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Energy-Efficient Sensor Network Design Subject to Complete Coverage and Discrimination Constraints

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Abstract*— In this paper, we develop a novel algorithm to deploy an energy-efficient wireless sensor network (WSN) for the target-location service that is one of the major applications of sensor networks. Such sensor network has to be designed to achieve complete coverage quality of surveillance and full discrimination for predetermined resolution on the sensor field. To extend lifetime of sensor network, the duplicate deployment approach is a simple and intuitive way. However, such approach will result in increasing too much deployment cost. In this paper, we propose a novel strategy to cope with the problem. We consider deploying K independent sets of sensors to monitoring the area in turn and locating the intruder together. With this strategy, the duty cycle of each sensor is only $1/K$ and the lifetime of the sensor network will be extended up to K times. Such sensor placement problem is a variant of the set K-cover problem, which is NP-complete. We formulate the problem as a 0/1 integer-programming problem. A Lagrangean relaxation based heuristic then is proposed for solving the optimization problem. The experimental results show that the proposed strategy gets a significant improvement in the lifetime of sensor network compared to the duplicate deployment approach under the deployment cost constraint. The proposed algorithm is highly effective in terms of the overall deployment cost. Furthermore, the algorithm is very efficient and scalable in terms of the solution time.

I. Introduction

The rapid growth in sensor technology and wireless communication has led to the development of wireless sensor networks (WSNs). A wireless sensor network comprises several sensors, sink nodes, and back-end systems. These tiny, low-cost, and low power sensors are deployed in an ad hoc manner in an interested area. These sensors collect physical information from the area, process and forward the information to the sink nodes. Afterward, the back-ends can obtain global views according to the information provided by the sink nodes [2], [3].

Both sensor deployment and energy conservation are key issues of WSNs [2]. A sensor network can be deployed in two ways: random or controlled placement [12]. When the environment is unknown, dangerous, or inhospitable, sensors cannot be deployed manually. The placement may be relied on aircrafts, cannons, and so on. By this way, the sensor nodes will be randomly placed on the sensor field. On the contrary, if the terrain of sensor field is predetermined, we can adopt the controlled approach that deploys sensors by carefully planning to meet a certain quality of service requirement, for example, surveillance [6], [7], target positioning [4], [5], [11], and target tracking. Obviously, to achieve the same quality of service requirement, the random placement approach wastes more resources than the controlled placement approach.

On the other hand, due to cost and environment concerns, the battery of sensor is not always rechargeable – particularly when the network operates in inhospitable or hostile fields. Once the sensors energy exhaust, the sensors fail to perform their jobs, it will result in the degradation of quality of surveillance on the sensor network. Therefore, how to design an energy-efficient sensor network is really a major challenge. For energy conservation, the sensors are designed to have active and sleeping states [13], [14]. Generally, the power consumption of sleeping sensors can be neglected. Hence, several papers explore the design of good sensor sleep schedules to provide network coverage [10]. Previous papers consider energy conservation issue in post-deployment phase [1], [15]. These papers propose many heuristics that select mutually exclusive sets of sensor nodes from a randomly deployed sensor network to prolong the lifetime of sensor network with coverage constraint.

In this paper, we focus on the problem of constructing an energy-efficient sensor network for target-positioning services using the controlled placement approach. The design goals are to achieve target positioning as well as to prolong sensor network lifetime. To support positioning functionality, the sensor field must be completely covered, is called complete surveillance, and each unit on the field must be distinguishable. It requires deploying more sensors than to support surveillance functionality. However, it is not necessary to keep all sensors in active to provide the target-positioning service if intrusion events occur infrequently. Actually, the surveillance service is enough as no intruders on the sensor field.

Therefore, we motivate to propose a novel strategy to cope with the problem. We deploy K independent sets of sensors to support positioning service on a sensor field. Each set, which is called a cover [1], [15], can provide complete coverage on the field. Each cover is activated in turn to monitor the field if there are no intrusions. The rest of the covers are inactive and

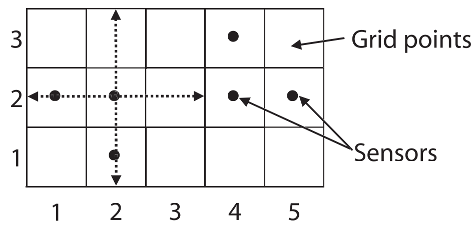

Fig. 1. A complete coverage/discrimination sensor field with 3 by 5 grids. (The detection radius of sensor is 1.)

TABLE I
THE WORKING MODES ON THREE STATES FOR THE SENSOR NODE. THE RADIO IS A DOMINANT POWER CONSUMER.

Power states of sensor node	Components of sensor node		
	Processor	Sensor	Radio
Active	active	on	Tx/Rx
Monitor	idle	on	Rx
Sleep	off	off	off

operate at the sleeping mode. As soon as the intrusion event occurs, all sets of sensors are activated and work together to locate the intruder. With this strategy, the duty cycle of each sensor is only $1/K$ and the lifetime of sensor can be effectively prolonged up to K times.

In this paper, we formulate the problem as a 0/1 integer programming problem where the objective function is the minimization of the total deployment cost subject to complete coverage and discrimination constraints under a given amount of cover K. The problem is a variant of the set K-cover problem and thus is NP-complete [1], [15]. In the proposed solution procedure, a Lagrangean relaxation based heuristic is developed to solve the optimization problem [8], [9].

From papers review, we find that this study differs from prior works in several points. First, we consider both the energy conservation and lifetime extending during the sensor deployment phase for target positioning. Second, we present a mathematical model to describe the optimization problem. This formulation can be used for an integer programming package for optimally solving a small-scale problem, and can be used to facilitate the development of an efficient and effective heuristic algorithm for solving large-scale problems. Third, a Lagrangean relaxation based heuristic is proposed to solve the problem. Finally, the relationship between the deployment cost and the maximum extension of system lifetime is investigated.

The rest of this paper is organized as follows. The problem and mathematic model then are described in sections II and III, respectively. Additionally, the solution procedure is presented in section IV. Furthermore, the computational results are discussed in section V, and conclusions are presented in section VI.

II. PROBLEM DESCRIPTION

A. Sensor Placement

In this paper, we use the grid-based placement approach to construct WSNs [4], [5], [6], [7], [11]. The sensor field can be represented as a collection of two-dimensional grid points and sensors are placed on candidate grid points, as illustrated in Fig. 1. The positioning resolution of system determines the granularity of grid point. The distance between two adjacent grid points is adopted as a length unit. Fig. 1 illustrates a sensor field with 5 by 3 grid points. On the field, there are six sensors place on grid points with coordinates (1, 2), (2, 1), (2, 2), (4, 2), (4, 3), and (5, 2).

This study assumes that the sensor detection model is 0/1 model [4], [5], [11]. The coverage is assumed to be complete (1) if the distance between the grid point and the sensor is no more than the detection radius of the sensor. Otherwise, the coverage is assumed to be incomplete (0). For example, the radius of the sensors illustrated in Fig. 1 is assigned to one. It is a homogeneous sensor network. Therefore, the sensor which is situated at (4, 3) covers grid points (4, 3), (3, 3), (5, 3), and (4, 2). Sensor locating at (2, 2) can cover grid points (1, 2), (2, 1), (2, 2), (2, 3), and (3, 2). If each grid point in a sensor field can be detected by at least one sensor, the field is called completely covered sensor field. The sensor network illustrated in Fig. 1 has complete coverage.

To locate an intruder, we define a unique power vector for each grid point. The power vector of a grid point is constructed according to the deployment of sensors. If a sensor covers the grid point, constituent of the power vector of the grid point, which is corresponding to the sensor, is set to 1, otherwise 0. For example, as illustrated in Fig. 1, the power vectors of grid point (1, 3) and (3, 2) are <1, 0, 0, 0, 0, 0> and <0, 0, 1, 1, 0, 0> corresponding to sensors at (1, 2), (2, 1), (2, 2), (4, 2), (4, 3), and (5, 2), respectively. The power vector for each grid point on the sensor field is stored on the database of the back-end systems. Once an intruder is detected, the sensors have to report the information to the sink nodes. According to the received information, the back-end can obtain a specific power vector to determine the position of the intruder. If each grid point has a unique power vector on a sensor field, the sensor field is completely discriminated. The sensor field in Fig. 1 is completely covered/discriminated by the sensor network, which can provide surveillance and target-positioning services.

B. Energy-Efficient Sensor Networks

The duplicate deployment approach is a simple and intuitive way to extend network lifetime in sensor deployment phase. With this approach, we can deploy K duplicate sensor networks on a sensor field. Each of them can provide surveillance and target-positioning services. These sensor networks operate in relay fashion and therefore can provide up to K times service time. However, the total cost is increased by K times accordingly.

In this section, we propose a novel energy-efficient strategy to cope with the problem. We deploy a sensor network such that it includes K independent covers to support positioning service on a sensor field. Each cover can only provide

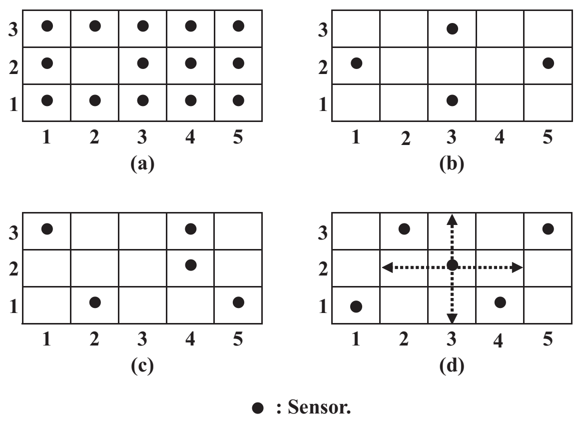

Fig. 2. An energy-efficient deployment on a 3 by 5 sensor field with 3 covers: (a) overall placement. (b) Cover 1. (c) Cover 2. (d) Cover 3. (The detection radius of each sensor is 1.)

complete coverage functionality on the field. Therefore, the amount of resource requirement for the proposed strategy is less than that for the duplicate deployment approach. From the network's viewpoint, sensors operate in two states: the surveillance and positioning states. When intrusions do not exist, the network operates in the surveillance state. Each cover is activated in turn to monitor the field. The rest of covers (i.e., $K-1$ covers) are inactive and operate at the sleeping mode. Once the intrusion event occurs, the network will transit to the positioning state. All of the sensors are activated and work together to locate intruder. At the period, all of the sensors on the network operate in active state.

As discussion above, a sensor node can operate in three power states [14]: active, monitor, and sleep states. Table I presents the working modes of the sensor's components corresponding to these power states of a sensor node. The main power consumption for a sensor node contains three domains: sensing, data processing, and communication. The communication depletes much more energy than the sensing and the processing, so the radio transceiver is a dominant power consumer in a sensor node. Hence, the energy consumption for sensing device and processor can be neglected. Moreover, power consumption of radio components at the off mode is much less than that for transmitting/receiving mode (typically, in one to four orders of magnitude) [13]. When the network operates in surveillance, sensor nodes operate in the monitor mode will consume more power than sensors operate in the sleeping mode. Consequentially, the power consumption for the sleeping mode can be neglected. With this strategy, if intrusion events occur infrequently, the duty cycle of each sensor is only $1/K$ and the lifetime of sensor can be effectively prolonged up to K times.

For example, if the lifetime of sensor network illustrated in Fig. 1 is prolonged by three times using duplicate deployment approach, the number of sensors will be increased to 18. Our algorithm required only 14 sensors in the same field and can prolong sensor network lifetime by three times. The whole

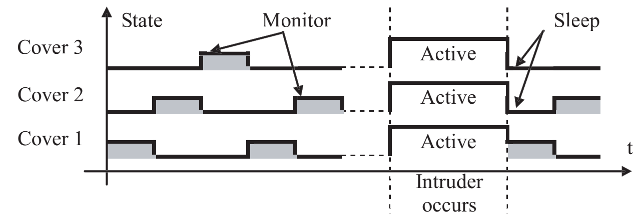

Fig. 3. The schedule of each cover in Fig. 2.

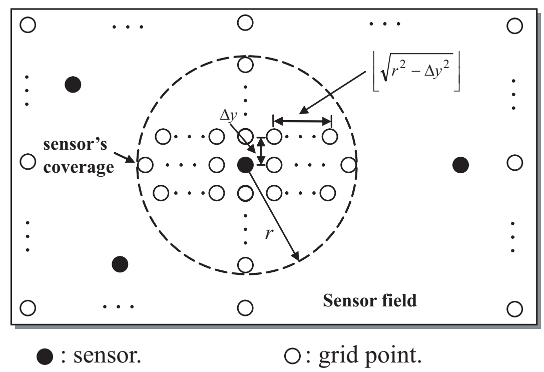

Fig. 4. Sensor and its coverage.

sensor network is composed of three covers (as illustrated in Fig. 2), each of which provides complete coverage. The duty-cycle of each cover is illustrated in Fig. 3. The average duty-cycle is reduced to $1/3$ such that the lifetime of network is extended. Obviously the proposed algorithm provides an economical solution to deploy an energy-efficient sensor network.

Afterward this study discusses the possible number of covers in a sensor network. First, we investigate the number of covering grid points of sensor with a specific detection radius. Then, Lemma 1 and 2 are obtained.

Lemma 1: Suppose a sensor has detection radius r, then the number of covering grids, G_r, for the sensor in an infinite sensor field can be represented as

$$G_r = 2r + 1 + 2\sum_{\Delta y=1}^{r}(2\lfloor\sqrt{r^2-\Delta y^2}\rfloor + 1)$$

, as shown in Fig. 4, where Δy is the distance from the sensor to a grid point in y axis.

Lemma 2: A grid point can be covered by a set of sensors. The maximum cardinality of the set exactly equals the number of covering grid points of a sensor that is allocated in the grid point.

For example, in an infinitive field, a sensor with detection radius 1 can cover 5 grid points at most. At the same time,

TABLE II
THE THEORETIC UPPER BOUND ON THE NUMBER OF COVERS IN THE 10 X 10 SENSOR FIELD.

r	1	2	3	4	5	6	7
Upper bound (U_r)	3	6	11	17	26	35	45

each grid point on the field can be covered by 5 sensors at most.

Generally, it is impractical to use an infinite sensor field. This study focuses on the case of the rectangular and finite sensor field. For a finite sensor field, an upper bound on the number of covers is determined by critical grid points in the field. A critical grid point is one that is covered by a sensor set with the smallest cardinality.

Lemma 3: On a rectangular sensor field with a finite area, the critical grid points are located at the corner of the field. Therefore, the upper bound of the number of covers, U_r, is

$$U_r = 2r + 1 + \sum_{\Delta y=1}^{r} \lfloor \sqrt{r^2 - \Delta y^2} \rfloor$$

, where r represents the detection radius of the sensor and Δy is the distance from the sensor to a grid point in y axis.

In a sensor field with 3 by 5 grid points, as illustrated in Fig. 2, the radius of sensor is assumed to be one, and the critical grid points are (1, 1), (1, 3), (5, 1), and (5, 3). According to Lemma 3, the upper bound of the number of covers, U_1, is 3. In Fig. 2, grid point (1, 3) can only be covered by the sensors placed in (1, 3), (1, 2), and (2, 3). Meanwhile, the corner grid points are all covered by 3 sensors maximally. In this case, the sensor network can be partitioned into the maximum number of covers, 3 covers. Clearly, as shown in Fig. 2, we can deploy three covers, using the minimum number of sensors, 14 sensors, for the 3 by 5 sensor field.

To achieve complete discrimination, the sensor radius must be smaller than a half of the diameter of the sensor field. Therefore, we vary the radius (from 1 to 7) to compute the theoretic upper bound of the number of covers U_r in a 10 by 10 sensor field. Table II shows the theoretic upper bound on the number of covers in the field. Theoretically, if radius is 7, we can deploy a sensor network with 45 covers such that its lifetime can be extended by 45 times.

The solution space of the problem is $O((K+1)^m)$. When field size, m, and the number of covers, K, increase gradually, the solution space increases rapidly. Hence, it is necessary to develop an efficient algorithm for this problem.

III. PROBLEM FORMULATION

The notations used to model the problem are listed as follows.

Given Parameters:

- $A = \{1, 2, ..., m\}$: The set of the indexes for candidate locations where sensor can be allocated.
- $B = \{1, 2, ..., n\}$: The set of the indexes for grid points that can be covered and located by the sensor network, $m \leq n$.
- K : The number of covers required for the sensor network.
- a_{ij} : Indicator which is 1 if grid point i can be covered by sensor j , and 0 otherwise.
- c_j : Cost function of sensor j.

Decision Variables:

- x_{jk} : 1 if sensor j is designated to cover k of the sensor network, and 0 otherwise.
- y_j : Sensor allocation decision variable, which is 1 if sensor j is allocated in the sensor network and 0 otherwise.

Problem (IP):

$$Z_{IP} = min \sum_{j=1}^{m} \sum_{k=1}^{K} c_j x_{jk} \qquad (IP)$$

$s.t.:$

$$\sum_{j=1}^{m} a_{ij} x_{jk} \geq 1 \qquad \forall i \in B,\ 1 \leq k \leq K \qquad (1)$$

$$\sum_{k=1}^{K} x_{jk} \leq 1 \qquad \forall j \in A \qquad (2)$$

$$y_j = \sum_{k=1}^{K} x_{jk} \qquad \forall j \in A \qquad (3)$$

$$\sum_{j=1}^{m} (a_{ij} - a_{\ell j})^2 y_j \geq 1 \quad \forall i, \ell \in B, i \neq \ell \qquad (4)$$

$$x_{jk} = 0 \ \ or \ \ 1 \qquad \forall j \in A,\ 1 \leq k \leq K \qquad (5)$$

$$y_j = 0 \ \ or \ \ 1 \qquad \forall j \in A. \qquad (6)$$

Physical meanings of the objective function and constraints are briefly described as follows. Problem (IP) presents that the objective is to minimize the total cost of sensors. Constraint (1) requires that each grid point must be covered in every cover of the sensor network. Constraints (2) and (3) ensure that each sensor only belongs to one cover of the sensor network. The discrimination constraint is $\sum_{j=1}^{m} (a_{ij} y_j - a_{\ell j} y_j)^2 \geq 1$ that requires the Hamming distance between each pair of grid points in the sensor network must be greater than one. And the discrimination constraint can be rewritten as Constraint (4). Constraints (5) and (6) require integer property of the decision variables with respect to x_{jk} and y_j.

IV. SOLUTION PROCEDURE

A. Lagrangean Relaxation

This section presents the algorithm for solving the proposed sensor placement problem. An approach based upon Lagrangean relaxation is considered. Lagrangean relaxation is a method for obtaining lower bounds (for minimization problems) as well as good primal solutions in integer programming problems [8], [9]. A Lagrangean relaxation is obtained by identifying in the primal problem a set of complicated constraints whose removal will simplify the solution of the primal problem. Each of the complicated constraints is multiplied by a multiplier and added to the objective function. This mechanism is known as dualizing the complicating constraints.

Using the Lagrangean relaxation, this investigation chooses to dualize Constraints (1), (3), and (4), and establishes the following Lagrangean relaxation problem.

Problem (LR):

$$
\begin{aligned}
&Z_D\ (u^1, u^2, u^3)\\
&\quad = min\{\sum_{j=1}^{m}\sum_{k=1}^{K} c_j x_{jk} + \sum_{i=1}^{n}\sum_{k=1}^{K} u_{ik}^1 (1 - \sum_{j=1}^{m} a_{ij} x_{jk})\\
&\quad + \sum_{j=1}^{m} u_j^2 (y_j - \sum_{k=1}^{K} x_{jk})\\
&\quad + \sum_{i=1}^{n}\sum_{\ell=1,\ell\neq i}^{n} u_{i\ell}^3 (1 - \sum_{j=1}^{m} (a_{ij} - a_{\ell j})^2 y_j)\} \qquad (LR)\\
&s\ .\ t.:\\
&\quad \sum_{k=1}^{K} x_{jk} \le 1 \qquad \forall j \in A \qquad (2)\\
&\quad x_{jk} = 0\ \ or\ \ 1 \qquad \forall j \in A,\ 1 \le k \le K \quad (5)\\
&\quad y_j = 0\ \ or\ \ 1 \qquad \forall j \in A. \qquad (6)
\end{aligned}
$$

The multipliers u^1, u^2, and u^3 are the vectors of $\{u_{ik}^1\}$, $\{u_j^2\}$, and $\{u_{i\ell}^3\}$, respectively. Notably, Constraints (1) and (4) are dualized such that the corresponding multipliers u^1 and u^3 are nonnegative.

(LR) can be decomposed into two independent and easily solvable subproblems, where only the decision variables x_{jk} are involved in the first subproblem and only the decision variables y_j are involved in the second subproblem. Note that, the constant terms, $\sum_{i=1}^{n}\sum_{k=1}^{K} u_{ik}^1$ and $\sum_{i=1}^{n}\sum_{\ell=1,\ell\neq i}^{n} u_{i\ell}^3$, were omitted from the objective function in the subproblems.

Subproblem 1: for x_{jk}

$$
\begin{aligned}
&Z_{sub1}(u^1, u^2)\\
&= min\{\sum_{j=1}^{m}\sum_{k=1}^{K} ((c_j - u_j^2) - \sum_{i=1}^{n} u_{ik}^1 a_{ij}) x_{jk}\} \quad (sub1)\\
&s\ .\ t.:\\
&\quad \sum_{k=1}^{K} x_{jk} \le 1 \qquad \forall j \in A \qquad (2)\\
&\quad x_{jk} = 0\ \ or\ \ 1 \qquad \forall j \in A,\ 1 \le k \le K. \quad (5)
\end{aligned}
$$

Subproblem 2: for y_j

$$
\begin{aligned}
&Z_{sub2}(u^2, u^3)\\
&= min \sum_{j=1}^{m} (u_j^2 - \sum_{i=1}^{n}\sum_{\ell=1,\ell\neq i}^{n} u_{i\ell}^3 (a_{ij} - a_{\ell j})^2) y_j\ (sub2)\\
&s\ .\ t.:\\
&\quad y_j = 0\ \ or\ \ 1 \qquad \forall j \in A. \qquad (6)
\end{aligned}
$$

Subproblem 1 comprises $|A|$ (one for each sensor) problems. To simplify descriptions of the procedures for solving Subproblem 1, p_{jk} is used to represent the following function:

$$p_{jk} = (c_j - u_j^2) - \textstyle\sum_{i=1}^{n} u_{ik}^1 a_{ij}.$$

For each sensor, first we assume sensor j is allocated and p_{jk} is calculated for each cover k. Then the minimal p_{jk} of sensor j in all cover ($min \quad p_{jk}$) is determined and the corresponding cover number k' can be obtained. If the minimal p_{jk} in all cover is negative, we assign $x_{jk'}$ to one. It means that sensor j is designated and belongs to cover k'. Otherwise, $x_{jk},\ \ 1 \le k \le K$, are assigned to 0.

Subproblem 2 also comprises $|A|$ problems. Let q_j be the coefficient of y_j in (sub2).

$$q_j = u_j^2 - \textstyle\sum_{i=1}^{n}\sum_{\ell=1,\ell\neq i}^{n} u_{i\ell}^3 (a_{ij} - a_{\ell j})^2.$$

For each sensor j, if q_j is negative, we assign y_j to one. Otherwise, let y_j be zero.

For any $(u^1, u^3) \ge 0$, using the weak Lagrangean duality theorem, the optimal objective function value of (LR), $Z_D(u^1, u^2, u^3)$, is a lower bound on Z_{IP} [8], [9]. The dual problem then is

$$Z_D = max_{(u^1,u^3)\ge 0} Z_D(u^1, u^2, u^3). \qquad (D1)$$

$(D1)$ is solved to find the highest lower bound. Several methods exist for solving the dual problem $(D1)$. One of the most popular methods is the subgradient method [8], [9]. Let a $(|B|*K+|A|+|B|*|B|)$ vector b represent a subgradient of $Z_D(u^1, u^2, u^3)$. In iteration t of the subgradient optimization procedure, the multiplier vector π is updated by

$$\pi^{t+1} = \pi^t + \xi^t b^t.$$

The step size ξ^t is determined by

$$\xi^t = \tfrac{\lambda(Z_{IP}^* - Z_D(\pi^t))}{\|b^t\|^2}$$

, where Z_{IP}^* represents an upper bound on the primal objective function value, obtained by applying a heuristic to (IP), and λ is a scalar satisfying $0 \le \lambda \le 2$.

B. Getting Primal Feasible Solutions

After optimally solving each Lagrangean relaxation problem, a set of decision variables can be found. Since some of the constraints are relaxed, the solutions are infeasible for the primal problem. However, efficient heuristic algorithms must be developed to adjust the optimal dual solutions. A set of feasible solutions of the primal problem (IP) then can be obtained. With increasing number of iterations, the better primal feasible solution is an upper bound (UB) of the problem (IP), while the Lagrangean dual problem provides the lower bound (LB) of the problem (IP).

In this section, we develop a heuristic for obtaining primal feasible solutions. The algorithm is shown as follows.

Step 1: Check Constraint (3) for each sensor. If $y_j = 1$ and $\sum_{k=1}^{K} x_{jk} = 0$ for sensor j, sensor j is added to cover k' such that $p_{jk'}$ is the minimum of p_{jk} for all covers on sensor j. If $y_j = 0$ and $\sum_{k=1}^{K} x_{jk} = 1$ for sensor j, then let y_j be one, and the sensor j is allocated.

Step 2: For each cover k of the sensor network, check coverage Constraint (1). If the coverage constraint is violated, then addition procedure or exchange procedure is repeated until the coverage constraint is satisfied. Then, we try to drop sensors that are redundant in terms of coverage constraint.

Step 3: Check the discrimination Constraint (4) for the whole sensor network. If Constraint (4) is violated, a lot of sensors are added to achieve the completely discriminated sensor network. Afterwards, this algorithm attempts to drop some sensors that are redundant for coverage and discrimination constraints.

V. Computational Results

To evaluate the performance of the proposed algorithm, we conduct a serial of experiments. The performance is assessed in terms of lifetime of sensor network, deployment cost, and computation time.

A. Scenario

The proposed algorithm is coded in C under a Microsoft Visual C++ 6.0 development environment. All the experiments are performed on a Pentium IV-1.4G Hz PC running Microsoft Windows XP. The algorithm is tested on a 10 by 10 sensor area. To achieve complete discrimination, the sensor radius must be smaller than a half of the diameter of the sensor area. The distance between two adjacent grid points is defined as a length unit. Hence, seven sets of experiments are conducted, which consider sensor radius r ranging from 1 to 7. According to Lemma 3, each set of experiments is investigated under a given K cover which ranges between 1 and the theoretic upper bound on maximum number of covers, U_r, $1 \leq r \leq 7$, as listed in Table III.

B. Results

1) Maximum Increasing Lifetime of Sensor Network: We first investigate whether the theoretic upper bound of covers, U_r, can be found. Based on our assumptions, the maximum lifetime of network is almost proportional to the number of covers that can be found. The experimental results are listed in Table III. In the first five cases, the sensor radius ranges from 1 to 5, and the proposed algorithm can always obtain the solution under the given upper bound of cover. Moreover, a little difference exists between the situations where the sensor radius is 6 and 7. The degradation of the solution quality is less than 4.4%. From this perspective, the proposed algorithm is very effective for maximizing the lifetime of network.

TABLE III
Comparison of U_r between the theoretical and the best found values.

r	1	2	3	4	5	6	7
U_r^*	3	6	11	17	26	35	45
U_r^{**}	3	6	11	17	26	34	43
D	0%	0%	0%	0%	0%	2.9%	4.4%

* : the theoretic upper bound.
** : the best found upper bound.
D : degradation.

TABLE IV
Selected sensor densities obtained in experiments.

K	Sensor radius (r) 1	2	3	4	5	6	7
1	0.40	0.28	0.25	0.19	0.22	0.25	0.25
3	0.80	0.40	0.29	0.22	0.22	0.25	0.25
6		0.76	0.42	0.31	0.25	0.26	0.26
11			0.76	0.50	0.36	0.31	0.33
17				0.79	0.61	0.42	0.40
26					0.97	0.65	0.53
34						0.97	0.75
43							0.97

K: number of covers.

2) Deployment Cost: This study shows the best found for the minimum deployment cost by the proposed algorithm. We assume all sensors have the same deployment cost, hence the overall deployment cost can be simplified as the number of deployed sensors. In this section, the sensor density is used to be a performance metric and it can be defined as follows:

$$\text{Sensor density } (\%) = \left(\frac{1}{n}\sum_{j=1}^{m}\sum_{k=1}^{K} x_{jk}\right) \times 100\%.$$

Table IV lists the selected results of experiments, which shows the sensor density requirements with specific sensor radius, r, and the number of covers, K. For example, the sensor radius is 1 in the first experiment, and the number of covers are 1, 2, and 3. In the first row of the Table (i.e., $K = 1$), we list the minimum required sensor density to support the target-positioning functionality. Furthermore, we can observe in the case $K = 1$ and $K = 3$ have the same deployment sensor-density when the sensor radius is 5. That means we can deploy a 3-cover sensor network using the same density for a single-cover network. Consequently, the lifetime of the sensor network can be extended up to 3 times. We can also get the same result when the sensor radius 6 and 7 in the same Table. These results indicate the proposed algorithm is very energy efficiency for deploying sensor networks. Due to the space limitation, the topologies of sensor networks obtained in each experiment are not illustrated in this paper.

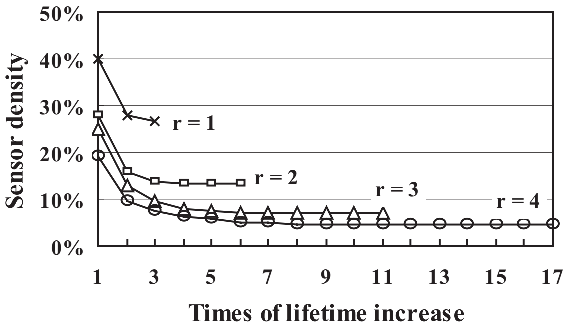

Fig. 5. Proportion of the lifetime extending times to average sensor density per cover.

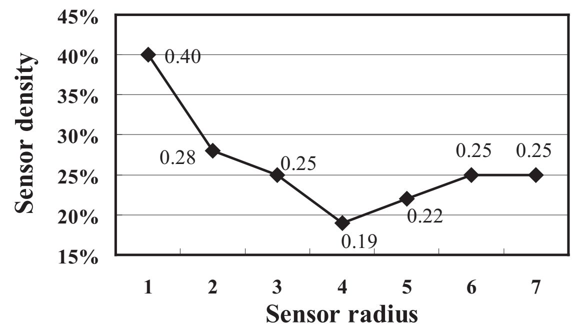

Fig. 6. Variant of the sensor radius and the corresponding density requirement. (K=1)

TABLE V

PERFORMANCE COMPARISON BETWEEN THE DUPLICATE DEPLOYMENT AND THE PROPOSED SENSOR PLACEMENT APPROACH.

	The duplicate deployment		The proposed approach	
r	# Duplication	Increased cost	# Cover	Increased cost
1	3	**3**	3	**2.00**
2	6	**6**	6	**2.71**
3	11	**11**	3	**3.04**
4	17	**17**	17	**4.16**
5	26	**26**	26	**4.41**
6	34	**34**	34	**3.88**
7	43	**43**	43	**3.88**

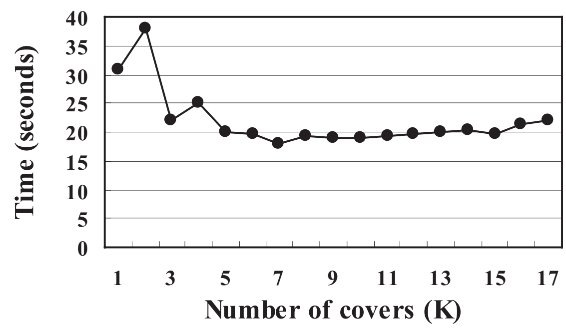

Fig. 7. The solution time for the 10x10 sensor area. (r = 4)

From average sensor density perspective, the average sensor density per cover is higher while the number of covers is few, as shown in Fig. 5. But while the cover quantity increases, the average sensor density per cover decreases progressively and achieves stable state. Therefore the proposed sensor placement algorithm is extremely effective for minimizing the sensor density requirement in extending lifetime.

Moreover, from the energy efficiency and deployment cost perspectives, the proposed algorithm demonstrates a significant improvement compared with the duplicate deployment approach. This study uses the required number of sensors for one cover as a base, then compare the times of lifetime extension and cost increase of duplicate deployment approach with that of the proposed approach, as listed in Table V. Obviously, the times of cost increase for the proposed approach is lower than that for duplicate deployment approach. For sensor radius 7, the required number of sensors is as low as 9% of duplicate deployment approach.

3) Solution Time: The study observes the computation time for the proposed algorithm. Table VI lists the maximum execution time of each set of experiments. The solution time of the algorithm is below 100 seconds in all cases. The efficiency of the algorithm thus can be confirmed.

TABLE VI

THE MAXIMUM SOLUTION TIME OF EACH EXPERIMENT.

Sensor radius (r)	1	2	3	4	5	6	7
Solution time (second)	43	61	85	38	25	91	51

4) Density vs. Different Radius: Now, we observe the experimental results that the number of covers is one, as illustrated in Fig. 6. The sensor radius varies from 1 to 7, and the sensor densities first decrease and then increase. When the sensor radius is 4, the sensor density requirement is the lowest in all cases. It is reasonable for sensors with smaller radiuses to have smaller covered areas. And thus, more sensors are required to cover the whole sensor field. Meanwhile, a larger sensor radius requires that more sensors are used to satisfy the discrimination constraint.

5) Scalability: Finally, we investigate the scalability of the proposed algorithm in terms of the solution time. First, we evaluate the solution time under various amount of covers, K. Fig. 7 shows the solution times for the 10 by 10 sensor field. In this experiment, the solution space ranges between $O(2^{100})$ and $O(18^{100})$. The results indicate that the solution times are very stable when K value increases. Actually, in all cases, the solution times are below 40 seconds.

The second experiment explores the solution time of the proposed algorithm under various sensor fields. The solution space extends from $O(2^{100})$ to $O(4^{200})$. The results show that the solution time increases very slowly, as shown in Fig. 8, when the solution space extends greatly. The maximum solution time in this experiment is only 542 seconds. These experiments indicate the proposed algorithm has excellent solution time and highly scalable.

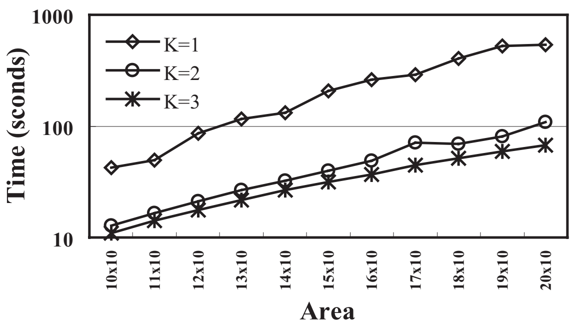

Fig. 8. The solution time for various sensor area. (r = 1)

VI. Conclusions

This investigation proposes a sensor placement algorithm to deploy an energy-efficient wireless sensor network for detecting and positioning. To the best of our knowledge, the proposed algorithm is truly novel and it has not been discussed in previous researches. This study first formulates the problem as a 0/1 integer programming problem, and then proposes a Lagrangean relaxation based heuristic for solving the optimization problem. The proposed approach can almost prolong the working life of a sensor network up to its theoretical upper bound without degrading quality of surveillance. The required average sensor density for one cover is effectively minimized and the deployment cost is just 9% of that using the duplicate sensor placement approach. Furthermore, using the same deployment density for a single-cover sensor network, we can deploy an energy-efficient sensor network such that it's lifetime can be extended up to 3 times. The computational results indicate that the sensor placement strategy is effective and the proposed algorithm is highly efficient, effective, as well as scalable. Obviously, this study contributes to deploy a sensor network for target positioning with maximum lifetime.

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