A Simulated Annealing Algorithm for Energy-Efficient Sensor Network Design

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Abstract

In this paper, we develop an algorithm to deploy an energy efficient sensor network such that it provides surveillance and target-positioning services. We consider to place K independent sets of sensors on a sensor field. These sets monitor the field in turn and work together when intrusion events occur. The lifetime of the sensor network is therefore prolonged up to K times. The problem is therefore a variant of the set K-cover problem, which is NP-complete. We formulate such sensor deployment problem as a 0/1 integer programming problem. A Simulated Annealing based heuristic then is proposed for solving the optimization problem. The experimental results show that the proposed algorithm indicates a significant improvement in the sensor lifetime compared to the intuitive approach. Furthermore, the proposed algorithm is highly effective and efficient in terms of the overall deployment cost.

1 Introduction

The evolutions in sensor technology and wireless communication have led to the development of wireless sensor networks (WSNs). A WSN comprises of a large number of tiny sensor nodes, which are low-cost and low power. Numerous sensors are ad hoc deployed in the interested area. These sensors collect physical information, process it and forward the local information to the sink nodes. Therefore the back-ends can obtain global views according to the information provided by the sensors [2, 3].

Many applications of sensor networks have been discussed in previous papers [2, 3]. Both surveillance and target positioning are the most important services of WSNs. However, the power efficiency, deployment cost, and some physical limitations influence surveillance quality on a sensor field. In the most scenarios, it is difficult to replace or recharge the sensor battery particularly when the network operates in inhospitable or hostile fields. The quality of surveillance will be degraded once the sensor energy becomes exhausted. Therefore how to design an energy-efficient sensor network is really a major challenge.

A sensor network can be deployed in two ways: random or controlled placement. When the environment is unknown, random placement must be used and sensors may be randomly dropped from aircraft. Using the controlled placement approach, sensors can be carefully deployed on a sensor field, if the terrain properties are predetermined. Consequently, the placement can be planned to meet the requirements of various levels of services; for example, surveillance, target positioning and target tracking. If the planning process is subject to some resource constraints (such as, the deployment cost) and to achieve some specific goals, the sensor deployment will be considered to be an optimization problem.

Both sensor deployment and energy conservation are key issues for WSNs [2]. This work considers the problem of constructing an energy-efficient sensor network for surveillance and target positioning services using the controlled placement approach. The design goals are to achieve target positioning as well as to prolong sensor network lifetime. To support positioning functionality, the sensor field must be completely covered and each unit in the field is discriminable. It requires to deploy more sensors than to support surveillance functionality. However, to keep all sensors in active to support the target positioning service is not necessary and waste sensors' energy if intrusion events occur infrequently. Actually, the surveillance service is enough when there isn't any intruder in the sensor field.

Hence, we try to deploy K independent sets of sensors to support positioning service on a sensor field. Each of which, is called cover, can provide complete coverage of the field. Each set is activated in turn to monitor the field when no any intruder has existed. Once the intrusion event occurs, all sets of sensors are activated and work corporately to locate the intruder. Generally, the power consumption for inactive sensors can be neglected, and the sensor lifetime can be effectively prolonged up to K times.

In this paper, we formulate the problem as a 0/1 integer programming problem where the objective function is the minimization of the total deployment cost required to complete coverage and discrimination constraints under a given amount of cover K. The problem is a variant of the set K-cover problem and thus is NP-complete [1, 11]. Then, the Simulated Annealing (SA) based heuristic is proposed to solve the optimization problem [8, 9].

The rest of this paper is organized as follows. Related works are reviewed in section 2. The problem and mathematic model then are described in sections 3 and 4, respectively. Additionally, the SA-based algorithm is presented in section 5. Furthermore, the computational results are discussed in section 6, and conclusions are presented in section 7.

2 Related work

In general, the goals of sensor placement include: reducing the deployment cost, extension the coverage, prolonging the system lifetime and so on. Dhillon et al. [7, 6] take account the probabilistic detection model and propose some algorithms to address the homogeneous sensor placement problem. The objective is to minimize the number of sensors required for adequate grid coverage under the terrain and coverage constraints. Their algorithms can construct a sensor network to support only surveillance service.

Chakrabarty et al. [4] formulated the heterogeneous sensor placement problem in terms of cost minimization under coverage constraints. Then the problem is solved by lpsolve package, and a divide-and-conquer approach is presented to cope with the large size problem. Moreover, they propose the sensor placement approaches based on identifying code theorem in graph theory to construct a sensor network providing target location service. However, their approaches cannot be applied to irregular sensor fields.

Our previous study formulated the sensor problem for target positioning as a min-max mathematical optimization model with accurate discrimination as the objective [5, 10].

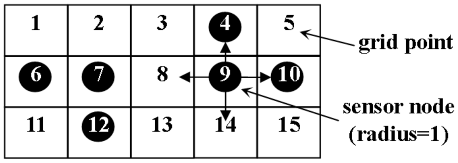

Figure 1. A complete coverage/discrimination sensor field.

From sensor placement perspective, the energy conservation strategy can be considered in "deployment phase" or "post-deployment phase". This study belongs to the former, and focuses on energy efficient sensor network deployment.

A lot of papers consider energy conservation issue in post-deployment phase. Slijepcevic and Potkonjak [11] propose a heuristic that selects mutually exclusive sets of sensor nodes from a randomly deployed sensor network, where the sensors of each of sets collaborate to achieve complete coverage. The problem is known as the set K-cover problem. The study does not take target positioning issue into account. In [1], the requirement of complete coverage of cover is relaxed. Abrams et al. design three algorithms to maximize the number of clusters.

From papers review, we find that this study differs from prior works in several points. First, we consider both the energy conservation and lifetime extending during the sensor deployment phase for target positioning. Second, we present a mathematical model to describe the optimization problem. Third, the SA-based algorithm is proposed to solve the problem. Finally, the relationship between the deployment cost and the maximum extension of system lifetime is investigated.

3 Problem description

A. Sensor Placement

In this paper, we use the controlled deployment method to construct WSNs. The sensor field can be represented as a collection of two- or three-dimensional grid points [4, 5, 6, 7, 10], as illustrated in Fig. 1. This approach is called grid-based placement. The positioning resolution requirement of application determines the granularity of grid point. In this paper, the distance between two adjacent grid points is adopted as a length unit. Therefore the sensor field illustrated in Fig. 1 has 5 by 3 grid points. And six sensors are placed on grid point 4, 6, 7, 9, 10, and 12.

This study assumes that the sensor detection model

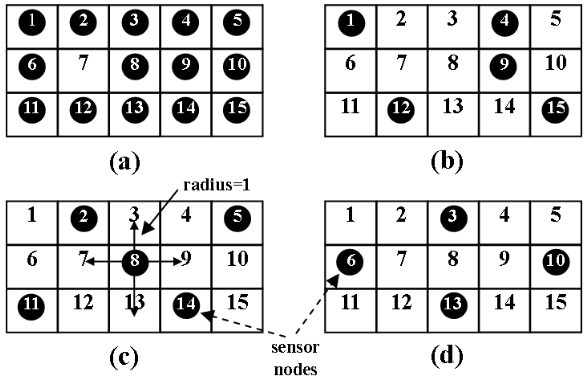

Figure 2. A grid-based sensor field with 3 covers: (a) overall placement. (b) Cover 1. (c) Cover 2. (d) Cover 3.

is 0/1 model [4, 5, 10]. The coverage is assumed to be complete (1) if the distance between the grid point and the sensor is less than the detection radius of the sensor. Otherwise, the coverage is assumed to be incomplete (0). For example, the radius of the sensors illustrated in Fig. 1 is assigned to one. Therefore sensor 4 covers grid point 3, 4, 5, and 9, sensor 7 covers grid point 2, 6, 7, 8, and 12, sensor 9 covers grid point 4, 8, 9, 10, and 14, and so on. If any grid point in a sensor field can be detected by at least one sensor, the field is called completely covered sensor field.

To locate an intruder, we give a power vector for each grid point. The power vector of a grid point is constructed according to the deployment of sensors. If a sensor covers the grid point, constituent of the power vector of the grid point, which is corresponding to the sensor, is set to 1, otherwise 0. For example, as illustrated in Fig. 1, the power vectors of grid point 1 and 8 are <0, 1, 0, 0, 0, 0> and <0, 0, 1, 1, 0, 0> corresponding to sensor 4, 6, 7, 9, 10, and 12, respectively. When the sensor deployment is finished, the power vectors for each grid point in the sensor field are constructed and store on the database at the back-end of the network. Once an intruder was detected, sensor has to report the information to the sink nodes. According to the received information, the back-end can obtain a power vector to determine the position of the intruder. If each grid point has a unique power vector in a sensor field, the sensor field is called completely discriminated. The sensor field in Fig. 1 is completely covered/discriminated by the sensor network, which can provide surveillance and target positioning services.

B. Energy-Efficient Sensor Networks

The duplicate placement approach is an intuitive method to extend sensor network lifetime by deploying K duplicate sensors at the same grid point. Sensors at the same grid point work in relays, the sensor network lifetime is therefore prolonged by K. However, the cost is increased by K times.

This study attempts to construct the sensor network such that it includes K mutually exclusive sets (number K is given). These sets are called cover [11]; they are activated in turn to keep complete coverage of the sensor field when no any intruder has existed in the field. Once the intrusion event occurs, all sensors are activated and work corporately to locate intruder. Generally, the power consumption for inactive sensors can be neglected, thus the sensor lifetime can be effectively prolonged by K times. For example, if the lifetime of sensor network illustrated in Fig. 1 is prolonged by three times using duplicate placement approach, the number of sensors will be increased to 18. Our algorithm requires only 14 sensors in the same field and can also prolong sensor network lifetime by three. These sensors are divided into 3 covers (as illustrated in Fig. 2), each of which provides complete coverage. Obviously the proposed algorithm provides an economical solution to deploy a energy efficient sensor network.

Afterwards this study discusses the possible number of covers in a sensor network. First the amount of covering grid points of sensor with a specific detection radius has been discussed and the following lemmas have been obtained.

Lemma 1: Suppose a sensor has detection radius r, then the number of covering grid points, G_r, for the sensor in an infinite sensor field can be represented as

$$G_r = 2r + 1 + 2\sum_{\Delta y=1}^{r}(2\lfloor\sqrt{r^2 - \Delta y^2}\rfloor + 1)$$

, as shown in Fig. 3.

Lemma 2: A grid point can be covered by a set of sensors. The maximum cardinality of the set exactly equals the number of covering grid points of a sensor that is allocated in the grid point.

For example, in an infinitive field, the radiuses of the sensor are 1, 2, 3, and the numbers of covering grid points are 5, 13, 29, respectively. Moreover, a grid point can be covered by 5, 13, 29 sensors at most.

Generally, it is impractical to use an infinite sensor field. This study focuses on the case of the rectangular sensor field with a finite area. For a finite sensor field,

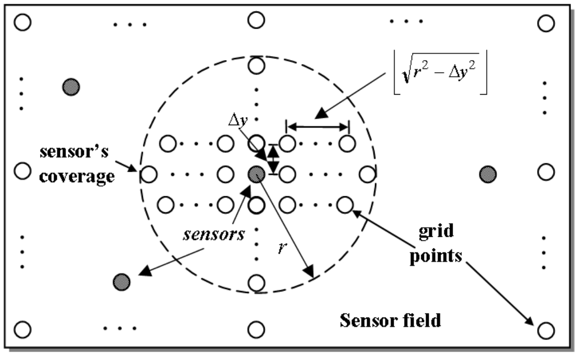

Figure 3. Sensor and its coverage.

the upper bound of the number of covers is determined by critical grid points in the field. A critical grid point is a grid point, which is covered by a sensor set with smaller cardinality than other sensor sets.

Lemma 3: On a rectangular sensor field with a finite area, the critical grid points are located at the corner of the field. Therefore, the upper bound of the amount of covers, U_r, is

$$U_r = 2r + 1 + \sum_{\Delta y=1}^{r} \lfloor \sqrt{r^2 - \Delta y^2} \rfloor$$

, where r represents the detection radius of the sensor and Δy is the distance from the sensor to a grid point in y axis.

In a sensor field with 3 by 5 grid points, as illustrated in Fig. 2, the radius of sensor is assumed to be 1, while the *critical grid points* are 1, 5, 11 and 15. According to Lemma 3, the upper bound of the number of covers, U_1, is 3. In Fig. 2, grid point 1 can only be covered by sensors placed in grid points 1, 2 and 6. Meanwhile, the corner grid points are all covered by 3 sensors maximally. In the case, the sensor network can be partitioned into a maximum of 3 covers. Clearly, as shown in Fig. 2, we can found the maximum covers for the 3 by 5 sensor network.

4 Problem formulation

The notations used to model the problem are listed as follows.

Given Parameters:

- $A = \{1, 2, ..., m\}$: The set of the index for candidate locations where sensor can be allocated.
- $B = \{1, 2, ..., n\}$: The set of the index for service points that can be covered and located by the sensor network, $m \leq n$.
- K : The number of covers required for the sensor network.
- a_{ij} : Indicator function which is 1 if service point i can be covered by sensor j and 0 otherwise.
- c_j : Cost function of sensor j.

Decision Variables:

- x_{jk} : 1 if sensor j is allocated on cover k of the sensor network.
- y_j : Sensor allocation decision variable which is 1 if sensor j is allocated in the sensor network.

Problem (IP):

$$Z_{IP} = \min \sum_{j=1}^{m} \sum_{k=1}^{K} c_j x_{jk} \qquad (IP)$$

Subject to :

$$\sum_{j=1}^{m} a_{ij} x_{jk} \geq 1 \qquad ,\forall i \in B,\ 1 \leq k \leq K \qquad (1)$$

$$\sum_{k=1}^{K} x_{jk} \leq 1 \qquad ,\forall j \in A \qquad (2)$$

$$y_j = \sum_{k=1}^{K} x_{jk} \qquad ,\forall j \in A \qquad (3)$$

$$\sum_{j=1}^{m} (a_{ij} - a_{\ell j})^2 y_j \geq 1 \quad ,\forall i, \ell \in B, i \neq \ell \qquad (4)$$

$$x_{jk}, y_j = 0 \ \ or \ \ 1 \qquad ,\forall j \in A,\ 1 \leq k \leq K \qquad (5)$$

Physical meanings of the constraints are briefly described as follows. Constraint (1) requires that each service point must be covered in every cover of the sensor network. Constraint (2) and (3) ensure that each sensor only belong to one cover of the sensor network. The discrimination constraint is $\sum_{j=1}^{m}(a_{ij}y_j - a_{\ell j}y_j)^2 \geq 1$ that requires the Hamming distance between each pair of service points in the sensor network must be greater than one. And the discrimination constraint can be rewritten as constraint (4). Constraint (5) requires integer property of the decision variables with respect to x_{jk} and y_j.

Table 1. The SA pseudo code for sensor placement.

```
1.  Let $x_{jk} \leftarrow 1, \forall j \in A$, if $(j \ mod \ K) + 1 = k$.
2.  Calculate $S_0,\ S_1$, and $s_{1k},\ 1 \leq k \leq K$.
3.  $z_{old} \leftarrow E$.
4.  Let $z_{min} \leftarrow z_{old}$ and save the current configuration as the solution.
5.  $t \leftarrow t_0,\ r \leftarrow r_0$.
6.  While $t > t_f$ do
7.    Repeat $r$ times
8.      If $S_0 \neq \emptyset$ then choose $s_{new} \in S_0$ randomly.
9.      Choose $s_{old} \in S_1$ randomly.
10.     Choose covers $c_1$ and $c_2$, $c_2 \neq c1$, in $[1,\ K]$ randomly.
11.     Choose action $ac,\ ac \in \{1,\ 2,\ 3\}$, randomly.
12.     If $ac \neq 1$ and $S_0 \neq \emptyset$ then $S_1 \leftarrow S_1 \cup s_{new},\ s_{1k}|_{k=c_2} \leftarrow s_{1k}|_{k=c_2} \cup s_{new},\ S_0 \leftarrow S_0 - s_{new}$.
13.     If $ac \neq 2$ and $S_1 \neq \emptyset$ then $S_1 \leftarrow S_1 - s_{old},\ s_{1k}|_{k=c_1} - s_{old},\ S_0 \leftarrow S_0 \cup s_{old}$.
14.     $z_{zew} \leftarrow E,\ \Delta E \leftarrow z_{new} - z_{old}$.
15.     Generate random variable $\gamma$ uniformly distributed in (0, 1).
16.     If $\Delta E \leq 0$ or $\gamma < e^{(-\Delta E/t)}$ then
17.       $z_{old} \leftarrow z_{new}$.
18.       If $z_{new} < z_{min}$ then $z_{min} \leftarrow z_{new}$, save the current configuration as the solution.
19.     else recover the change for $S_0$, $S_1$, and $s_{1k},\ 1 \leq k \leq K$, that were made in step (12) and (13).
20.    End
21.    $r \leftarrow r * \beta$; If $z_{old} < m$ then $t \leftarrow t * \alpha_1$ else $t \leftarrow t * \alpha_2$.
22. End
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5 Algorithm

Simulated annealing (SA) is a highly reliable method for solving hard combinatorial optimization problems. The concept of SA is applied to derive an efficient method for solving the problem approximately. To simplify the solution procedure, we try to relax the coverage and discrimination constraints (i.e., constraints (1) and (4)) by penalizing objective function Z_{IP}.

The penalty for constraint (1) is $1 + p\sum_{k=1}^{K}\sum_{i=1}^{n} g_{ik}$, where $p,\ p \geq m$, is a constant. Variable $g_{ik},\ \forall i \in B,\ 1 \leq k \leq K$, indicates whether grid point i was covered by sensors in cover k. g_{ik} can be calculated as following:

$$g_{ik} = \begin{cases} 1, \text{ if } \sum_{j=1}^{m} a_{ij} x_{jk} = 0. \\ 0, \text{ otherwise.} \end{cases}$$

The penalty for constraint (4) is $1 + p^2(1 - d_{min})$. Variable d_{min} represents the minimum Hamming distance between each pair of service points,

$$d_{min} = \begin{cases} 1, \text{ if } \min\limits_{\forall i,\ \ell \in B,\ i \neq \ell} \sum_{j=1}^{m} (a_{ij} - a_{\ell j})^2 y_j \geq 1. \\ 0, \text{ otherwise.} \end{cases}$$

Hence, energy, E, can be defined as

$$\begin{aligned} E =& (1 + p\sum_{k=1}^{K}\sum_{i=1}^{n} g_{ik}) \\ & (1 + p^2(1 - d_{min}))\sum_{j=1}^{m}\sum_{k=1}^{K} c_j x_{jk}. \end{aligned}$$

Table 1 shows a pseudo code of the algorithm. We will use the following symbols regarding any feasible solution x:

$$\begin{aligned} S_0 =& \{j | \sum_{k=1}^{K} x_{jk} = 0,\ \forall j \in A\}, \\ s_{1k} =& \{j | x_{jk} = 1,\ \forall j \in A\}, \\ S_1 =& \bigcup_{k=1}^{K} s_{1k}. \end{aligned}$$

Initially, we assume the sensors are deployed at all grid points. In each loop, one of three actions (i.e., remove, add, and exchange) will be chosen randomly. Each action attempts to change the deployment status of one sensor. The solution with the minimum energy, z_{min}, is saved as the best found solution. While frozen temperature, t_f, is reached, the algorithm stops. If $z_{min} \leq m$, the best found solution is the feasible solution to this problem.

Table 2. Comparison of U_r between the theoretical and best found values.

r	1	2	3	4	5	6	7
U_r^*	3	6	11	17	26	35	45
U_r^{**}	3	6	11	17	26	34	44
D	0%	0%	0%	0%	0%	2.9%	2.2%

* : the theoretic upper bound.
** : the best found upper bound.
D : degradation.

The SA-based algorithm is a stochastic process. Computing time for the algorithm is relative to the selection of the cooling schedule, the desired solution quality, and the optimization problem itself. Hence, as the size of the problem increases, the time complexity of the problem will become increasingly irrelevant [9]. In the proposed problem, the evaluation for discrimination constraint (i.e., constraint (4)) dominates the solution time of the algorithm. To check whether the constraint is satisfied, we have to calculate the Hamming distance of power vectors for each pair of grid points in a sensor field. Therefore, the time complexity for the evaluation is $O(n^3)$, where n is number of grid points in the field.

6 Computational results

The algorithm was tested on a 10 by 10 service area. The sensor radius r ranges between 1 and 7. According to Lemma 3, each set of experiments is investigated under a given K cover which ranges between 1 and the theoretic upper bound on maximum number of covers, U_r, as listed in Table 2.

The parameters of the cooling schedule are $\alpha_1 = 0.5$, $\alpha_2 = 0.75$, and $\beta = 1.3$. The initial values of r and t are $20n$ and 100, respectively; and n is the number of grid points in the sensor field. The frozen temperature, t_f, is set to $t_0/10000$. p is m, $m = n$, and the cost of sensor, c_j, $\forall 1 \leq j \leq m$, is set to one.

In this paper, we evaluate the performance of the SA algorithm in terms of sensor lifetime as well as deployment cost. We first investigate whether the theoretic upper bound of covers, U_r, can be found. The experimental results are listed in Table 2. In the first five cases, the SA algorithm can always obtain the solution under the given upper bound of cover. Moreover, in the last two cases, the degradation of the solution quality is less than 2.9%. From this perspective, the proposed algorithm is very effective for maximizing the network lifetime.

Table 3. The increased deployment cost by the SA approach (compared to the deployment with one cover).

r	1	2	3	4	5	6	7
K	3	6	11	17	26	34	44
C_I	3	6	11	17	26	34	44
C_{SA}	2.00	3.17	3.85	5.18	4.63	4.67	4.57

K: number of covers.
C_I: the increased cost by intuitive approach.
C_{SA}: the increased cost by the SA approach.

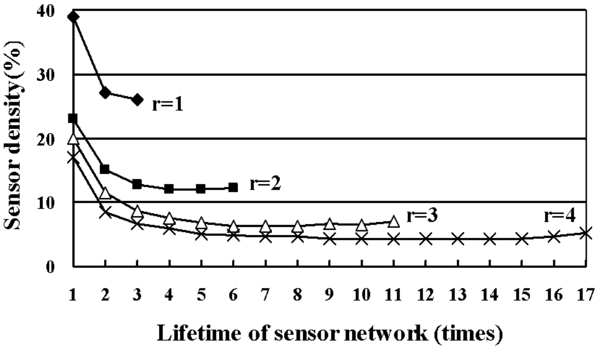

Figure 4. Proportion of the lifetime extending times to average sensor density per cover.

Second, this study shows the best found for the minimum deployment cost by the proposed algorithm. Since all sensors have the same deployment cost, the overall deployment cost can be simplified as the amount of deployed sensors. This section instead uses a normalized term, average sensor density, as a performance metric. The average sensor density (%) is $\frac{1}{K}(\frac{1}{n}\sum_{j=1}^{m} y_j) \times 100\%$. Due to the space limitation, we only illustrates parts of experimental results in Fig. 4. From average sensor density perspective, the average sensor density per cover is higher while the number of covers is few. But while the cover quantity increases the average sensor density per cover decreases progressively and achieves stability. Therefore the proposed sensor placement algorithm is extremely effective for minimizing the sensor density increase in extending lifetime.

Moreover, from the energy efficiency and deployment cost perspectives, the proposed algorithm can significantly reduce the deployment cost when the amount of covers are increased, as listed in Table 3. Compared with an intuitive approach that duplicates K sets of sensor for prolonging network lifetime K times.

Each set of sensors can completely support the coverage/discrimination on the sensor field. The times of cost increase for the proposed approach is lower than for the intuitive approach. For sensor radius 7, the required number of sensors is as low as 10.39% of the intuitive approach.

7 Conclusions

In this paper, a SA-based algorithm is proposed to design an energy efficient sensor deployment problem. The proposed algorithm is truly novel as has not been discussed in any previous research. The proposed approach can almost prolong the lifetime of sensor network up to the theoretical upper bound without degrading quality of surveillance. The required average sensor density of a cover is effectively minimized. Moreover, the deployment cost is just 10.39% of that using the duplicate sensor placement approach. The computational results indicate that the sensor placement approach is effective and the proposed algorithm is highly efficient and effective. Obviously, this work contributes to deploying a sensor network for target positioning with maximum lifetime.

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