The Path-Based Minimum Power Broadcast Problem in Static Wireless Networks

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Abstract—**The crucial design challenge in broadcasting is how to save energy, because each individual node only has a small battery as a power source. Thus, the objective of this paper is to find the optimal radii range for each node in static wireless networks so that the total power consumption can be minimized. The problem is formulated as a minimum-power broadcast tree constructed based on paths, instead of links or nodes. Since this problem is NP-complete, we adopt Lagrangian Relaxation (LR) to decompose it and independently solve the sub-problems. The LR dual-mode problem ensures the objective lower bound value. The primal-mode problem is solved via our proposed approximation heuristic, which takes prompts from a set of LR multipliers, to obtain the upper bound's objective value. We present experimental results from randomly generated networks and show that our proposed algorithm saves more than 30%, 5%, and 10% energy compared to the Prim's minimum spanning tree (PMST), the Broadcast Incremental Power (BIP), and another proposed Greedy Incremental Broadcast Tree (GIBT) algorithms, respectively**.

I. Introduction

Many wireless techniques, such as Wi-Fi, Bluetooth, HiperLAN, and sensor networks, are currently in use. Basically, these techniques can be categorized into two types of system for wireless networks. One is the infrastructure, namely, base-station (BS) oriented, and the other is the non-infrastructure, namely, ad hoc wireless network [5]. Here we are interested in ad hoc wireless networks with adjustable power broadcasting to save total power consumption in multicasting (or broadcasting) messages, such as global information flooding, information diffusion, and query messages to a sub-region in sensor networks [13].

Our research problem differs from other studies, such as [3], [6], and [18], which explore the relationship between the power range and topology control and minimize the maximum power utilized by any node such that the resulting graph is connected. Meanwhile, some studies focus on ways to build minimum-energy networks via the shortest path algorithm, measuring the cost of the edge by its power level [3], [14], [19], and [21]. However, to send the same message to several nodes by broadcasting, specifically because wireless transmission inherently reaches several nodes with a single transmission so that all nodes within range of a relay node's transmission radius receive a packet in one broadcast transmission cycle. For example, in Fig. 1, since distance d_1 is larger than d_2, node 1 has to broadcast with power radius to d_1 in order to cover nodes 2 and 3.

In this paper we address the problem of multicasting in multiple hop networks from the viewpoint of energy efficiency. If the relay node in multiple hop wireless networks is not assigned, messages flooding wastes energy on the duplicating packets. We nominate the relay node to forward packets and construct a multicast tree, even if all nodes need to receive the broadcast packet. Our objective is to determine the minimum-power multicast tree solution. As no localized methods can approximate the minimum-energy broadcast tree within a constant factor [22], a centralized algorithm, i.e., an LR-based approach, is used to optimize our objective.

The most important aspects of this problem are (i) how to select the relay nodes and (ii) how to assign the power radii of the selected relay nodes. With regard to the selection of the relay nodes, many researchers have presented the problem as a minimum-power broadcast tree problem, which has been proven to be NP-complete [20]. Since each wireless device controls the power radius, as shown in Fig. 2, the goal is to reduce the power consumption exponentially in order to reach the area of next hop wireless devices.

Therefore, we calculate the required power range of each device to send the multicast message and minimize the total power consumption. Fig. 1 shows an example of selecting nodes 1, 2, 3, 5, and 7 and their power radii, to forward a message from source node 1 to the receiving nodes 8, 9, 10, 11, and 12. The aggregation of the relay nodes' optimal power consumption is minimized.

Some researchers have studied the relationship between energy-efficiency and multicast trees, e.g., Wieselthier et al. proposed a series of heuristics to solve this problem [10], [11], and [12]. These heuristics (e.g., BIP) belong to the pruned heuristics or greedy heuristics that change link by link or node by node. Others belong to Prim's minimum spanning tree algorithm, which converted them to gain the "multicast wireless advantage". Other enhance algorithms includes *r*-shrink [2] and EWMA [15] to execute shrink and outer post sweep procedures, which involve the monotonic decreasing and increasing function of an input tree, respectively. In other words, if there is a gain, it changes the tree structure to accommodate it; otherwise, it leaves the tree intact. In this paper, the shortest path algorithm is used to solve the sub-problem and obtain a primal feasible solution.

In [1], the authors proposed an integer programming models to judge the quality of the optimization solutions. Since the link-based mathematical formulation is complex, it

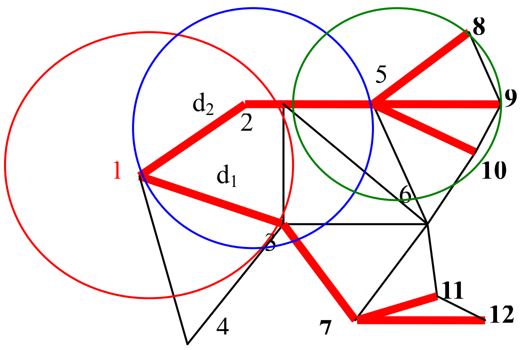

Figure 1. Minimum-energy broadcasting tree

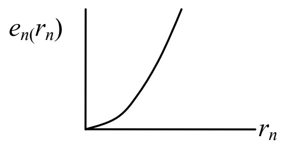

Figure 2. The exponential function of power consumption with power radius

is difficult to solve in exponential time with large scale wireless networks. Here, a path-based approach is adopted to solve the problem.

Therefore, the problem is formulated as a nonlinear optimization-based problem with only three decision variables, transmission radius, links, and paths. To fulfill the timing and the quality requirements of the optimal decisions, the LR method, which has been successfully adopted to solve many famous NP-complete problems [16], is used. As for further computational experiments, our proposed routing algorithm is expected to be effective in dealing with this complex optimization problem.

The remainder of this paper is organized as follows. In Section II, we briefly describe the minimum-power consumption, the mathematical formulation, and the proposed solutions; in Section III, the detailed procedure of the optimal energy-efficient routing algorithm is described; in Section IV, the primal-mode feasible routing algorithm for solving the optimal mathematical problem is illustrated; in Section V, we construct the experimental environment to prove that our approach approaches the optimal solution; finally, in Section VI, we present our conclusions.

II. Problem Formulation

The minimum power multicast through broadcasting problem is modeled as a graph, $G(V, L)$, where V vertices represent wireless nodes distributed on a two-dimensional plane and direct link L (e.g., nk indicates that node k is covered by n's radius). Each wireless node has an omni-directional antenna. Accordingly, we have developed a mathematical model to deal with the problem as a dynamic minimum-power consumption routing problem in order to minimize the total energy consumption for multicasting (or broadcasting) packets. Tables 1 and 2 list the given parameters and the decision variables, respectively.

TABLE 1
Notation for Given Parameter

Notation	Description
V	The set of nodes.
L	The set of links. $(nk) \in L$.
s	The specific source node.
D	The set of destination nodes.
P_{od}	The set of paths from the source (o) to destinations (d).
R	The maximum transmission range.
H_d	The maximum hop count to the destinations d.
d_{nk}	The distance between node n and k.
$\delta_{p(nk)}$	Indication function, which denote link (nk) on the path p.

TABLE 2
Notation for Decision Variables

Notation	Description
γ_n	Transmission radius of node n.
y_{nk}	1 if link (nk) is used, and 0 otherwise.
$e(\gamma_n)$	Energy consumption for node n to transmit one unit of information up to distance γ_n.
x_p	1 if path p is used, and 0 otherwise.

The equations of primal problem (IP) are listed as follows:

Objective function

$$\min \sum_{n\in V} e(\gamma_n) \quad \text{(IP)}$$

subject to

$$d_{nk} y_{nk} \le \gamma_n \qquad \forall n,k \in V \qquad (1)$$

$$\sum_{k\in V} y_{kn} \le 1 \qquad \forall n \in V \qquad (2)$$

$$\sum_{k\in V} y_{sk} \ge 1 \qquad (3)$$

$$\sum_{k\in V} y_{kd} \ge 1 \qquad \forall d \in D \qquad (4)$$

$$\sum_{p\in P_{sd}} x_p \delta_{p(nk)} \le y_{nk} \qquad \forall n,k \in V, d \in D \qquad (5)$$

$$\sum_{n,k\in V} \sum_{p\in P_{sd}} x_p \delta_{p(nk)} \le H_d \qquad \forall d \in D \qquad (6)$$

$$\sum_{p\in P_{sd}} x_p = 1 \qquad \forall d \in D \qquad (7)$$

$$y_{nk} = 0 \text{ or } 1 \qquad \forall n,k \in V \qquad (8)$$

$$x_p = 0 \text{ or } 1 \qquad \forall p \in P_{sd}, d \in D \qquad (9)$$

$$\gamma_n < R \qquad \forall n \in V. \qquad (10)$$

The objective function (IP) of this problem is to minimize the total power consumption of all relay nodes subject to:

Constraint (1): when the link (nk) is selected, the power of node n must be large enough to cover all the neighbors, k.

Constraint (2): limit the in-degree branch equal to 1 as a tree constraint. The symbol '≤' is used to fulfill the negative LR multiplier property.

Constraint (3): enforce the source node, s, has to select at least one link to send out its message.

Constraint (4): enforce at least one relay node to cover a destination.

Constraint (5): once the path, p, is selected and the link (nk) is on the path, then the decision variable, y_{nk}, must be set to 1.

Constraint (6): to limit the hop count to less than the given value, H_d. We adopt the Bellman-Ford algorithm to solve this constraint.

Constraint (7): any original-destination (OD) pair must only exist on one path.

Constraint (8) and (9): enforce the integer property of the decision variables.

Constraint (10): the selected power level is less than the given maximum power limited, which comprise in the distances between other neighbors within the maximum power range.

III. Solution Procedure

For wireless networks, the minimum-power multicast through broadcasting problem is a difficult because scalable solutions are not readily available. It is necessary, therefore, to develop heuristics to solve the problem. We adopt an LR-based approach that not only achieves the near-optimal solution, but also obtains the lower bound (LB) to quantify the minimum power consumption. In the following, we describe the paradigm of the approach and the decomposed sub-problems.

A. Lagrangian Relaxation (LR)

In the 1970s, an LR-based approach was first used to solve large scale linear programming problems [16]. In brief, it is a flexible solution strategy that permits us to exploit the fundamental structure of possible optimization problems by relaxing complicated constraints into the objective function with Lagrangian multipliers [4], [16], and [4]. Accordingly, the primal-mode problem can be transformed into a dual-mode problem. Furthermore, we can decompose complex mathematical models into stand-alone sub-problems and use a proper algorithm to optimally solve each sub-problem. By the nature of decomposition, it can effectively lesson the complexities and difficulties comparing to the origin problem.

Here, we transform the above primal problem (IP) into the following LR problem, where Constraints (1) and (5) are relaxed. For a vector of non-negative Lagrangian multipliers (i.e., μ_{nk}^1 and μ_{nkd}^2), the LR problem is given by:

$$Z_{LR}(\mu_{nk}^1, \mu_{nkd}^2) = \min \sum_{n\in V} e(\gamma_n) + \sum_{n\in V}\sum_{k\in V} \mu_{nk}^1 \left[d_{nk} y_{nk} - \gamma_n \right] + \sum_{n\in V}\sum_{k\in V}\sum_{d\in D} \mu_{nkd}^2 \left[\sum_{p\in P_{sd}} x_p \delta_{p(nk)} - y_{nk} \right] \quad \text{(LR)}$$

subject to (2), (4), (6), (7) , (8), (9), and (10).

To solve this problem, we can decompose (LR) into the following three independent and solvable sub-problems, (SUB1), (SUB2), and (SUB3).

Sub-problem (SUB1), related to decision variables γ_n and $e(\gamma_n)$.

Objective function

$$Z_{SUB1} = \min \left\{ \sum_{n\in V} (e(\gamma_n) - \gamma_n \sum_{k\in V} \mu_{nk}^1) \right\} \quad \text{(SUB1)}$$

subject to (10).

This sub-problem is related to the decision variables γ_n and can be further decomposed into $|V|$ sub-problems. Since the decision variable γ_n is the radius range, which refers to the discrete distance between neighbors, and the energy model is $e_n(\gamma_n) = \gamma_n^{\alpha} + c$ with parameter $\alpha = 2{\sim}4$ [9], we need only to choose the suitable range to all destinations for each node and optimally solve the problem.

Fig. 3 shows the convex power consumption curve of this sub-problem's objective function. We adopt a simple line search, or let the first differential equal zero to determine the minimal radius range. Accordingly, the objective values of each node are calculated and sorted. We then select all negative objective value nodes to set their radius range. If the summation of the previous selected nodes' range is smaller than the longest OD-pair, we select the minimum residual positive objective value such that the selected range is large enough to reach the furthest destination. The node with this minimum objective value is then selected to reach the destinations. Finally, (SUB1) is optimally solved and its minimum objective value is summarized as the 1st part of the dual-mode value.

Sub-problem (SUB2): (related to decision variables y_{nk})

Objective function

$$Z_{SUB2} = \min \left\{ \sum_{n\in V}\sum_{k\in V} (\mu_{nk}^1 d_{nk} - \sum_{d\in D} \mu_{nkd}^2) y_{nk} \right\} \quad \text{(SUB2)}$$

subject to (2), (3), (4), and (8).

This sub-problem is related to decision variable y_{nk}, which can be further decomposed into $|V|^2$ sub-problems. Let θ_{nk} denotes the weight of link (nk), we get

$$\theta_{nk} = \mu_{nk}^1 d_{nk} - \sum_{d\in D} \mu_{nkd}^2$$

Here, two conditions that must be satisfied to determine the value of y_{nk}:

Condition I: If $\theta_{nk} < 0$, then assign $y_{nk} = 1$.

Condition II: If $\theta_{nk} \geq 0$, then assign $y_{nk} = 0$.

With Constraints (3) and (4), we initially arrange θ_{nk} in increasing order according to their values. Then, we consider the value of y_{nk} in Conditions I and II. Thus, in Condition I, for all $\theta_{nk} < 0$ in increasing order, we assign the first $y_{nk} = 1$. As to the remaining θ_{nk}, we first check the node, n, which can have at most one incoming link. If there already exists one relay link, then we assign $y_{nk} = 0$; otherwise, we assign $y_{nk} = 1$ (i.e., with respect to Constraint (2)). In Condition II, for all $\theta_{nk} > 0$ in increasing order, we should first check for destination d, which must have one incoming link for a data packet to be delivered correctly. If there is no relay link to this destination,

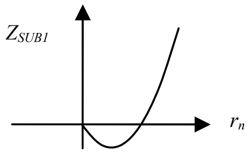

Figure 3. The objective function convex curve of the sub-problem (SUB1)

then we assign y_{nk} = 1; otherwise, we assign y_{nk} = 0 (i.e., with respect to Constraint (4)). Finally, the objective value of sub-problem (SUB2) is summarized as the 2nd part of the dual-mode value.

Sub-problem (SUB3): (related to decision variable x_p)

Objective function

$$Z_{SUB3} = \min\left\{\sum_{n\in V}\sum_{k\in V}\left(\sum_{d\in D}\left(\mu_{nkd}^2 \sum_{p\in P_{sd}} x_p \delta_{p(nk)}\right)\right)\right\} \quad \text{(SUB3)}$$

subject to (6), (7), (9).

This sub-problem is related to "to-be-determined" OD-pair, x_p, and can be further decomposed into $|D|$ sub-problems. Each sub-problem is a shortest path problem with a hop count constraint, where μ_{nkd} is the link cost and H_d is the hop constraint for each OD-pair to determine the path to reach d. This is a classic shortest path problem and can be easily solved by the Bellman-Ford algorithm. Finally, the objective value of sub-problem (SUB3) is summarized as the 3rd part of the dual-mode value.

B. The Dual-Mode Problem and the Subgradient Method

According to the weak Lagrangian duality theorem [16], for any $\mu_{nk}^1, \mu_{nkd}^2 \geq 0$, Z_D is lower bound of Z_{IP}. The following dual-mode problem (D) is then constructed to calculate the tightest lower bound.

Dual-mode Problem (D) is

$$Z_D = \max Z_D\left(\mu_{nk}^1, \mu_{nkd}^2\right) \quad \text{(D)}$$

subject to

$$\mu_{nk}^1, \mu_{nkd}^2 \geq 0 . \quad (11)$$

There are several methods for solving the dual-mode problem (D). One of the most popular is subgradient method[7]. Let a path vector, p, be a subgradient of $Z_D\left(\mu_{nk}^1, \mu_{nkd}^2\right)$. Then, in iteration k of the subgradient optimization procedure, the multiplier vector $\pi^k = (\mu_{nk}^1, \mu_{nkd}^2)$ is updated by $\pi^{k+1} = \pi^k + t^k g^k$. The step size, t^k, is determined by $t^k = \delta \cdot (Z_{IP}^h - Z_D(\pi^k)) / \left\|g^k\right\|^2$, where Z_{IP}^h is the primal-mode objective function value for a heuristic solution (an upper bound on Z_{IP}), and δ is a constant, $0 < \delta \leq 2$.

IV. Obtaining a Primal Feasible Solution

The primal-mode feasible solution is an upper bound (UB) of the problem (IP), while the Lagrangian dual-mode solution guarantees the lower bound (LB). Iteratively, both solving Lagrangian dual-mode problem and obtaining primal-mode feasible solutions, we get the LB and UB, respectively. The gap between LB and UB, computed by (UB - LB) / LB * 100%, illustrates the optimality of the problem solution.

The composition of power radius set $\{\gamma_n\}$, which represents how far a signal can reach, is large and difficult to solve. The value of y_{nk} is also hard to solve because it oscillates between 0 and 1. Thus, the decision variable x_p is the best choice to find the primal-mode feasible solutions, because once $\{x_p\}$ is determined, the decision variables y_{nk} and γ_n are also determined. We have developed a heuristic for routing policy adjustment based on $\{x_p\}$. First, we adjust the arc weight $c_{nk} = \mu_{nk}^1 d_{nk}^2$ to ensure that each OD pair perceives the same arc weight on the same link. In other words, the transmission graph will be a multicast tree, which meets the requirement of Constraint (2). Then, we run the Bellman-Ford algorithm to get the solution set of $\{x_p\}$. However, this set may duplicate relay nodes, which would lead to higher power consumption. Thus, we also adopt the *sweep*() [12] to adjust the previous father nodes to a common one and turn off or reduce the others' radius range.

Fig. 4 shows the procedure of obtaining primal-mode feasible solution to solve this problem. Steps 1 and 2 initial find out the multicast tree hint from LR multipliers. Step 3 runs the *sweep*() to adjust multicast tree. Fig. 5 shows an example of the trend line for getting the primal problem solution values (UB) and dual-mode problem values (LB). The UB curve tends to decrease to get the minimum feasible solution. In contrast, the LB curve tends to increase and cove-

Step 1 We use the shortest path algorithm (SPA) to find the initial primal UB value.

Step 2 Adjust the arc weight $c_{nk} = \mu_{nk}^1 d_{nk}^2$ and run the Bellman-Ford algorithm to get the solution set of $\{x_p\}$.

Step 3 Once $\{x_p\}$ is determined, y_{nk} and γ_n are also determined. We map each value of γ_n from the path and set R.

Step 4 Now that we have a multicast tree, we check Constraint (2) of the multicast tree, which get from **Steps 2** and **3**. For each node n, we execute the *sweep*() [12] to adjust the multicast tree.

Step 5 Iteratively execute the **Steps 2~4** with LR multipliers, which are updated from dual-mode problem.

Figure 4. The heuristic for obtaining a primal solution and a routing policy adjustment

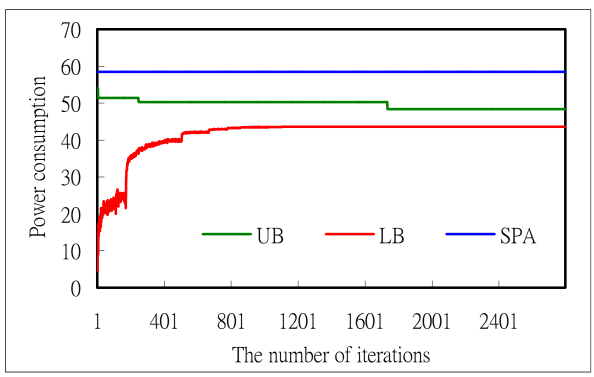

Figure 5. A sample of LR execution results (The number of nodes is 20, which random deploy within 10 * 10 area, and the maximum radius range is 4. The duality gap is about 8.79%)

rge rapidly to reach the optimal solution. The LR-based method ensures the optimization results between the UB and LB, so we keep the gap as small as possible in order to enhance our solution quality and achieve near optimization.

V. Experimental Results

We evaluate our proposed heuristics and compare them with the MSPT (Minimum Shortest Path Tree), PMST, and BIP [12] algorithms. The BIP algorithm is similar in principle to Prim's algorithm for the formation of MSPTs, in the sense that new nodes are added to the tree once at a time (on a minimum-cost basis) until all nodes are included in the tree. We also compare to our another proposed GIBT algorithm (refer to APPENDIX A), which finds the dominant set T to contain the nodes on the shortest path first and then iteratively find the shortest path from all other destinations to reach any one of the current dominant set T. We distribute V nodes in a uniformly random fashion over a field of size 10 * 10. Each node has a maximum transmission range $R = 5$. For a uniformly random deployment, the network connectivity is only a function of the average number of neighbors of a node.

In this paper, the following three experiments are evaluated: (i) varying the numbers of the nodes; (ii) varying the maximum power radius or parameter R; and (iii) varying the number of OD-pairs. In (i), the experimental variable varies the total number of nodes V, i.e., their densities, within the same transmission area. In (ii), the experimental variable varies the maximum power radius R, which affects the number of out-degrees of each node. All nodes of these two experiments are set as destinations. In other words, all nodes must receive the broadcast message from a specific source. In (iii), the experimental variable varies the number of OD-pairs, which means not all nodes have to receive multicast messages.

Fig. 6 shows the experimental results of (i) for the MSPT, PMST, GIBT, BIP, and EWMA heuristics. The gap of the LR-UB (solved by our proposed primal-mode feasible solution) and LR-LB (solved by our dual-mode problem) is less than 30%, which is caused by duality gap because Constraint (1) is relaxed. The LR-based heuristic is better than other heuristics, such as BIP, by at least 5%.

Fig. 7 shows the experimental results of (ii) to compare the gap between the UB and the LB. When the radius range increases, the gap decreases. In this experiment the optimal maximum radius range is between 3~5 in static networks.

Fig. 8 shows the experimental results of (iii) to compare with the MSPT, PMST, GIBT, BIP, and EWMA heuristics. When the number of OD-pair increases, the total power consumption increases. But our approach is more efficient than other heuristics and achieves more than 5% improvement in power consumption. The gap is decrease to 5% to ensure the approach achieve near optimization when the number of OD-pairs is increase. Although the second proposed GIBT algorithm is not better than the BIP algorithms, the solution results keep in 10%.

VI. Conclusions

Dynamic power control is one of most important ways to reduce energy consumption. We have constructed a path-based minimum-energy multicast tree problem based on a mathematical formulation that differs from other link-based or node-based approaches. In this paper, the problem is solved by an LR approach to obtain the LB and UB of objective function, thus ensuring the optimal solution value remains within bounds. The experimental results show that the gap is small, which means our proposal heuristic achieves the near optimization. Our heuristic is also better than other heuristics, such as MSPT, PMST, BIP, and GIBT. Specifically, it improves energy consumption, which it does so by approximately 5% over BIP.

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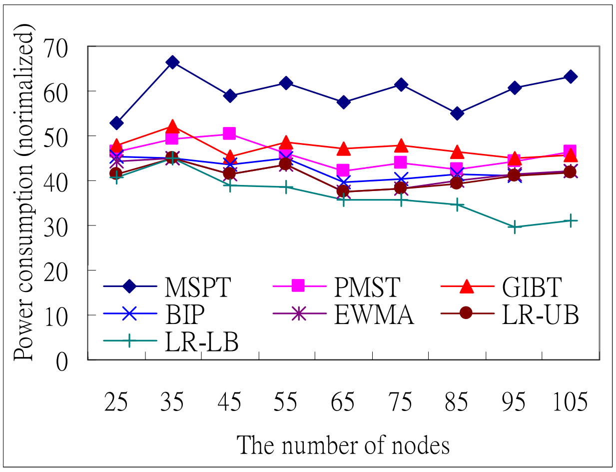

Figure 6. The experimental results of Case (i)

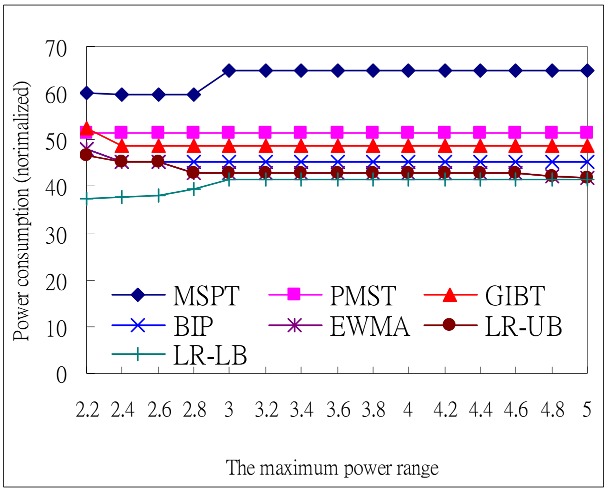

Figure 7. The experimental results of Case (ii)

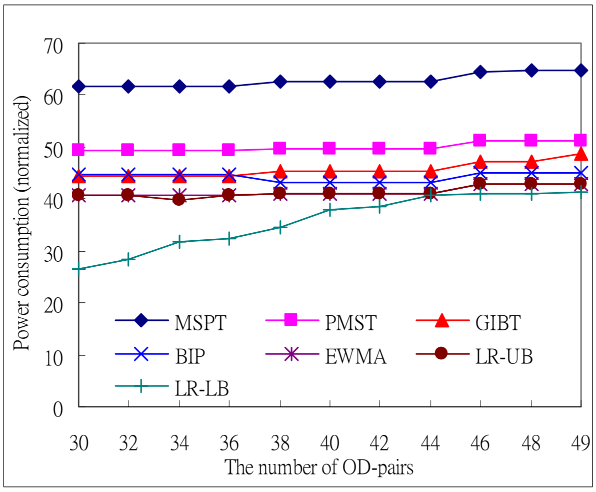

Figure 8. The experimental results of Case (iii)

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APPENDIX A: PSEUDO CODE FOR THE GIBT ALGORITHM

The following pseudo code describes the proposed algorithm GIBT (Greedy Incremental Broadcast Tree), which is implemented in the same manner as the straightforward version of the Dijstra's shortest path algorithm, with the destination nodes to any dominant set nodes of in progress broadcasting tree, as indicated in Fig. 9.

```
Algorithm GBIT (G,, s, D)
Input: G = (V, L) (a weighted directed graph where v ∈ V, and (u, v) ∈ L), s
        (the source node), and D (the set of destinations).
Output: The broadcasting tree T[v], which recorded the previous node of each
        relay node v and destination d, source from the specific s; v.SP denote
        as the shortest path cost from destination d to the current broadcasting
        sub-tree.
        {All link cost (u, v) are assumed to be nonnegative.}
begin
  for all vertices v do
    T[v] := -1;
    v.mark := FALSE;
    v.SP := INFINITE;
  end-for
  T[s] := -2;
  s.mark := TRUE;
  s.cost := 0;
  while all node d is unmarked do
    w := d;
    while the current shortest cost node u, which is unmarked, do
      T[u] := w;
      u.mark ;= TRUE;
      for all edges (u, v) such that v is unmarked do
        if u.SP + cost (u, v) < v.SP then
          v.SP := u.SP +cost (u, v);
          T[v] := u;
        end-if
      end-for
      w := u;
    end-while
  end-while
end.
```

Figure 9. The pseudo code for the GIBT