl}$ for link l of multicast group g.

Step 2 Sort the negative coefficient $a_l - \sum_{d\in D_g}\beta_{gdl}$ from the smallest value to the largest value

Step 3 According the sorted sequence. <i> assigns the corresponding m_{gl} to the maximum traffic requirement in the multicast group and adds to the sum value until the total amount of maximum traffic requirements on link l is less than the capacity of link l. <ii> assign the boundary negative coefficient of m_{gl} to the difference between the capacity on link l and the sum value of m_{gl}, <iii> assign the other coefficients of m_{gl} to 0.

According to the weak Lagrangean duality theorem [7], for any β_{gdl}, $\theta_{gl} \ge 0$, $Z_D(\beta_{gdl}, \theta_{gl})$ is a lower bound on Z_{IP}. The following dual problem (D) is then constructed to calculate the tightest lower bound.

Dual Problem (D):

$$Z_D = \max Z_D(\beta_{gdl}, \theta_{gl})$$

subject to:

$$\beta_{gdl}, \theta_{gl} \ge 0$$

There are several methods for solving the dual problem (D). The most popular is the subgradient method, which is employed here [11]. Let a vector s be a subgradient of $Z_D(\beta_{gdl}, \theta_{gl})$. Then, in iteration k of the subgradient optimization procedure, the multiplier vector is updated by $\omega^{k+1}=\omega^k+t^k s^k$. The step size t^k is determined by $t^k=\delta(Z^h_{IP} - Z_D(\omega^k))/\|s^k\|^2$. Z^h_{IP} is the primal objective function value for a heuristic solution (an upper bound on Z_{IP}). δ is a constant and $0< \delta \le 2$.

4.2 Getting Primal Feasible Solutions

After optimally solving the Lagrangean dual problem, we get a set of decision variables. However, this solution would not be a feasible one for the primal problem since some of constraints are not satisfied. Thus, minor modification of decision variables, or the hints of multipliers must be taken, to obtain the primal feasible solution of problem (IP). Generally speaking, the better primal feasible solution is an upper bound (UB) of the problem (IP), while the Lagrangean dual problem solution guarantees the lower bound (LB) of problem (IP). Iteratively, by solving the Lagrangean dual problem and getting the primal feasible solution, we get the LB and UB, respectively. So, the gap between UB and LB, computed by (UB-LB)/LB*100%, illustrates the optimality of problem solution. The smaller gap computed, the better the optimality.

To calculate the primal feasible solution of the minimum cost tree, the solutions to the Lagrangean Relaxation problems are considered. The set of $\{x_{gpd}\}$obtained by solving (Subproblem 1) may not be a valid solution to problem (IP) because the capacity constraint is relaxed. However, the capacity constraint may be a valid solution for some links. Also, the set of $\{y_{gl}\}$ obtained by solving (Subproblem 2) may not be a valid solution because of the link capacity constraint and the union of $\{y_{gl}\}$ may not be a tree.

Here we propose a comprehensive, two-part method to obtain a primal feasible solution. It utilized a Lagrangean based modified T-M heuristic, followed by adjustment procedures. While solving the Lagrangean relaxation dual problem, we may get some multipliers related to each OD pair and links. According to the information, we can make our routing more efficient. We describe the Lagrangean based modified T-M heuristic below.

[Lagrangean-multiplier-based Modified T-M Heuristic]

1) Use $a_l - \sum_{d\in D_g}\beta_{gdl}$ as link l's arc weight and run the M-T-M heuristic.
2) After getting a feasible solution, we apply *the Lagrangean-multiplier-based adjustment procedure* (LAP) to adjust the result.

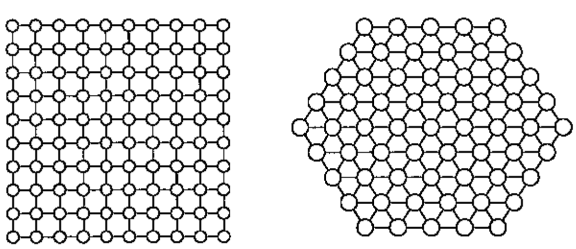
[Lagrangean-Multiplier-based Adjustment Procedure (LAP)]

1) Compute the aggregate flow of each link.
2) Sort the links by the difference between aggregate flow of each link and the link capacity in descending order.

3) Choose the first link. If the difference value of the link is positive, go to Step 4, otherwise Step 6.
4) Choose the group, which have the minimal sensitivity value $a_l - \sum_{d\in D_g} \beta_{gdl}$ on that link, to drop and use $a_l - \sum_{d\in D_g} \beta_{gdl}$ as link l's arc weight and run the M-T-M heuristic to re-add it to the tree. Consider the following possible adding measures and set the best one to be the final tree. Either adds the dropping node to the source node, or to other nodes having the same hop count, or to the nodes having a hop count larger or smaller by one.
5) If a feasible solution is found, go to Step2, otherwise Step 6.
6) Stop.

5. Experimental Results

In this section, computational experiments on the Lagrangean based heuristic and the simple heuristics are reported. The heuristics are tested on three kinds of networks- regular networks, random networks, and scale-free networks. Regular networks are characterized by low clustering and high network diameter, and random networks are characterized by low clustering and low diameter. The scale-free networks, which are power-law networks, are characterized by high clustering and low diameter. Reference [12] shows that the topology of the Internet is characterized by power laws distribution. The power laws describe concisely skewed distributions of graph properties such as the node degree.



(a) Grid Network (b) Cellular Network

Figure 1: Regular Networks

Two regular networks shown in Figure 1 are tested in our experiment. The first one is a grid network that contains 100 nodes and 180 links, and the second is a cellular network containing 61 nodes and 156 links.

Random networks tested in this paper are generated randomly, each having 500 nodes. The candidate links between all node pairs are given a probability following the uniform distribution. In the experiments, we link the node pair with a probability smaller than 2%. If the generated network is not a connected network, we generate a new network.

Reference [13] shows that the scale-free networks can arise from a simple dynamic model that combines incremental growth with a preference for new nodes to connect to existing ones that are already well connected. In our experiments, we applied this preferential attachment method to generate the scale-free networks. The corresponding preferential variable (m_0, m) is (2, 2). The number of nodes in the testing networks is 500.

For each testing network, several distinct cases, which have different pre-determined parameters, are considered. The traffic demands for each destination are drawn from a random variable uniformly distributed in pre-specified categories {1, 2, 5, 10, 15, 20}. The link costs are randomly generated between 1 and 5. The group number of each tested cases is 20. The cost of the multicast tree is decided by multiplying the link cost and the maximum bandwidth requirement on a link. We conducted 500 experiments for each kind of network. For each experiment, the result was determined by the group source, destinations and link costs generated randomly. Table 2 summaries the selected results of the computational experiments.

Table 2: Selected Results of Computational Experiments

C#	N#	SA	UB	LB	GAP	Imp.
Grid Network						
A	5	9,045	8,825	8,685.48	1.61%	2.49%
B	5	10,507	9,639	9,425.01	2.27%	9.01%
C	10	16,476	14,691	13,906.21	5.64%	12.15%
D	10	16,805	15,318	15,147.38	1.13%	9.71%
E	20	23,978	21,133	20,791.90	1.64%	13.46%
F	20	N/A	22,910	19,884.47	15.22%	∞
G	50	40,167	36,241	32,476.30	11.59%	10.83%
H	50	N/A	34,708	30,964.02	12.09%	∞
Cellular Network						
A	5	5,248	4,965	4,890.18	1.53%	5.70%
B	5	4,628	4,281	4,070.81	5.16%	8.11%
C	10	8,928	8,238	7,936.96	3.79%	8.38%
D	10	9,874	9,253	8,904.63	3.91%	6.71%
E	20	15,375	14,750	13,067.21	12.88%	4.24%
F	20	N/A	13,912	12,271.44	13.37%	∞
G	50	N/A	25,160	20,557.85	22.39%	∞
H	50	N/A	25,973	21,261.94	22.16%	∞
Random Networks						
A	5	3,984	3,763	3,487.12	7.91%	5.87%
B	5	3,952	3,465	3,421.11	1.28%	14.05%
C	10	6,765	5,862	5,474.57	7.08%	15.40%
D	10	8,790	8,360	7,300.52	14.51%	5.14%
E	20	14,465	12,782	11,558.87	10.58%	13.17%
F	20	13,266	11,811	9,364.91	26.12%	12.32%
G	50	28,690	24,555	21,540.62	13.99%	16.84%
H	50	28,833	25,774	21,864.95	17.88%	11.87%
Scale-Free Networks						
A	5	5,503	5,176	4,853.17	6.65%	6.32%
B	5	3,939	3,801	3,603.81	5.47%	3.63%
C	10	9,109	8,485	8,051.22	5.39%	7.35%
D	10	9,649	8,847	8,580.09	3.11%	9.07%
E	20	16,361	15,143	14,533.04	4.20%	8.04%
F	20	14,831	13,459	13,107.20	2.68%	10.19%
G	50	30,676	27,737	25,813.31	7.45%	10.60%
H	50	N/A	28,239	25,068.99	12.65%	∞

C#: Case Number
N#: Number of destinations within a group
SA: The result of the simple heuristic
UB and LB: Upper and lower bounds of the Lagrangean based modified T-M heuristic
GAP: Bound difference {(UB-LB)/LB}
Imp.: The improvement ratio of the Lagrangean based modified T-M heuristic {(SA - UB)/ UB}

For each testing network, the maximum improvement ratio between the simple heuristic and the Lagrangean based heuristic is 13.46 %, 8.83%, 15.40 %, and 10.60%, respectively. In general, the Lagrangean based heuristic performs well compared to the simple heuristic, even when the simple algorithm can not find a feasible solution, such as the case F and H of grid network and the case F, G, and H. There are two main reasons of which the Lagrangean based heuristic works better than the simple algorithm. First, the simple algorithm routes the group in accordance with fixed link cost and residual capacity merely, whereas the Lagrangean based heuristic makes use of the related Lagrangean multipliers. The Lagrangean multipliers include the potential cost for routing on each link in the topology. Second, the Lagrangean based heuristic is iteration-based and is guaranteed to improve the solution quality iteration by iteration. Therefore, in a more complicated testing environment, the improvement ratio is higher.

To claim optimality, we also depict the percentile of gap in Table 2. The results show that 72% of the regular and scale free networks have a gap of less than 10%, but the result of random networks show a larger gap. We also found that the simple heuristic perform well in many cases, such as the case A of grid network and case B of scale-free network.

6. Conclusions

In this paper, we attempt to solve the problem of capacitated min-cost multicast routing for multirate multimedia distribution. Our achievement of this paper can be expressed in terms of mathematical formulation and experiment performance. In terms of formulation, we propose a precise mathematical expression to model this problem well. In terms of performance, the proposed Lagrangean based heuristic outperforms the simple heuristics.

Our model can also be extended to deal with the QoS multicast routing problem for multirate multimedia distribution by adding QoS constraints. Moreover, the min-cost model proposed in this paper can be modified as a max-revenue model, with that objective of maximizing total system revenues by totally, or partially, admitting destinations into the system. These issues will be addressed in future works.

References

[1] D. Wu, Y. T. Hou, W. Zhu, T. Q. Zhang, and J. Peha, "Streaming video over the Internet: approaches and directions", *IEEE Transactions on Circuits and Systems for Video Technology*, Vol. 11, no. 3, pp. 282-300, March 2001.

[2] G. Karlsson and M. Vetterli, "Packet video and its integration into the network architecture", *IEEE Journal on Selected Areas in Communications*, Vol. 7, pp. 739–751, 1989.

[3] E. Amir, S. McCanne, and H. Zhang, "An application level video gateway", *Proceedings of ACM Multimedia*, pp. 255-265, Nov. 1995.

[4] T. Turletti and J. C. Bolot, "Issues with multicast video distribution in heterogeneous packet networks", *Proceedings of the 6th International Workshop on Packet Video*, 1994.

[5] N. F. Maxemchuk, "Video Distribution on Multicast Networks", *IEEE Journal on Selected Areas in Communications*, Vol. 15, no. 3, pp. 357-372, April 1997.

[6] M. Charikar, J. Naor, B. Schieber, "Resource Optimization in QoS Multicast Routing of Real-Time Multimedia," *IEEE/ACM Transactions on Networking*, Vol. 12, no. 2, pp. 340-348, April 2004.

[7] M. L. Fisher, "The Lagrangian relaxation method for solving integer programming problems", *Management Science*, Vol. 27, pp. 1-18, 1981.

[8] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, Network Flows: Theory, Algorithms, and Applications, Prentice Hall, ISBN0-13-027765-7, 1993.

[9] H. Takahashi and A. Matsuyama, "An approximate solution for the Steiner problem in graphs," *Math Japonica*, Vol. 6, pp. 573-577, 1980.

[10] A. M. Geoffrion, "Lagrangean relaxation and its uses in integer programming," *Math. Programming Study*, Vol. 2, pp. 82-114, 1974.

[11] M. Held and R. M. Karp, "The Traveling salesman problem and minimum spanning trees: Part I," *Operations Research*, Vol. 18, pp. 1138-62, 1970.

[12] C. Faloutsos, P. Faloutsos, and M. Faloutsos, "On power-law relationships of the Internet topology," *Proceedings of the ACM SIGCOMM*, Sept. 1999.

[13] A.-L. Barabasi and R. Albert, "Emergence of scaling in random networks," *Science*, Vol. 286. pp. 509-512, Oct. 1999.

[14] B. Li and J. Liu, "Multirate Video Multicast over the Internet: An overview," *IEEE Network*, Vol. 17, no. 1, pp. 24-29, Jan./Feb. 2003.

[15] K. Kar, S. Sarkar, and L. Tassiulas, "A scalable, lower-overhead rate control algorithm for multirate multicast sessions," *IEEE Journal on Selected Areas in Communications*, Vol.20, no. 8, pp. 1541-1557, 2002.

[16] W.-H. Wang, M. Palaniswami, and S. H. Low, "Necessary and sufficient conditions for optimal flow control in multirate multicast networks," *IEE Proc. Communication*, Vol. 150, no. 5, pp. 385-390, Oct. 2003.