A multicast tree aggregation algorithm in wavelength-routed WDM networks

Hsu-Chen Cheng, Chin-Chun Kuo and Frank Yeong-Sung Lin

National Taiwan University, 50, Lane 144, Keelung Rd., Sec. 4, Taipei, Taiwan, R.O.C.

ABSTRACT

Wavelength division multiplexing (WDM) has been considered a promising transmission technology in optical communication networks. With the continuous advance in optical technology, WDM network will play an important role in wide area backbone networks. Optical wavelength switching, compared with optical packet switching, is a more mature and more cost-effective choice for optical switching technologies. Besides, the technology of time division multiplexing in optical communication networks has been working smoothly for a long time. In the proposed research, the problem of multicast groups aggregation and multicast routing and wavelength assignment in wavelength-routed WDM network is studied. The optical cross connect switches in the problem are assumed to have limited optical multicast/splitting and TDM functionalities. Given the physical network topology and capacity, the objective is to maximize the total revenue by means of utmost merging multicast groups into larger macro-groups. The groups in the same macro-group will share a multicast tree to conduct data transmission. The problem is formulated as an optimization problem, where the objective function is to maximize the total revenue subject to capacity constraints of components in the optical network, wavelength continuity constraints, and tree topology constraints. The decision variables in the formulations include the merging results between groups, multicast tree routing assignment and wavelength assignment. The basic approach to the algorithm development for this model is Lagrangean relaxation in conjunction with a number of optimization techniques. In computational experiments, the proposed algorithms are evaluated on different network topologies and perform efficiently and effectively according to the experiment results.

Keywords: WDM, TDM, Multicast, Network Planning, Routing, Wavelength Assignment, Optimization, Lagrangean Relaxation Method, Mathematical Programming.

1. INTRODUCTION

It has been widely accepted that optical networks will form the building blocks for the next generation Internet. In the last several years, there has been a growing excitement in the area of optical Dense Wavelength Division Multiplexing – DWDM, or simply, WDM networks. WDM operates by sending multiple lightwaves across a single optical fiber. Information is carried by each wavelength, which is called a channel. Current development activities indicate that WDM technology will be deployed mainly in a backbone network for large regions. WDM also can enhance an optical network's capacity without expensive re-cabling and can reduce the cost of network upgrades. Current optical technology demonstrations have shown the feasibility of up to 160 channels, each operation at 10 Gbps, per fiber[1].

Wavelength routing is defined to be the selective routing of optical signals according to their wavelengths as they travel through the network elements between source and destination with or without wavelength converters. The importance of the reconfigurable optical cross-connect switch (OXCs), and the closely related optical add-drop multiplexer (OADMs), is that they allow the optical network to be reconfigured on a wavelength-by-wavelength basis to optimize traffic, congestion, and network growth[2].

In the wavelength-routed network, the granularity of switching is wavelength. The problem of wasting in bandwidth is arising from not-fully-loaded-lightpaths because the free bandwidth of the wavelengths in these lightpaths cannot be used by others. The bandwidth allocation problem in the wavelength-routed networks has been widely investigated[3, 4]. Several schemes using technologies of Optical Time Division Multiplexing (OTDM) have been proposed in the literature to achieve higher bandwidth utilization.

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Many broadband services such as video conferencing and distance learning employ multicasting for data delivery. The support of multicast is therefore essential for these applications. The multicast routing and wavelength assignment (MC-RWA) problem[5] is: given a limited number of wavelengths and a set of multicast calls, maximize the number of multicast calls admitted, or equivalently, minimize the call blocking probability under the constraint that each multicast tree can be assigned only one wavelength. It has been proved that the MC-RWA problem in circuit-switched multi-hop networks is NP-complete[6]. Therefore, the problem is complicated and hard to solve. Obtaining the optimal solution in such kinds of problem is intractable.

If the multicast groups of the same source are merged by means of OTDM technologies, the MC-RWA problem will be more complicated. A new approach[7] called tree-shared multicast (TS-MCAST) is proposed in optical burst switching networks and a multicast sharing class (MSC) associated with a shared tree is also defined. Most proposed work assumed that the OXCs in WDM networks are quipped with *full range power splitters* and/or wavelength converters, which may not be true in practice. In this paper, more physical resources constraints are taken into consideration. Besides, the tree topologies are not given in advance and the network capacity is not assumed to be as large as total traffic demands. As a result, some groups may not be admitted in due to the capacity constraint. In addition to maximize the revenue by TDM based groups aggregation, we further try to minimize the needed cost of supporting these groups.

In this paper, a multicast tree aggregation algorithm is proposed. It is not easy to represent the mergence between multicast groups in mathematical equations. As a result, "Macro-Group" is introduced. Macro-groups are constructed with the same amount of multicast groups to be considered and the problem of aggregation could be transformed into assignment problem. The transformation is illustrated in Fig. 1.

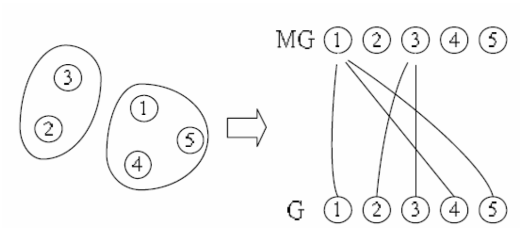

Figure 1: Transforming aggregation problem to assignment problem

After aggregating groups, a super-lighttree is constructed for each macro-group to which at least one multicast group being assigned. The problem is modeled as an optimization problem. This problem is an integer linear programming problem and the Lagrangean relaxation method and the subgradient method will be applied to solve this problem.

The rest of this paper is organized as follows. In Section 2, we formally define the problem being studied, as well as a mathematical formulation of max-revenue optimization is proposed. Section 3 applies Lagrangean relaxation as a solution approach to the problem. Section 4 illustrates the computational experiments. Finally, in Section 5 we present our conclusions and the direction of future research.

2. PROBLEM FORMULATION

The problem to be solved is which multicast group could be admitted in and to which macro-group it should be merged such that the total revenues are maximum. The optimal solution to the constrained multicast RWA should also be answered.

Consider the network in Fig. 2 with node 1 as the source of Group 1 and 2, and nodes 6 and 7 as the destinations of Group 1, and nodes 6 and 7 as the destinations of Group 2, respectively. If the node 5 is equipped with a full range splitter, Group 1 and Group 2 can be merged into a macro-group whose destinations are node 6, 7, and 8. The construction of a lighttree is simple.

If the splitting compatibility of node 5 is limited to be 1-to-2, the solution is slightly different and is shown in Fig. 3 and a different placement of splitter is also shown in Fig. 4.

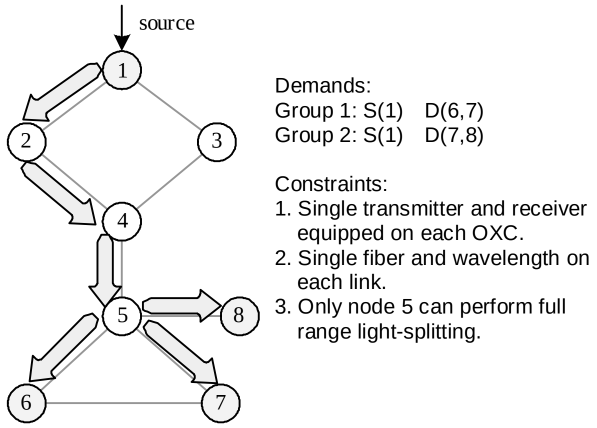

Figure 2: A simple scenario with a full range splitter.

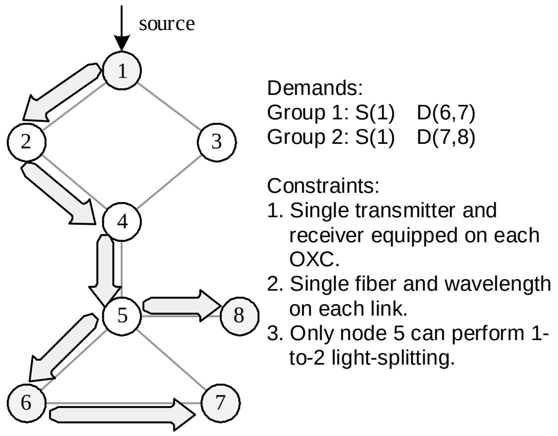

Figure 3: A simple scenario with a limited splitting capacity.

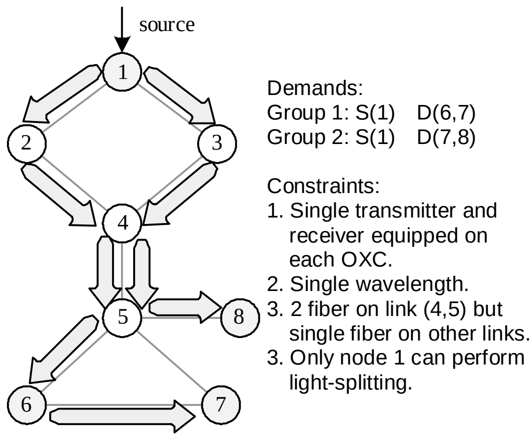

Figure 4: A simple scenario with different placement of splitter.

The physical topology is modeled as a directed graph $\mathcal{G} = \mathcal{G}(V, L)$. Physical links are represented by the directed edge set L, while the node set V represents the OXCs. Each link is equipped with a certain amount of unidirectional fibers. A multicast group is an application requesting for transmission in the network, which has one source and one or more destinations. The number of macro-group to be constructed is equivalent to the number of multicast groups to be considered. Each group should be assigned to at most one macro-group. Then, the constrained multicast RWA problems are solved only for those macro-groups having destinations. We now formalize the problem definition.

Assumptions:

1. The basic architecture used is a WDM network.
2. All OXCs used in the optical network have wavelength routing function but lack the capability of wavelength conversion.
3. All OXCs used in the optical network have TDM capability but the routing function is based on optical wavelength switching rather than time-slot or optical burst switching.

Given:

1. The optical network topology.
2. The number of fibers on each link and the wavelength channel cost on it.
3. The number of optical transmitters and receivers equipped on and splitting capability of each OXC.
4. The traffic demand of each multicast group in terms of time-slot and the revenue it can bring in.
5. The set of available wavelengths on each fiber.
6. The number of time-slots supported in a TDM frame.

Objective:

To maximize the total revenue.

Subject to:

1. Only the multicast groups originating at the same source node could be merged together
2. Each multicast group should be merged to at most one macro-group.
3. Capacity of components in the network.
4. Splitting Capability of each OXC.
5. Each macro-group is supported by one Super-Lighttree.
6. Wavelength continuity of each Super-Lighttree.

To determine:

1. Which group should be admitted in and the mergence result.
2. Routing and wavelength assignment (Super-Lighttree topology) of each macro-group.

Table 1: Description of notations

Given Parameters	
Notation	**Definition**
$\mathcal{G} = \mathcal{G}(V,L)$	Directed graph representing an optical network;
V	The set of OXCs;
L	The set of WDM links;
L_v^+	The set of outgoing links of node v;
L_v^-	The set of incoming links of node v;
$Dest(l)$	The destination node of link l;
C_l	The number of unidirectional fibers on link l;
B_l	The cost of link l;
SP_v	The splitting capability of node v;
Tx_v	The number of optical transmitters at node v;
Rx_v	The number of optical receivers at node v;
TS	Number of time-slots in a TDM frame;
G	The set of all multicast groups;
ts_g	Traffic demand of group g in terms of time-slots;
W	The set of available wavelength on each link;
A_g	The revenue of the multicast group g;
MG	The set of macro-groups;
G_v	The set of groups whose source node are v;
T_v	The set of macro-groups whose source node are v, T_v=G_v
D_t	The set of possible destination nodes of macro-group t;
o_g	The source node of group g;
o_t	The source node of macro-group t;
P_{gv}	Candidate path set from the source node of group g to node v;
P_{tv}	Candidate path set from the source of macro-group t to node v, which is identical to P_{gv} if the sources of g and t are the same node;
σ_{vg}	1 if node v is a destination of group g, and 0 otherwise;
δ_{pl}	1 if link l is on path p, and 0 otherwise.
Decision Variables	
Notation	**Descriptions**
m_{gt}	1 if group g is assigned to macro-group t; otherwise 0;
x_{tvp}	1 if path p is used for macro-group t to reach node v; otherwise 0;
y_{lk}^t	The number of fibers on link l with wavelength k used by macro-group t;
z_{tk}	1 if wavelength k is selected for macro-group t; otherwise 0.

An equivalent formulation of Problem is given by Optimization problem (IP):

$$Z_{IP} = \max \sum_{g \in G} \sum_{t \in MG} A_g m_{gt} - \sum_{l \in L} B_l \sum_{t \in MG} \sum_{k \in W} y_{lk}^t \qquad \text{(IP)}$$

subject to:

$$m_{gt} = 0 \text{ or } 1 \qquad \forall g \in G, t \in MG \qquad (1)$$

$$m_{gt} = 0 \qquad \forall g \in G, t \in MG, o_g \neq o_t \qquad (2)$$

$$m_{gt} = 0 \qquad \forall g \in G, t \in MG, t > g \qquad (3)$$

$$\sum_{t \in MG} m_{gt} \leq 1 \qquad \forall g \in G \qquad (4)$$

$$\sum_{g \in G} ts_g m_{gt} \leq TS \qquad \forall t \in MG \qquad (5)$$

$$x_{tvp} = 0 \text{ or } 1 \qquad \forall t \in MG, v \in V, p \in P_{tv} \qquad (6)$$

$$\sum_{p \in P_{tv}} x_{tvp} \leq 1 \qquad \forall t \in MG, v \in V \qquad (7)$$

$$\sum_{g \in G} m_{gt} \sigma_{vg} \leq \left| \{G_u \mid u = o_t\} \right| \times \sum_{p \in P_{tv}} x_{tvp} \qquad \forall t \in MG, v \in V \qquad (8)$$

$$y_{lk}^t \in \{0, 1, 2, ..., C_l\} \qquad \forall t \in MG, k \in W, l \in L \qquad (9)$$

$$\sum_{v \in V} \sum_{p \in P_{tv}} x_{tvp} \delta_{pl} \leq |D_t| \times \sum_{k \in W} y_{lk}^t \qquad \forall t \in MG, l \in L \qquad (10)$$

$$\sum_{t \in MG} y_{lk}^t \leq C_l \qquad \forall l \in L, k \in W, \qquad (11)$$

$$\sum_{l \in L_v^+} y_{lk}^t \leq SP_v \times \sum_{l \in L_v^-} y_{lk}^t \qquad \forall v \in V, t \in MG, t \notin T_v, k \in W, \qquad (12)$$

$$\sum_{k \in W} y_{lk}^t \leq \min\{C_l, \left\lceil \frac{1}{SP_v} \times |D_t| \right\rceil\} \qquad \forall t \in MG, l \in l, v = Dest(l) \qquad (13)$$

$$z_{tk} = 0 \text{ or } 1 \qquad \forall t \in MG, k \in W \qquad (14)$$

$$\sum_{k \in W} z_{tk} \leq 1 \qquad \forall t \in MG \qquad (15)$$

$$\sum_{g \in G} m_{gt} \leq \left| \{G_u \mid u = o_t\} \right| \times \sum_{k \in W} z_{tk} \qquad \forall t \in MG \qquad (16)$$

$$\sum_{k \in W} z_{tk} \leq \sum_{g \in G} m_{gt} \qquad \forall t \in MG \qquad (17)$$

$$\sum_{l \in L_v^+} y_{lk}^t \leq SP_v \times z_{tk} \qquad \forall v \in V, t \in T_v, k \in W, \qquad (18)$$

$$\sum_{k \in W} \sum_{l \in L_v^-} y_{lk}^t = 0 \qquad \forall t \in MG \qquad (19)$$

$$\sum_{t \in T_v} \sum_{k \in W} Z_{tk} \leq Tx_v \qquad \forall v \in V \qquad (20)$$

$$\sum_{t \in MG} \sum_{p \in P_{tv}} x_{tvp} \leq Rx_v \qquad \forall v \in V \qquad (21)$$

Constraints (1), (2), (4), (4), and (5) are group aggregation constraints. Each multicast group g should be merged to at most one macro-group, and the aggregated demands can not exceed the number of time slogs supported in a TDM frame. Constraint (2) requires that two multicast groups can be merged together only if they originating from the same source node. Equation (3) is a redundant constraint which is added to reduce computation time. A more detailed explanation is presented in section 3.

Constraints (6), (7), (8), (9), (10), (12), (13), and (19) are routing and wavelength channel allocation constraints. If a multicast group g is merged to macro-group t, all destination nodes of g should become t's destination. Constraint (8) requires that if node v is a destination node of macro-group t, there must be a simple path starting from t's source to it. Constraint (9) is a capacity constraint restricting the link usage on l by macro-group t assigned to wavelength k. Constraint (10) requires that if link l is on the path used for macro-group t to reach any node v, the link usage of t with wavelength k on that link should be greater than zero. Constraint (12) is a splitting constraint which requires that the total outgoing link usage of macro-group t with wavelength k on node v should be less than or equal to the product of splitting capability of v and total incoming link usage of t on node v with the same wavelength. Equation (13) and Equation (19) are redundant constraints added to restrict solution range in the relaxed problem.

Constraints (14), (15), (16), (17), and (18) are wavelength assignment constraints. For each macro-group t, it can be assigned to at most one wavelength, which implies wavelength continuity. If any multicast group g is merged to it, Equation (16) ensures that it will be assigned a wavelength. On the other hand, Constraint (17) requires that macro-group t should not be assigned a wavelength if no group is merged into it. Equation (18) is also a splitting constraint special for the source node of macro-group t.

Constraints (11), (20), and (21) are optical transceiver and capacity constraints. Constraint (11) requires that, for each wavelength, the total wavelength channels used on link l should not exceed the number of fibers on that link. For each OXC in the network, the total number of super- lighttree rooted at it should not exceed the amount of optical transmitter equipped on it. The number of being destination of any macro-group should not exceed the number of optical receiver equipped on the OXC.

The number of variables and constraints used in our formulation are both $O(|G|^2+|G|\times|V|\times|P|+|G|\times|L|\times|W|)$, where P is the candidate paths set between all pairs of nodes in the network. The size of P is $O(2^{|L|})$ for that each link l in the network may or may not be in a given path. However, with a slight modification, the space complexity of our formulation will not grow exponentially with network size in terms of links. The variable x_{tvp}, which represents whether source node of macro-group t reaching node v by path p, can be replaced by two 0-1 variables: x'_{tv} and x'_{tvl}. The former represents whether macro-group t use any path to reach node v while the later decides whether link l is on the path used by t from it's source to v.

The reason the modification can be made in such a way is that the variable x_{tvp} in all equations of the proposed formulation is almost represented in an aggregated form ($\sum_{p\in P_v} x_{tvp}$) except for Constraint (6) and Constraint (10). For each pair of macro-group t and node v, Constraint (7) requires that there is at most one path connecting t's source to v. As a result, the path p used by t to reach v can always be recovered from the information recorded in x'_{tv} and x'_{tvl}. Therefore, the space complexity of our formulation is reduced to $O(|G|^2+|G|\times|L|\times(|V|+|W|))$ where the exponential term $O(|P|)=O(2^{|L|})$ is replaced by $O(|L|)$.

3. SOLUTION PROCEDURE

3.1 Lagrangean relaxation

By using the Lagrangean Relaxation method[8, 9], we can transform the primal problem (IP) into the following Lagrangean Relaxation problem (LR) where constraints (5), (8), (10), (11), (12), (16), (17), and (18) are relaxed.For a vector of Lagrangean multipliers, a Lagrangean Relaxation problem of (IP) is given by Optimization problem **(LR):**

$$
\begin{aligned}
Z_d(u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8) = \min & -\sum_{g\in G}\sum_{t\in MG} A_g m_{gt} + \sum_{l\in L} B_l \sum_{t\in MG}\sum_{k\in W} y_{lk}^t + \sum_{t\in MG}\sum_{v\in V} u_{1tv}\left[\sum_{g\in G} m_{gt}\sigma_{vg} - \left|\{G_u \mid u=o_t\}\right| \times \sum_{p\in P_{tv}} x_{tvp}\right] \\
& + \sum_{t\in MG}\sum_{l\in L} u_{2tl}\left[\sum_{v\in V}\sum_{p\in P_{tv}} x_{tvp}\delta_{pl} - |D_t| \times \sum_{k\in W} y_{lk}^t\right] + \sum_{t\in MG} u_{3t}\left[\sum_{g\in G} m_{gt} - \left|\{G_u \mid u=o_t\}\right| \times \sum_{k\in W} z_{tk}\right] \\
& + \sum_{v\in V}\sum_{t\in T_v}\sum_{k\in W} u_{4vtk}\left[\sum_{l\in L_v^+} y_{lk}^t - SP_v \times z_{tk}\right] + \sum_{l\in L}\sum_{k\in W} u_{5lk}\left[\sum_{t\in MG} y_{lk}^t - C_l\right] \\
& + \sum_{v\in V}\sum_{t\in\{MG-T_v\}}\sum_{k\in W} u_{6vtk}\left[\sum_{l\in L_v^+} y_{lk}^t - SP_v \times \sum_{l\in L_v^-} y_{lk}^t\right] + \sum_{t\in MG} u_{7t}\left[\sum_{g\in G} ts_g m_{gt} - TS\right] + \sum_{t\in MG} u_{8t}\left[\sum_{k\in W} z_{tk} - \sum_{g\in G} m_{gt}\right]
\end{aligned}
$$

subject to: (1)(2)(3)(4)(6)(7)(9)(13)(14)(15)(19)(20)(21)

where u_1, u_2, u_3, u_4, u_5, u_6, u_7, and u_8 are the vectors of non-negative Lagrangean multipliers $\{u_{1tv}\}$, $\{u_{2tl}\}$, $\{u_{3t}\}$, $\{u_{4vtk}\}$, $\{u_{5lk}\}$, $\{u_{6vtk}\}$, $\{u_{7t}\}$, and $\{u_{8t}\}$,. To solve (LR), we decompose the problem into the following four independent and easily solvable optimization subproblems.

Subproblem 1: (related to decision variable m_{gt})

$$Z_{sub1}(u_1,u_3,u_7,u_8) = \min -\sum_{g\in G}\sum_{t\in MG} A_g m_{gt} + \sum_{t\in MG}\sum_{v\in V}\sum_{g\in G} u_{1tv} m_{gt}\sigma_{vg} + \sum_{t\in MG}\sum_{g\in G} u_{3t} m_{gt}$$
$$+\sum_{t\in MG} u_{7t}\sum_{g\in G} ts_g m_{gt} - \sum_{t\in MG} u_{8t}\sum_{g\in G} m_{gt}$$
$$= \min \sum_{t\in MG}\sum_{g\in G}\left(\sum_{v\in V} u_{1tv}\sigma_{vg} + u_{3t} + u_{7t} ts_g - u_{8t} - A_g\right)$$

subject to: (1)(2)(3)(4).

Subproblem 1 can be further decomposed into $|V|$ problem because the groups will be aggregated together only if they root at the same nodes. A redundant constraint (3) is added to the problem in order to avoid oscillation of decision variable iteration by iteration. A formal proof is given below.

Lemma 1
Constraint (3) is a redundant constraint.

Proof
The lemma is proved by construction. A simple permutation and re-labeling technique can be applied to all possible assignments between groups and macro-groups to satisfy Constraint (3). Given an aggregated macro-group t, it can be relabeled to the smallest ID among all groups assigned to it. Because each group can be aggregated to at most one macro-group, no macro-group will come into collision with others in terms of ID. As a result, the assignment between groups and relabeled macro-groups satisfies the constraint (3).

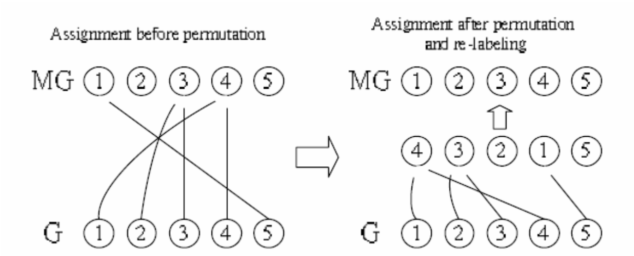

Figure 5: Permutation and re-labeling on macro-groups

An example is illustrated in Fig. 5. The macro- groups are permutated according to the lowest ID among groups being assigned to them and relabeled according to the new order. For example, macro-group 4 is relabeled to 1 because multicast group 1 is assigned to it. The macro-group 3 is relabeled in this way as well. According to the computational experiments, the running time of the algorithm will be shortened and the lower bound will be slightly higher with this redundant constraint. For each group g, it will be aggregated to the macro-group t with lowest coefficient $\sum u_{1tv}+u_{3t}+u_{7t}ts_g-u_{8t}-A_g$. If the lowest coefficient is greater than 0, group g is dropped; otherwise g is aggregated to macro-group t and the corresponding variable m_{gt} is set to be 1.

Subproblem 2: (related to decision variable x_{tvp})

$$Z_{sub2}(u_1,u_2) = \min -\sum_{t\in MG}\sum_{v\in V}\sum_{p\in P_{tv}} \left|\{G_u \mid u = o_t\}\right| \times u_{1tv} x_{tvp} + \sum_{t\in MG}\sum_{l\in L}\sum_{v\in V}\sum_{p\in P_{tv}} u_{2tl} x_{tvp}\delta_{pl}$$
$$= \min \sum_{t\in MG}\sum_{v\in V}\sum_{p\in P_{tv}}\left(\sum_{l\in L} u_{2tl}\delta_{pl} - \left|\{G_u \mid u = o_t\}\right| \times u_{1tv}\right) x_{tvp}$$

subject to: (6)(7)(21).

Subproblem 2 is composed of $|MG|$ shortest path tree problems for each macro-group t, where u_{2tl} is the arc weight of link l. For each pair t and v, if the cost of the result shortest path p is less than the threshold value $|\{G_u|u=o_t\}|\times u_{1tv}$, set x_{tvp} to be 1, otherwise set it to be 0. If Constraint (21) is violated by some node v, sort values of shortest path cost minus threshold value in ascending order, and followed by setting the first Rx_v corresponding x_{tvp} to be 1.

Subproblem 3: (related to decision variable y^t_{lk})

$$Z_d(u_2,u_4,u_5,u_6) = \min \sum_{t\in MG}\sum_{k\in W}\sum_{l\in L}\left[B_l + u_{5lk} - |D_t|\times u_{2tl}\right]y^t_{lk} + \sum_{k\in W}\sum_{v\in V}\sum_{t\in T_v}\sum_{l\in L_v^+} u_{4vtk}y^t_{lk}$$
$$+\sum_{k\in W}\sum_{v\in V}\sum_{t\in\{MG-T_v\}}\sum_{l\in L_v^+} u_{6vtk}y^t_{lk} - \sum_{k\in W}\sum_{v\in V}\sum_{t\in\{MG-T_v\}}\sum_{l\in L_v^-} u_{6vtk}SP_v y^t_{lk}$$

subject to: (9)(13)(19).

Subproblem 3 can be further decomposed into $|MG|$ problems. The link usage and wavelength for each macro-group $t\in MG$ should be decided. In order to minimize the objective function, the wavelength k with smallest coefficient is select for each link.

Subproblem 4: (related to decision variable z_{tk})

$$Z_d(u_3,u_4,u_8) = \min -\sum_{t\in MG}\sum_{k\in W}\left|\{G_u \mid u=o_t\}\right|u_{3t}z_{tk} - \sum_{v\in V}\sum_{t\in T_v}\sum_{k\in W} u_{4vtk}SP_v z_{tk} + \sum_{t\in MG}\sum_{k\in W} u_{8t}z_{tk}$$
$$= \min -\sum_{v\in V}\sum_{t\in T_v}\sum_{k\in W}\left(|T_v|u_{3t} + u_{4vtk}SP_v z_{tk} - u_{8t}\right)$$

subject to: (14)(15)(20).

Subproblem 4 can be further decomposed into $|V|$ problems. For each node v, every macro-group rooted at v has to be assigned a wavelength. According to Lagrangean multipliers $\{u_{4vtk}\}$, the wavelength k with largest value of u_{4vtk} is chosen for macro-group t if the coefficient $|T_v|\times u_{3t}+u_{4vtk}SP_v-u_{8t}$ is larger than 0. Otherwise, macro-group t is skipped. For each node v, if the number of macro-groups rooted at it is larger than the number of transmitters on it, macro-groups are sorted according to their coefficients in ascending order and then the first T_{xv} corresponding z_{tk} are set to be 1.

According to the weak Lagrangean duality theorem[10], for any u_1, u_2, u_3, u_4, u_5, u_6, u_7, and $u_8\geq 0$, $Z_D(u_{1tv}, u_{2tl}, u_{3t}, u_{4vtk}, u_{5lk}, u_{6vtk}, u_{7t}, u_{8t})$ is a lower bound on Z_{IP}. The following dual problem (D) is then constructed to calculate the tightest lower bound.

Dual Problem (D):

$$Z_D = \max Z_D(u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8)$$

subject to: $u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8 \geq 0$.

There are several methods for solving the dual problem (D). The most popular is the subgradient method[9], which is employed here. Let a vector s be a subgradient of $Z_D(u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8)$. Then, in iteration k of the subgradient optimization procedure, the multiplier vector is updated by $\omega^{k+1}=\omega^k+t^k s^k$. The step size t^k is determined by $t^k=\delta(Z^h_{IP}-Z_D(\omega^k))/\|s^k\|^2$. Z^h_{IP} is the primal objective function value for a heuristic solution (an upper bound on Z_{IP}). δ is a constant and $0 < \delta \leq 2$.

3.2 Getting primal feasible solutions

After optimally solving the Lagrangean dual problem, we get a set of decision variables. However, this solution would not be a feasible one for the primal problem since some of constraints are not satisfied. Thus, minor modification of decision variables, or the hints of multipliers must be taken, to obtain the primal feasible solution of problem (IP). Generally speaking, the best primal feasible solution is an upper bound (UB) of the problem (IP), while the Lagrangean dual problem solution guarantees the lower bound (LB) of problem (IP). Iteratively, by solving the Lagrangean dual problem and getting the primal feasible solution, we get the LB and UB, respectively. So, the gap between UB and LB, computed by (UB-LB)/LB*100%, illustrates the optimality of problem solution. The smaller gap computed, the better the optimality.

Owing to the complexity of the primal problem, here we propose a comprehensive, two-part method to obtain a primal feasible solution: the group aggregation heuristic and the constrained multicast routing and wavelength assignment (RWA) heuristic. The first one determines which group can be admitted in, the memberships between admitted-in multicast groups and the destinations of all macro-groups. After the aggregation of multicast groups and memberships of macro-group are determined, we solve constrained multicast RWA subproblem for each macro-group.

[Lagrangean Multipliers-based Group Aggregation Heuristic]

1. Based on { m_{gt} }, identify the set of un-admitted- in groups, denoted by U_g.
2. Based on { m_{gt} }, calculate the aggregated demands of time-slots of all macro-groups and identify the set of multicast group which are assigned to it, denoted by G_t.
3. Identify the set of macro-groups whose aggregated demand exceeds the number of available time-slots in a TDM frame (*TS*), denoted by T_m.
4. Remove one $m_{gt} \in$, calculate contribution ratio for each group g in G_t.
5. Drop $G \in G_t$ with lowest contribution ratio and insert g into U_g.
6. Repeat step 5 until the aggregated demands in terms of time-slots of t is less than or equal to *TS*.
7. Repeat step 4, 5, and 6 until T_m becomes empty.
8. For each nonempty macro-group, identify the destinations, calculate the revenue and insert it to the set T_r, which is the set of macro-group to be routed.
9. Based on revenue, sort $T \in T_r$ in descending order.

After applying the algorithm described above, we get the set of macro-groups for which we solve multicast RWA in next heuristic.

[Lagrangean Multipliers-based Multicast RWA Heuristic]

1. Select a macro-group $t \in T_r$ with highest revenue it can earn, and check whether residual transmitter on source node and receiver on destination nodes of *t* are enough or not.
2. If any residual resources needed by *t* are not enough, drop all groups in G_t and insert into U_g. Repeat step1.
3. Else, run SPH-J algorithm |W| times for each wavelength. Select wavelength *k* with lowest routing cost.
4. If no such wavelength *k* exists, drop all groups in G_t and insert them into U_g.
5. Else, decrease the residual transmitter on source node of *t*, the residual receiver on destinations of *t*, and the residual link capacity with wavelength *k* of all used link according to the link usage calculated in step 3. Remove *t* from T_r and repeat step 1 until T_r becomes empty.

[Algorithm SPH-J]

1. Insert source node to set *TreeNodeSet*, and insert all destinations to set *DestSet*.
2. Calculate the link cost: $SPHCost(l) = B_l + \sum_{t \in MG} \sum_{k \in W} u_{6vtk}$, where *v=Dest(l)*.
3. Choose $d \in DestSet$ with feasible path and lowest cost to any node $tn \in TreeNodeSet$, degree constraint of each node and capacity constraint of each link on the acyclic path should be checked. If no such *d* exists, terminate this algorithm and return fail.
4. Remove *d* from *DestSet* and insert it with other nodes all the way in the path into *TreeNodeSet*.
5. Repeat step 3 until *DestSet* is empty.
6. Return success.

The time complexity of iterations of the proposed Lagrangean relaxation based algorithm is composed of three parts: solving Lagrangean relaxation subproblems, solving the Lagrangean dual problem, and getting primal feasible solutions. The third part dominates others because the high complexity of solving constrained multicast RWA problem. The worst case time complexity is $O(|G| \times |W| \times |D_t^2| \times |L| \times \log|V|)$.

4. COMPUTATIONAL EXPERIMENTS

In this section, computational experiments on the Lagrangean relaxation based heuristic and other primal heuristics are reported. Because of the complexity of the multicast group aggregation and constrained multicast routing and wavelength assignment problems, it is not easy to get a tighter lower bound by solving the Lagrangean relaxation problem iteration by iteration. But this powerful methodology provides a lot of hints to help us get a primal feasible solution. In order to demonstrate the difference of solution quality between the algorithms proposed in this paper and other primal heuristics, a simple algorithm is implemented to compare with the Lagrangean relaxation based algorithms.

In section 3, the problem is decomposed into two subproblems: the group aggregation subproblem and the constrained multicast routing and wavelength assignment (RWA) subproblem. Without implications of the Lagrangean multipliers, memberships and demands of groups are the only information we can rely on to solve the group aggregation problem. Two groups can be merged into one macro-group if they have sufficient overlap in terms of destination nodes and the aggregated demands do not exceed the number of available timeslots in a TDM frame. The simple algorithm is described as follow.

[Algorithm SA]

Step 1 (Initialization):

Read configuration file to construct WDM network and generate multicast traffic demands.

Step 2 (Group Aggregation):

For each group g rooted at node v, merge g to a macro-group t ($t < g$) with highest extent of overlap (at least 66%) in terms of destination nodes without violating capacity constraint. If no such t exists, assign g to the macro- group with ID g.

Step 3 (Constrained Multicast RWA):

Applying the same algorithms described in section 4.2 to determine the routing and wavelength assignment problem for each macro-group.

Step 4 (Termination):

Calculate the result value from step 3 and terminate this algorithm.

The network topology used for our numerical experiments are 14-node 42- link NSFNET network (Fig. 6) and 12-node 50-link GTE network (Fig. 7). The number beside each link indicates the cost of the link.

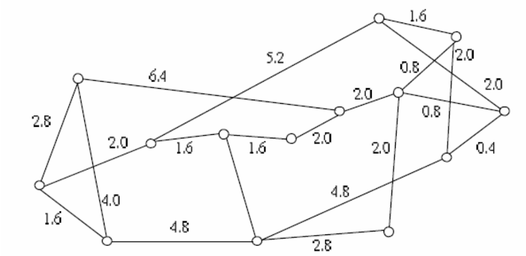

Figure 6: 14-nodes 42-links NSFNET network.

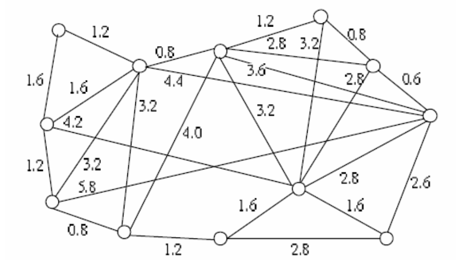

Figure 7: 12-nodes 50-links GTE network.

First, we experiment different aggregation levels in Experiment-I. The number of transmitters Tx on each node is calculated from dividing 40 (half of average number of groups tested) by the number of nodes in the network. The number of receivers Rx is calculated from multiplying the number of transmitters by the average group size. Each case is tested on two topologies presented above. Table 2 summaries the selected results of the computational experiments.In general, the results of LR are all better than SA. For each testing network, the maximum improvement ratio between the simple heuristic and the Lagrangean based heuristic is 30.48 %, and 21.56%, respectively. To claim optimality, we also depict the percentile of gap in Table 2. As we mention before, because of the complexity of the multicast group aggregation and constrained multicast routing and wavelength assignment problems, it is not easy to get a tighter lower bound. The gap between upper bound and lower bound may be a duality gap, because we relax several constraints.

Second, the relationship between the reduced cost and increased splitting capability are examined in Experiment-II. In this case, we try to compare the costs between different splitting capabilities conditions. To achieve fair comparison, the number of groups being admitted in should be the same or we can compare the costs between different numbers of groups being routed in the network. As a result, the offered loads in terms of the number of groups are set to be small to make all of them being admitted in. Table 3 summaries the computational experiments of Experiment-II. In general, the Lagrangean based heuristic performs well compared to the simple heuristic, even when the simple algorithm can not find a feasible solution, such as the case A with 10 groups and no splitting capability. We also found that if the nodes have higher splitting capability, the cost would be reduced.

There are three main reasons that LR works better than SA. First, the SA makes groups aggregation decision only based on the destination nodes set and residual capacity in terms of time-slots, whereas LR makes use of the related Lagrangean multipliers. The Lagrangean multipliers include the potential cost for routing and wavelength assignment on each link in the formed tree topology. Second, LR solves the constrained multicast routing and wavelength assignment problem based on the modified link cost, which takes the splitting capability of destination node of link into consideration. As a result, LR can find a feasible solution with higher possibility comparing to SA. The solution quality is better too. Last, LR is iteration-based and is guaranteed to improve the solution quality iteration by iteration. Therefore, in a more complicated testing environment which the extent of mergence between groups is higher, the improvement ratio is higher.

Table 2: Selected results of experiment I

CASE	SP.	G#	SA	UB	LB	GAP	Imp.
NSFNET Network						**Max Imp. Ratio: 30.48 %**	
A	8	50	2553	3227.8	4629.9	43.41%	26.43%
		70	2872.4	3748	6524.45	74.08%	30.48%
		90	3233	4001.3	8415	110.31%	23.76%
		110	3569.4	4172.4	10266	146.05%	16.89%
B	12	50	2691.8	3353.2	4624.25	37.91%	24.57%
		70	2991	3740.4	6521.74	74.36%	25.06%
		90	3301.6	4128.2	8412.6	103.78	25.04%
		110	3643.2	-4469	10254.6	129.46%	22.67%
GTE Network						**Max Imp. Ratio: 21.56 %**	
A	8	50	3696.2	4463.3	5071.49	13.63%	20.75%
		70	4288.8	5173.9	7122.5	37.66%	20.64%
		90	4884	5836.55	9148.73	56.75%	19.5%
		110	5278.5	6416.57	11235.52	75.1%	21.56%
B	12	50	3726.3	4360.3	5067.45	16.22%	17.01%
		70	4421	5307.3	7106.7	33.9%	20.05%
		90	4858.2	5891.1	9161.43	55.51%	21.26%
		110	5374.5	6236.8	11182.75	79.3%	16.04%

SP: Additional Splitting Capabilities
SA: The result of the simple heuristic
UB and LB: Upper and lower bounds of the Lagrangean based heuristic
GAP: The error gap of the Lagrangean relaxation
Imp.: The improvement ratio of the Lagrangean based heuristic

Table 3: Results of experiment II

CASE	SP	SA Cost	LR Cost	Imp.
Group Number=10				
A	0	N/A	247.2	N/A
	8	218	170.8	-21.65%
	16	203.2	160	-21.26%
	24	194	148.4	-23.51%
B	0	225.6	224.4	-0.53%
	8	175.6	164	-6.61%
	16	170.8	152.8	-10.54%
	24	166.4	143.2	-13.94%
Group Number =20				
A	0	N/A	N/A	N/A
	8	364.8	350.8	-3.84%
	16	352.4	318.8	-9.53%
	24	343.2	299.6	-12.7%
B	0	N/A	N/A	N/A
	8	355.6	282.4	-20.58%
	16	329.2	278.8	-15.31%
	24	324.4	261.2	-19.48%

5. CONCLUSION

The achievement of this paper can be expressed in terms of mathematical formulation and experiment performance. In terms of formulation, we propose a precise mathematical expression to well model the problems of multicast tree/group aggregation, constrained multicast routing and wave length assignment on wavelength-routed WDM networks. The overall problem is modeled as an integer linear programming problem. In terms of performance, the proposed Lagrangean relaxation and subgradient based algorithms outperform the primal heuristics with acceptable computation time.

Different network topologies are tested in experiments, including NSFNET network and GTE network. And different parameters setting, including different number of wavelength available in fiber-optics, different size of multicast groups, and different demands in terms of time-slots have been tested to make this research more generic. As a result, we suggest that network operators apply the proposed Lagrangean relaxation based algorithms when dealing with network design problems related to supporting multicast communications in resource constrained WDM networks.

In this paper, Quality of Service (QoS) measurements are not taken into consideration. In the future, the QoS requirements can be added to the proposed flexible formulation. For example, delay bound, jitter, and the hop count constraint can be easily added to the formulation. Due to the variety of services carried on the networks, different group aggregation admission policies can also be added to the mathematical model to fulfill different service requirements.

Besides, the feasibility of the lighttree approach depends obviously on the relative cost of optical OXCs, transmitters and receivers at different capability. Today, these costs are rapidly changing due to the rapid evolution of electrical and optical technologies. The resources placement in WDM networks is an important issue as well. If the related technologies get mature, different tree sharing schemes can be taken into consideration such as aggregating groups originating from different source nodes. Concatenation of two shared tree is also a possible way.

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