Optimal Energy-Efficient Routing for Wireless Sensor Networks

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Abstract

The network lifetime for wireless sensor network plays an important role to survivability. Thus, we indicate the importance of routing protocol to network lifetime, and model the expected retransmission time as a convex function with respect to aggregate flow on each sensor node. Thus we formulate the optimal energy-efficient routing as a non-linear min-max programming problem with convex product form, which can be optimally solved by optimal routing framework. Based on the optimal routing framework, we propose Lagrangean-based algorithm and primal optimal algorithm. By the combination of these two algorithms, we can optimally and efficiently get the routing assignment to maximize the network life in the sensor network. From experiments, we observe that when the optimal network lifetime increases as the number of sensor nodes increase. While the shortest path-based heuristic algorithm can only achieve about 48% network lifetime compared to our solution approach.

1. Introduction

In the wireless sensor network, the energy aware routing (EAR) protocol was presented to extend the network life time [4] [18]. The EAR requires hardware support which is the capability of knowing the battery status, e.g., how many watts the node still remains. In [3], this concept was enhanced by introducing "altruist", the node having surplus energy to forward traffic, into wireless sensor networks. Through properly exchanging battery status between neighbors, the routing policy is energy efficient and the network life time extends significantly.

Since there are only a few OD (Original Destination) pairs needed to be recorded in each node's routing table, we can compute several candidate routing paths for each OD pair. The benefit of multiple candidate routing paths was argued in [11]. We apply the features of optimal routing [14] when selecting the candidate paths and scaling traffic flow between paths until the optimality conditions are satisfied.

In this paper, we choose table-driven routing policy and apply distance vector based algorithm, e.g. distributed Bellman-Ford (DBF) algorithm. To take the advantage of the asynchronous convergence property of DBF [7], we build a routing protocol implemented with distributed fashion which is indispensable in practice for sensor networks. To apply optimal routing features on DBF, we have to define the link length as sophisticated parameters which are capable of affecting network life time as in [1] and [16].

Based the analysis of the expected retransmission time and the collision probability in [9], we model the expected retransmission time as a convex function of aggregate traffic load on the node then take it into the proposed routing algorithm. In [5], the author proposed that min-max node lifetime objective function tends to find longer path resulting to decrease average node lifetime. Our algorithm contributes to keep balance between minimum node lifetime and average node lifetime in this stage.

The sensor deployment component is for topology determination [6]. Note that the sensor network topology is non-regular and usually randomly spread as [8]. We formulate the energy efficient routing problem as a nonlinear optimization problem. To fulfill the timing and the quality of the optimal decisions, the solution approach to the mathematical problem is Lagrangean relaxation method. In the further computational experiments, our proposed routing algorithm is expected to be efficient and effective to deal with each complexity problems.

The remainder of this paper is organized as follows. In Section 2, we briefly describe the optimal energy routing problem and present the problem formulation. In Section 3, the solution approach is presented. In Section 4, illustrate algorithms to solve the optimal mathematical problem and compare with other methods with experimental results in section 5. Finally, we present our conclusions in Section 6.

2. Problem description

In sensor networks, the interference is a significant effect on communication, which will affect bit-error rate and retransmission. To get an over-estimated retransmission model, consider the pure-aloha MAC formulation which can be taken as a performance lower-bound of those MAC layers in practice:

$$h = Ge^{-2G} \tag{1}$$

here h is the throughput of the transmitter node, defined as aggregate flow divided by wireless channel capacity.

Notation G is the traffic load including retransmission for the transmitter node. Then the expected retransmission time R is:

$$R = E[retran_time] = \sum_{k=1}^{\infty} kP_s (1 - P_s)^{k-1} = e^{2G}$$
(2)

where P_s is the successful transmission probability. From the deduction in [2], the expected retransmission time in pure-Aloha system is e^{2G} . Give the deduction as following, first we apply Tyler expansion at G=0 (then R=1) on f(R)=ln(R):

$$h = \frac{\ln R}{2R} = -\frac{R}{4} + 1 - \frac{3}{4R}$$
(3)

After applying the quadratic equation formula, the expected retransmission time R is a function of throughput h:

$$\begin{cases} R(h) = 2 - 2h - \sqrt{4h^2 - 8h + 1} & \text{if } h \le 0.133\\ R(h) = R(0.133) + R'(0.133)(h - 0.133) & \text{if } h > 0.133 \end{cases}$$
(4)

Then, we develop a mathematical model to deal with the energy efficient routing problem in order to maximize the network life time in the system. Table 1 lists the given parameters and Table 2 lists the decision variables. According the above discussions, we formulate the energy efficient routing problem as a complex nonlinear programming.

Objective function:

$$Z = \max\min_{n} t_n \tag{PB1}$$

Subject to:

$$\sum_{w \in W} r_{iw} = R_i \qquad i \in V \qquad (5)$$

$$\sum_{p \in P_{w}} f_{wp} = \sum_{i \in V} r_{iw} \qquad \forall w \in W \qquad (6)$$

$$\sum_{w \in W} \sum_{p \in P_w} f_{wp} \delta_{np} = g_n \qquad \forall n \in N \qquad (7)$$

$$h_n = g_n / c_n \qquad \forall n \in N \qquad (8)$$

$$2g \leq (1-q_n)c \qquad \forall n \in N \qquad (9)$$

$$\sum_{n=1}^{\infty} \left(1 - \frac{q_n}{q_n} \right) = 0$$

$$R(n_n) = 2 - 2n_n - \sqrt{4n_n^2 - 8n_n + 1} \quad \forall n \in N, \ n_n \le 0.133 \ (10)$$

$$R(h_n) = 1.6518 + 40.1924(h_n - 0.123) \quad \forall n \in N, \ h_n \le 0.123 \ (11)$$

$$\frac{1}{g_n R(h_n)e_n + a_n(1-q_n) + d_n q_n} = t_n \qquad \forall n \in \mathbb{N}$$
(12)

$$0 \le q_n \le 1 \qquad \forall n \in N \quad (13)$$
$$f \ge 0 \qquad \forall w \in W \; n \in P \quad (14)$$

$$f_{wp}^{-} = 0 \qquad \forall w \in \{N - W\}, p \in P_w \quad (15)$$

 $r_{iw} \ge 0 \qquad \qquad \forall w \in W, i \in V \quad (16)$

$$r_{iw}^{-} = 0 \qquad \qquad \forall w \in \{N - W\}, i \in V \quad (17)$$

The objective function is to maximize the network lifetime of the given wireless sensor network configuration. The network lifetime is related to the routing policy and passive mode management, which are

 Table 1. Notation descriptions for given parameters

Notation	Descriptions	
Ν	The set of wireless sensor nodes;	
V	The set of events subscribed by the application layer	
	services in the sensor network;	
Ш/	The node set being capable of sensing subscribed	
W	events;	
D	The set of specific destination nodes;	
	The traffic flow source from subscribed event i,	
R_i	measured by Kbps (kilobytes per second), assuming	
	it is constant bit rate;	
D	The candidate paths to the destination and origin	
P_w	from node $w, w \in W$;	
E_n	The initial energy level of node <i>n</i> , measured by Watt;	
	The transmission energy required by node n to	
e_n	transmit an information unit, measured by Watt per	
	kilobyte;	
C_n	The capacity on node <i>n</i> , measured by Kbps;	
~	The energy required by node <i>n</i> to retain active mode,	
a_n	measured by Watt per second;	
d_n	The energy required by node n to retain passive	
	mode, measured by Watt per second;	
	The expected retransmit time until success. It is a	
$R(h_n)$	convex function related to the channel throughput of	
	node <i>n</i> , measured by kilobytes per second.	
8	Indicator function which is a 0-1 variable. If node n is	
Unp	in path p then set to 1, otherwise 0;	

Table 2. Notation descriptions for decision variables

Notation	Descriptions
	The traffic flow source from subscribed event <i>i</i> and
r_{iw}	sensed by node w, measured by measured by Kbps.
ſ	The traffic flow source from node w and route through
J_{wp}	path <i>p</i> , measured by measured by Kbps;
g_n	Aggregate flow on node <i>n</i> ;
h_n	Channel throughput of node <i>n</i> ; $h_n = g_n / c_n$
	The portion that node n is in passive mode of it self's
q_n	node life time. It is in [0, 1].
t_n	The time duration of node <i>n</i> to exhaust its energy;

the decision variables in our formulation. Constraint (5) ensures that the event-driven traffic can be fully dispatched to the corresponding sensors. Constraint (6) is the path-oriented routing requirement constraint. Constraint (7) calculates the aggregate flow on node n. Constraint (8) calculates the channel throughput according to the aggregate flow on node n. Constraint (9) is bandwidth constraint on wireless sensor nodes. Constraints (10) and (11) are both convex functions modeling the expected number of retransmission time related to the channel throughput of node n. Constraint (12) calculates the node lifetime concerning the aggregate traffic flow on node n, the energy consumption rate and the frequency that node n is in passive mode. Constraint (13) enforces the portion that node n is in passive mode of its node lifetime is between 0 and 1. Constraints (14)-(17) ensure the traffic flow are positive or zero.

Because the node lifetime t_n must be positive, the original objective function can be rewritten as following:

$$Z = \max\min_{n} t_n = \min\max_{n} (1/t_n)$$

$$= \min \max_{n} [g_{n}R(h_{n})e_{n} + a_{n}(1-q_{n}) + d_{n}q_{n}]/E_{n} \quad (18)$$

Also, at the optimum, the passive mode must be fully utilize to achieve the best energy-efficient. Constraint (9) is active and $q_n = 1 - 2g_n/c_n$. Thus an equivalent formulation of Problem (PB1) is:

Objective function:

$$Z = \min_{n} \frac{g_n R(h_n) e_n + (a_n - d_n) \frac{2g_n}{c_n} + d_n}{E_n} \quad (PB2)$$

Subject to: $(5) \sim (17)$ except (9), (12), and (13).

This re-formulation eliminates Constraints (9), (12), and (13) as well as decision variables q_n by merging them into the objective function and the problem becomes a single decision variable programming problem.

3. Solution approach

The optimal energy-efficient routing problem (OEERP) in wireless sensor networks is a nonlinear programming problem with convex product form. We apply Lagrangean Relaxation (LR), which has been successfully adopted to solve many famous NP-complete problems [13], to solve the optimal energy-efficient routing problem. By Lagrangean strong duality theorem, the tightest lower bound attained by Lagrangean dual problem is exactly the primal feasible objective function [15]. We also conduct a primal algorithm to get the optimal routing assignment resulting to maximize network lifetime.

We transform the maximization problem to minimization without loss of correctness.

Let
$$s = \max[g_n R(h_n)e_n + 2g_n(a_n - d_n)/c_n + d_n]/E_n$$
, then

an equivalent formulation of Problem (PB2) is: Objective function:

$$Z = \min s \tag{PB3}$$

Subject to:

$$(5) \sim (17)$$
 except (9), (12), and (13).
 $s > 0$ (19)

 $g_n R(h_n)e_n + 2g_n(a_n - d_n)/c_n + d_n \le sE_n \quad \forall n \in N$ (20) Constraint (19) ensures the equality with original problem (PB1). Constraint (20) defines the minimum node lifetime in the network. By using the LR method, the primal problem can be transformed into the following LR problem where Constraint (20) is relaxed. For a vector of non-negative Lagrangean multipliers, the LR problem is given by optimization problem as:

Objective function:

 $Z_{LR}(\alpha) = \min s +$

$$\sum_{n \in N} \alpha_n \left[g_n R(h_n) e_n + 2g_n (a_n - d_n) / c_n + d_n - sE_n \right]$$
(LR)

Subject to: $(5) \sim (19)$ except (9), (12), (13) and (18).

In this formulation, α is the vector of $\{\alpha_n\}$, which are Lagrangean multipliers and $\alpha_n \ge 0$. To solve this problem,

we can decompose the problem into the following two independent and solvable optimization Subproblems.

We add a restricted upper-bound of s in Constraint (SUB1.1) and it should not change the optimal solution value in (PB3). The meaning of s is the upper bound above the reciprocal of node lifetime, equal to the lower bound on node lifetime. Then Subproblem (SUB1) is:

Subproblem (SUB1): related with decision variable *s*. Objective function:

$$Z_{SUB1}(\alpha) = \min(1 - \sum_{n \in N} \alpha_n E_n)s$$
 (SUB1)

Subject to: $0 < s \le \overline{s}$

Sub-problem 2 (SUB2) is the bottleneck of all sub-problems. To apply standard nonlinear optimization techniques, we reformulate Problem (SUB2) as (SUB2-2) with single path-oriented decision variable x_{ip} modeling the routing assignment form subscribed event *i* to destination through path *p*. The objective function and constraints are as follow:

Subproblem (SUB2): related with decision variables r_{iw} and f_{wp} .

Objective function:

$$Z_{SUB2}(\alpha) = \min \sum_{n \in \mathbb{N}} \alpha_n [g_n R(h_n) e_n + 2g_n (a_n - d_n) / c_n + d_n] \text{ (SUB2)}$$

Subject to: (5) ~ (7), (10) ~ (11), and (14) ~ (17).

Subproblem(SUB2-2): related with decision variables x_{ip} . Objective function:

$$Z_{SUB2-2}(\alpha) = \min \sum_{n \in \mathbb{N}} \alpha_n \left[g_n R(h_n) e_n + 2g_n (a_n - d_n) / c_n + d_n \right]$$
(SUB2-2)

Subject to: (10), (11) and

$$\sum_{p \in P} x_{ip} = R_i \qquad i \in V \quad (SUB2-2.1)$$

$$\sum_{i \in V} \sum_{p \in P_w} x_{ip} \delta_{np} = g_n \qquad \forall n \in N \qquad (SUB2-2.2)$$
$$x_{in} \ge 0 \qquad \forall i \in V, p \in P \qquad (SUB2-2.3)$$

Problem (SUB2-2) is a minimum cost flow problem with a convex cost function and multi-commodities routing requirement. We solve this problem with optimal routing framework which is a variation of projection methods for convex cost routing problem [14]. According to the weak Lagrangean duality theorem [17], for any $\alpha_i \ge 0$, $Z_{LR}(\alpha_i)$ is a lower bound on Z_{IP} . The following dual problem (21) is then constructed to calculate the tightest lower bound.

$$Z_D = \max_{\alpha_i \ge 0} Z_{LR}(\alpha_i) \tag{21}$$

There are several methods to solve the dual problem, among which the subgradient method is the most popular and is employed here. Computational performance and theoretical convergence properties of the subgradient method are discussed in [12]. In this dual problem, let a vector χ be a subgradient of problem $Z_{LR}(\alpha_i)$. In iteration k of the subgradient optimization procedure, the multiplier vector π is updated by $\pi^{k+1} = \pi^k + \zeta^k \chi^k$. The step size ζ^k is determined by $\zeta^k = \zeta \cdot Z^h - Z_D(\pi_k) / \|\chi^k\|^2$, where Z^h is the

primal objective function value for a heuristic. It is an upper bound on Z_D .

We describe the solution procedure of optimal energy-efficient routing as Fig. 1 and the detailed description of each procedure is:

a. Computing first derivative length

By definition, the cost function of Subproblem (SUB2-2) that the object is to minimize the sum of reciprocal of node lifetime weighted by Lagrangean multipliers, and the FDL is the first derivation with path flow. We get the first derivative length as following:

$$\sum_{n \in N} \alpha_n \left[R(h_n) e_n + g_n R'(h_n) \frac{dh_n}{dg_n} e_n + \frac{2(a_n - d_n)}{c_n} \right]$$
(22)

Given the valuation of first derivative length as mentioned above, at every iteration we compute each node's lifetime according the current routing assignment, and recognize the bottleneck node in the network. Then update the first derivative length of all the paths. Finally we shift flow between paths based on the first derivative length computed earlier. Repeat these steps until algorithm converge.

b. Finding MFDL path

Our goal is to find the most economic path to shift positive flow iteration by iteration. By the lemmas of optimal routing framework, the most economic and energy-efficient path must with the minimum sum of Node-FDL. Thus we directly apply Dijkstra shortest path algorithm to find the MFDL path for each OD-pair iteration by iteration, where the length computed for shortest path is the node-FDL.

c. Path flow adjustment

We shift positive amount of flow form the maximum FDL path to the minimum one by applying Newton method on line search. Let β^* be the step size that minimizes $D[x + \beta (x^2 - x)]$ over all β is between 0 and 1, that is,

$$D[x + \beta^{*}(x' - x)] = \min_{\beta \in [0,1]} D[x + \beta^{*}(x' - x)]$$
(23)

The procedure above is a special case of the so-called Frank-Wolfe method for solving nonlinear programming problems with convex constraint sets.

Step 1:	Initialization . Set the iteration counter k to be 1. Pre-calculate		
	all the candidate paths of each OD-pairs and init any arbitrary		
	one of feasible routing assignment set.		
Step 2:	Stopping. If k is greater than a pre-specified counter limit then		
	stop.		
Step 3:	Computing. Update the aggregate flow on each node in the		
	network according to the current routing assignment.		
Step 4:	Updating. Compute node-FDL and path-FDL according to the		
	up-to-date aggregate flow.		
Step 5:	Shifting flow. Shift flow between paths according to		
path-FDL. Increase k by 1 and go to Step 2.			
Figure 1. An optimal energy-efficient routing			
algorithm			

4. Energy-efficient routing algorithm

The primal optimal algorithm is similar with that we use to solve Lagrangean Subproblem (SUB2). However, the procedures including computing path FDL phase, finding path phase, and flow adjustment phase are different. This difference is resulted from the min-max behavior and the differential property. We will give a detailed description of min-max Dijkstra algorithm as followings:

a. Finding minimum capacity cut fath

Because the objective function in our primal problem is a min-max function, but the general Dijkstra shortest path algorithm is not suit for finding MFDL path. Thus, we modify the Dijkstra algorithm to find such a min-max shortest path and the pseudo code is showed as Figure 2.

The Single_Source_Minimum_Cut_Paths (SSMCP) algorithm finds the path with minimum capacity cut form the given OD-pair. However, this algorithm only returns the minimum cut capacity value but does not record the path. Figure 3 shows an example of two paths with the same capacity cut.

The two paths are with the same minimum capacity cut which value is *c*. In general, we choose the right path with fewer hops as a better energy-efficient path than



Figure 2. Single_Source_Minimum_Cut_Path algorithm



the left. But the SSMCP algorithm may still possible return the left path. So based on the output of SSMCP algorithm, we apply modified Dijkstra algorithm to find the shortest and minimum cut path. Figure 4 shows the pseudo-code of Capacity_ Constrained_Shortest_Path (CCSP) is for this purpose.

b. Polya's method on path flow adjustment

The basic idea to adjust path flow as the bottleneck node lifetime of path p is the minimum in the sensor network, and then we decrease the fraction of path flow on p according to specific step sizes and add back the same amount of flow to another path with larger bottleneck node lifetime. The step of flow adjustment procedure is given as Figure 5.

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Algorithm Capacity Constraind Shortest Paths(G, s, d)
Input: G=(V, E) (a weighted directed graph), s (the source node), d
       (the destination node), and UC (the upper bound of
       constrained capacity).
Output: for destination node, d.SP is the length of the shortest path
       with the given minimum capacity cut from s to d;
       {All lengths are assumed to be nonnegative.}
begin
    for all vertices w do
         w.mark := false:
         w.SP := Infinite;
         s.SP := 0;
    end-for
    while the node d is unmarked do
         let w be an unmarked node such that w.SP is minimal;
         w.mark := true:
         for all edges (w,z) such that z is unmarked do
              if w.SP+z.length < z.SP and z.capacity < UC then
                   z.SP := w.SP + z.length;
              end-if
         end-for
    end-while
end
```

Figure 4. Capacity_Constrained_Shortest_Paths algorithm

r	
Step 1:	Initialization . Set the iteration counter k to be 1. Compute
	the path bottleneck lifetime according to any given feasible
	routing assignment.
Step 2:	Stopping. If k is greater than a pre-specified counter limit
-	then stop.
Step 3:	Finding. Find the paths with minimum and maximum
-	bottleneck lifetime.
Step 4:	Selection. Select path with minimum bottleneck lifetime
	and denote its flow f . Shift the fraction of f by a positive
	stepsize t_{c}^{k} . More precisely, we shift flow with amount of
	$f * t_f^k$ form the path with minimum bottleneck lifetime
	to the path with minimum bottleneck lifetime.
Step 5:	Computing. Calculate the bottleneck node lifetime of each
-	path.
Step 6:	Next-loop. Increase k by 1 and go to Step 2.
Figure	E. The step of flow adjustment presedure

Figure 5. The step of flow adjustment procedure

5. Experimental results

The experimentation variable δ is defined as the ratio between the edge lengths of grid area and the sensor's communication radius. The experimental result is given as Table 3. In wireless sensor networks, the communication radius is about 12.5 meters and when $\delta = 8$ the area size is 100×100 meters. We adopt the energy consumption parameter of EYES-nodes [19] in our study. For each sensor node, the parameters are as follows:

• Wireless channel capacity is 10 kbps.

- Initial battery capacity of each sensor node is between 1300 and 1600 Watts.
- Energy consumption rate on receiving (transmitting) is 0.2 Watts per byte.
- Energy consumption rate to retain in active mode (passive mode) is 50 (10) Watts per second respectively.

In our computational experiments, we generate several system scenarios with different (1) average package length and (2) sensor node density. Then we apply the primal optimal algorithm introduced in Section 3 and Section 4 to compute the maximum network lifetime.

To experiment (1) average package length, we set up two cases with different parameters. In Case 1, the area size ratio δ is 4. Here we set the number of nodes in Case 1 is 27, which is 1.5 times the minimum number of sensor nodes from Table 3. And traffic demand is set as 5 which is 0.2 times the number of sensor nodes. In Case 2, the area size ratio δ is 8, and we set up parameters according to the same logic as in Case 1. The experimental results are given in Table 4.

To experiment (2) sensor node density, we set up two cases with different parameters. In Case 3, the area size ratio δ is 4. Here traffic demand is fix at 6 and average packet length is 200 bytes. The numbers of nodes are 1.5, 2.5, 3.5, and 4.5 times the minimum number of sensor nodes satisfying 1-connectivity. So they are 27, 45, 63, and 81 respectively. In Case 4, the area size ratio δ is 8. Those parameters in both Cases are list below, and the experimental results are given in Table 5.

In Table 4, we evaluate the effect of the expected retransmission time function which is consistent with our convex function to the aggregate flow on each sensor node Table 5 shows how the connectivity and the number of sensor node affect the network lifetime. It is clear that if the number of sensor nodes increases, it is with higher probability the network topology has higher connectivity. Thus the average network lifetime increase.

Figure 6 shows the network lifetime comparison between our primal optimal algorithm (PO) and Short-based algorithm (SP), which adopt the Dijkstra algorithm. Even though the path is the shortest path from the point of view with node lifetime and energy-efficiency, it is still fragile if any one bottleneck node exhaust it battery life. While our PO algorithm use multiple paths to route traffic between every OD-pairs, and the network lifetime increase as the number of sensor nodes increases.

We measure the primal optimal algorithm with time-complexity. The |N|, |W|, and |P| denote the number of sensor nodes, the number of OD-pairs, the number of candidate paths of each OD-pair, respectively. Table 7 shows the number of operations required and time complexity of optimal routing framework for one iteration.

6. Conclusions

In this paper, we address the importance of routing protocol on energy efficiency. Our proposed LR-based algorithm can efficiently get the near-optimal solution and the primal algorithm can optimally solve the problem but spending much more time. Both of them are variations of optimal routing framework, which is to optimally solve the multi-commodities routing problem. We appraise the marginal cost of candidate paths by MFDL in LR-based algorithm and by capacity cut in primal optimal algorithm respectively. Using the quantity we adjust flow between paths for each OD-pair until the two optimal conditions are satisfied. Then we eventually achieve the optimal routing which is energy-efficient weighted load-balancing.



Figure 6. Comparing network lifetime between PO and SP

Table 3. Minimum number of sensor nodes to achieve 1-connectivity by different ratio δ

Ratio δ	4	8	12	16
Number of sensors	18	131	477	881

Table 4. Experimental results for average packet length

Packet length	Network lifetime		
	Case 1	Case 2	
100	14925	665.9	
300	4075	50.4	
500	844	17.0	

Table 5. Experimental results for sensor node number

Sensor nodes	Network lifetime		
	Case 3	Case 4	
27	23194	123	
45	23901	418	
63	28544	1035	
81	28714	1388	

Table 6. The time complexity of optimal routing framework

Optimal routing framework			
Procedure	Number of operations required	Time complexity	
Updating node flow	N + W P N	O(W P N)	
Finding MFDL path	$ W N ^2$	$O(W N ^2)$	
Computing path FDL	W P N	O(W P N)	
Flow adjustment	$ W ^{*}(P +1)$	O(W P)	

7. References

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