AN OPTIMIZATION-BASED DESIGN ALGORITHM FOR CATV NETWORKS

K. W. Peng*, Y. S. Lin, J. F. Kwang
National Taiwan University, Lunghwa University of Science and Technology*
No.1, Sec. 4, Roosevelt Rd., Taipei, 106, Taiwan
d6725005@im.ntu.edu.tw

ABSTRACT

Although an increasing number of new services are running on CATV networks, the quality of traditional CATV network systems can not fully support the new services. Thus, operators of CATV network systems need to improve or expand their network equipments and capacity. But the most economical way to construct a CATV network that meets all requirements has yet to be found.

In this paper, we propose a near-optimal two-way CATV network design algorithm that minimizes total installation costs. First, we introduce a mathematical model to describes CATV networks. Because of the nature of CATV such networks, the nonlinear property is unavoidable and must therefore be dealt with. By applying some optimization methods, including geometric programming, surrogate functions, linear relaxation, and the steepest decent method, we have successfully developed an optimization-based algorithm.

In the numerical experiments, we use some network examples to test the algorithm. For the test examples, the optimal designs are found by exhaustive searching and compared with designs generated by our algorithm. The cost differences range from 0 to 2.5%, and the results generated by our algorithm are close to the optimal solutions.

KEY WORDS

Network modelling and simulation, Network planning, and Optimization-based algorithm

1. Introduction

An increasing number of new services are now running on CATV networks. The earliest CATV (Community Antenna Television) systems were constructed in small towns or semi-rural areas, where off-air television reception was poor or unavailable [4]. Because of their popularity and high bandwidth, CATV such networks have become one of the most popular technologies for providing a "last-mile" communication platform.

However, the quality of traditional CATV network systems may not be able to fully support new interactive services, like Movies-On-Demand (MOD) and Voice-over-IP (VoIP)[3]. Thus the operators of CATV network systems need to improve their current network equipments and capacity to achieve adequate QoS (Quality of Service) to their subscribers. Otherwise, the complaints from customers would not be surprising.

On the other hand, there are few papers concerning about CATV network planning and capacity expansion issues [2,9]. The reason is obvious: This kind of problems is too complicated. First, with the government regulations and network complexity, the CATV systems need to be designed carefully to meet all requirements mandated. Furthermore, the "noise-funneling" effect makes this problem more complicated [7]. Therefore, it is hard to build a quantitative model to describe this problem, not even to solve it effectively. Without this kind of models and solving procedures, the designers today can only try based on their own experiences. There are some CATV CAD tools; however, their functions are very limited [1].

Our goal is to develop a near-optimal design algorithm to minimize total installation costs of a two-way CATV network, subject to the performance requirements [6]. In section 2, the nature of CATV networks is described. Based on the observation, we present our formulation in section 3. Because of the nature of the problem, the nonlinear property is unavoidable and must therefore be dealt with. Our algorithm is based on mathematical programming, relaxation techniques, and heuristics [4]. The solution procedure is described in section 4. Finally, we present our numerical experiments and findings in section 5 and 6.

2. CATV NETWORKS

A typical CATV network topology and a number of key transmission components are shown in Figure 1. In order to construct a high-quality CATV system, we must first explore the relationships among the components and their combined effect on end-to-end performance.

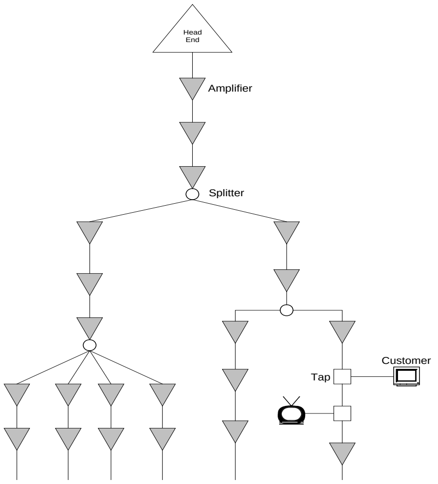


Fig. 1. The Topology of CATV Networks

From the figure, it is clear that a CATV system contains many physical parts and devices. Our goal is to develop an intelligent design algorithm so that designers can build feasible, near-optimal systems that minimize total deployment costs.

We now present an example of an end-to-end path to demonstrate how the end-to-end CNR, X-MOD, CSO, and CTB are calculated using related parameters of the intermediate network components. In Fig. 2, an end-to-end path from a head end to a user is described in links and components.

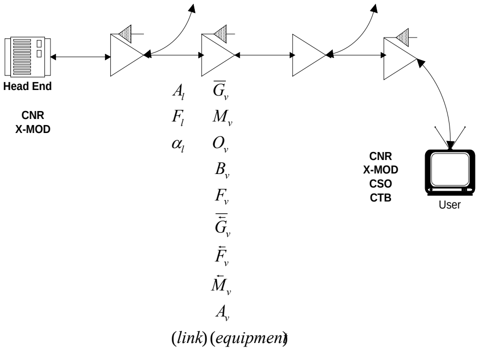


Fig. 2. An End-to-end Path

In Fig. 2, the components could be amplifiers or other devices. Let S_i be the input signal, N_i be the input noise from the head end, S_o be the output signal, and N_o be the output noise to the user. In addition, let G_v be the gain of the component, F_v be the noise figure of the component, α_l be the cable splitting factor, and A_l , A_v be the attenuation factor. The factors that effect end-to-end performance can be formulated as Fig. 3.

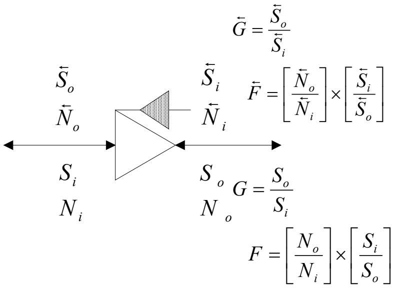


Fig. 3. The factors effect end-to-end performance

As shown in Figure 3, four factors affect end-to-end performance on the downstream path. The first is the gain G_v of an amplifier, which typically increases the power levels of both the input signal and the input noise. The second factor is the noise figure, F_v, of an amplifier and the noise figure, F_l, of a link, which indicate the amplitude of internal noise that the amplifier or link introduces. The third factor is the attenuation, A_l, of a link and/or the insertion loss, A_v, of a passive device, which reduce the power level of the input signal and the amount of noise. The fourth factor is the splitting factor, α_l , which monitors the effect of power reduction due to transmission cable branching.

Meanwhile, three factors affect end-to-end performance on the upstream path. The first is the gain, $\overleftarrow{G}$, of an upstream amplifier, which typically increases the power levels of both the upstream input signal and the upstream input noise. The second is the noise figure, $\overleftarrow{F}_v$, of an amplifier and the noise figure, F_l, of a link, which indicate the amplitude of internal noise that the amplifier or link introduces. The third factor is the attenuation, A_l, of a link and/or insertion loss, A_v, of a passive device, which reduce the power level of the input signal and the amount of noise respectively.

The signal originated from a head end or a user is affected by every component on the path. In other words, we can formulate the end-to-end performance constraints by calculating the accumulated effect along a path. Based on this technique, we propose a mathematical model to minimize the total construction cost with all end-to-end performance constraints satisfied.

3. Mathematical Formulation

Here we construct mathematical expressions of the CATV network design problem. A legend of the notations used is given in TABLE I.

TABLE I
LEGEND OF NOTATIONS

Notation	Description
L	The set of links
C_l	Installation cost of link *l*
Φ_l	Cost function of link *l*
V	The set of intermediate nodes
Φ_α	Cost function of active component (downstream amplifier)
$\Phi_{\overleftarrow{\alpha}}$	Cost function of active component (upstream amplifier)
Φ_β	Cost function of passive component
W	The set of user pairs
S_i	Input signal strength from the head end
P_w	The set of candidate paths for user pair *w*
H_{pc}	The number of class-*C* components on path *p*
λ_{pi}	Node *v* as the i^{th} equipment of path *p*
κ_{pi}	Link *l* as the i^{th} link of path *p*
N_i	Input noise strength from the head end
$\overleftarrow{S}_i$	Input signal strength from the user tap
$\overleftarrow{N}_i$	Input noise strength from the user tap
$\overleftarrow{C}_{sys}$	Upstream CNR performance requirement
$\lvert W \rvert$	The cardinality of *W*
$\overleftarrow{X}_{sys}$	Upstream cross-modulation (X-MOD) performance requirement
L_{out}^v	The set of outgoing links for node *v*
L_{in}^v	The set of incoming links for node *v*
δ_{pl}	The indicator function, which is 1if link *l* is on path *p* and 0 otherwise
F_v^L	Lower bound of the downstream noise figure for component on *v*
F_v^U	Upper bound of the downstream noise figure for component on *v*
$\overleftarrow{F}_v^L$	Lower bound of the upstream noise figure for component on *v*
$\overleftarrow{F}_v^U$	Upper bound of the upstream noise figure for component on *v*
F_l^L	Lower bound of the noise figure for link *l*
F_l^U	Upper bound of the noise figure for link *l*
M_v^L	Lower bound of the cross modulation intercept for component *v*
M_v^U	Upper bound of the cross modulation intercept for component *v*
O_v^L	Lower bound of the composite second order intercept for component *v*
O_v^U	Upper bound of the composite second order intercept for component *v*
B_v^L	Lower bound of the composite triple beat order intercept for component *v*
B_v^U	Upper bound of the composite triple beat order intercept for component *v*
$\overline{G}_v^L$	Lower bound of the downstream full gain for component *v*
$\overline{G}_v^U$	Upper bound of the downstream full gain for component *v*
$\overline{\overleftarrow{G}}_v^L$	Lower bound of the upstream full gain for component *v*
$\overline{\overleftarrow{G}}_v^U$	Upper bound of the upstream full gain for component *v*
$\overleftarrow{M}_v^L$	Lower bound of the upstream cross modulation intercept for component *v*
$\overleftarrow{M}_v^U$	Upper bound of the upstream cross modulation intercept for component *v*
K_e	The set of possible configurations for equipment *e*
K_c	The set of possible configurations for cable *c*
F_t^A	The set of available downstream noise figures for component *t*
$\overleftarrow{F}_t^A$	The set of available upstream noise figures for component *t*
F_c^A	The set of available noise figures for link *c*
$\overline{G}_t^A$	The set of available downstream full gains for component *t*
$\overline{\overleftarrow{G}}_t^A$	The set of available upstream full gains for component *t*
$\overleftarrow{M}_t^A$	The set of available upstream cross modulations for component *t*
M_t^A	The set of available cross modulations for component *t*
O_t^A	The set of available composite second orders for component *t*
B_t^A	The set of available composite triple beats for component *t*
A_t^A	The set of available attenuation factors for component *t*
A_c^A	The set of available attenuation factors for cable *c*
ε	Threshold considered in the projection method
Decision Variables:	
y_l	Binary decision variable, which is 1 if link *l* is installed and 0 otherwise
z_v^α	Binary decision variable, which is 1 if component *v* is installed as a downstream active component and 0 otherwise
$z_v^{\overleftarrow{\alpha}}$	Binary decision variable, which is 1 if component *v* is installed as an upstream active component and 0 otherwise
z_v^β	Binary decision variable, which is 1 if component *v* is installed as a passive component and 0 otherwise
A_l	Attenuation factor of cable link *l*

F_l	Noise figure of cable link *l*
F_v	Downstream noise figure of equipment *v*
$\overline{G}_v$	Downstream full gain of equipment *v*
M_v	Downstream cross modulation intercept parameter of equipment *v*
B_v	Composite triple beat intercept parameter of equipment *v*
O_v	Composite second order intercept parameter of equipment *v*
$\overleftarrow{F}_v$	Upstream noise figure of equipment *v*
$\overline{\overleftarrow{G}}_v$	Upstream full gain for component *v*
$\overleftarrow{M}_v$	Upstream cross modulation intercept parameter of equipment *v*
x_p	Binary decision variable, which is 1 if path *p* is used and 0 otherwise
G_v	Downstream gain of equipment *v*
α_l	Cable splitting factor of link *l*
$\overleftarrow{G}_v$	Upstream gain of equipment *v*

The following are the proposed mathematical expressions of the CATV network design problem.

$$\min \sum_{l\in L}[y_l C_l + y_l \Phi_l(A_l, F_l)] + \sum_{v\in V}[z_v^{\alpha}\Phi_{\alpha}(F_v, \overline{G}_v, M_v, B_v, O_v) + z_v^{\overleftarrow{\alpha}}\Phi_{\overleftarrow{\alpha}}(\overleftarrow{F}_v, \overline{\overleftarrow{G}}_v, \overleftarrow{M}_v) + z_v^{\beta}\Phi_{\beta}(A_v)] \qquad \text{(IP1)}$$

s.t.

$$\sum_{p\in P_w} x_p \sum_{n=1}^{H_{pc}}\left(\frac{z^{\alpha}_{\lambda pn}F_{\lambda pn}}{\prod_{i=1}^{n}G_{\lambda pn}A_{\lambda pn}\prod_{j=1}^{n}A_{\kappa pn}\alpha_{\kappa pn}} + \frac{F_{\kappa pn}}{\prod_{i=1}^{n}G_{\lambda pn}A_{\lambda pn}\prod_{j=1}^{n}A_{\kappa pn}\alpha_{\kappa pn}}\right) \le N\times 10^{\frac{\frac{S}{N}-C_{sys}}{10}} - N \qquad \forall \mathrm{w}\in \mathrm{W} \qquad (1.1)$$

$$\sum_{p\in P_w} x_p \sum_{n=1}^{Hpc}[z^{\alpha}_{\lambda pn}10^{\frac{M_{\lambda pn}}{20}}\prod_{i=1}^{n}G_{\lambda pi}A_{\lambda pi}\times\prod_{j=1}^{n}A_{\kappa pj}\alpha_{\kappa pj}] \le \frac{10^{\frac{-M_{sys}}{20}}}{S} \qquad \forall w\in W \qquad (1.2)$$

$$\sum_{p\in P_w} x_p \sum_{n=1}^{Hpc}[z^{\alpha}_{\lambda pn}10^{\frac{B_{\lambda pn}}{20}}\prod_{i=1}^{n}G_{\lambda pi}A_{\lambda pi}\times\prod_{j=1}^{n}A_{\kappa pj}\alpha_{\kappa pj}] \le \frac{10^{\frac{-B_{sys}}{20}}}{S} \qquad \forall w\in W \qquad (1.3)$$

$$\sum_{p\in P_w} x_p \sum_{n=1}^{Hpc}[z^{\alpha}_{\lambda pn}10^{\frac{O_{\lambda pn}}{10}}\prod_{i=1}^{n}G_{\lambda pi}A_{\lambda pi}\times\prod_{j=1}^{n}A_{\kappa pj}\alpha_{\kappa pj}] \le \frac{10^{\frac{-O_{sys}}{10}}}{S} \qquad \forall w\in W \qquad (1.4)$$

$$\frac{\sum_{w\in W}\sum_{p\in P_w} x_p \sum_{n=1}^{H_{pc}}(z^{\overleftarrow{\alpha}}_{\lambda pn}\overleftarrow{F}_{\lambda pn}\prod_{i=1}^{n}\overleftarrow{G}_{\lambda pn}A_{\lambda pn}\prod_{i=1}^{n-1}A_{\kappa pn} + \overleftarrow{F}_{\kappa pn}\prod_{i=1}^{n}\overleftarrow{G}_{\lambda pn}A_{\lambda pn}\prod_{i=1}^{n-1}A_{\kappa pn})}{\sum_{p\in P_w} x_p \prod_{i=1}^{H_{pc}}\overleftarrow{G}_{\lambda pn}A_{\lambda pn}\prod_{i=1}^{H_{pc}}A_{\kappa pn}} \le \overleftarrow{N}\times 10^{\frac{\frac{\overleftarrow{S}}{\overleftarrow{N}}-\overleftarrow{C}_{sys}}{10}} - \overleftarrow{N}\times|W| \qquad \forall w\in W \qquad (1.5)$$

$$\sum_{p\in P_w} x_p \sum_{n=1}^{Hpc}[z^{\overleftarrow{\alpha}}_{\lambda pn}10^{\frac{M_{\lambda pn}}{20}}\prod_{i=n}^{H_{pc}}\overleftarrow{G}_{\lambda pi}A_{\lambda pi}\times\prod_{j=n+1}^{H_{pc}}A_{\kappa pj}\alpha_{\kappa pj}] \le \frac{10^{\frac{\overleftarrow{X}_{sys}}{20}}}{\overleftarrow{S}} \qquad \forall w\in W \qquad (1.6)$$

$$G_v \le \overline{G}_v \qquad \forall v\in V \qquad (1.7)$$

$$\overleftarrow{G}_v \le \overline{\overleftarrow{G}}_v \qquad \forall v\in V \qquad (1.8)$$

$$\sum_{l\in L^v_{out}} \alpha_l \le 1 \qquad \forall v\in V \qquad (1.9)$$

$$0 \le \alpha_l \le 1 \qquad \forall l\in L \qquad (1.10)$$

$$\sum_{l\in L^v_{in}} y_l = 1 \qquad \forall v\in V \qquad (1.11)$$

$$y_l = 0 \; or \; 1 \qquad \forall l\in L \qquad (1.12)$$

$$\sum_{w\in W}\sum_{p\in P_w}\delta_{pl}x_p \le y_l\,|W| \qquad \forall l\in L \qquad (1.13)$$

$$\sum_{p\in P_w} x_p = 1 \qquad \forall w\in W \qquad (1.14)$$

$$z_{\alpha} \le A_v \le 1 \qquad \forall v\in V \qquad (1.15)$$

$$(A_l, F_l)\in K_c \qquad \forall l\in L \qquad (1.16)$$

$$\frac{G_v}{z_v^{\alpha}} \le \frac{1}{\varepsilon} \qquad \forall v\in V \qquad (1.17)$$

$$\varepsilon \le z_v^{\alpha} \le 1 \qquad \forall v\in V \qquad (1.18)$$

$$\frac{\overleftarrow{G}_v}{z_v^{\overleftarrow{\alpha}}} \le \frac{1}{\varepsilon} \qquad \forall v\in V \qquad (1.19)$$

$$\varepsilon \le z_v^{\overleftarrow{\alpha}} \le 1 \qquad \forall v\in V \qquad (1.20)$$

$$\varepsilon \le z_v^{\beta} \le 1 \qquad \forall v\in V \qquad (1.21)$$

$$z_v^{\alpha} + z_v^{\beta} \le 1 \qquad \forall v\in V \qquad (1.22)$$

$$z_v^{\overleftarrow{\alpha}} \le z_v^{\alpha} \qquad \forall v\in V \qquad (1.23)$$

The primal problem, IP1, is the objective function that contains the cost summation of all equipment and links installed in the CATV network. Constraint (1.1) enforces the downstream CNR performance requirement of each OD pair. Constraint (1.2) enforces the downstream X-MOD performance of each OD pair. Constraint (1.3) enforces the downstream CTB performance of each OD pair. Constraint (1.4) enforces the downstream CSO performance of each OD pair. Constraint (1.5) enforces the upstream CNR performance requirement of each OD pair. Constraint (1.6) enforces the upstream XMOD performance requirement of each OD pair. With the limitation of the paper length, we do not list those constraints concerning about the feasible range of variables. For example, the binary decision variable x_p

must be 1 or 0. There is a constraint, $x_p = 1 \text{ or } 0, \forall p \in P_w, w \in W$ for variable x_p.

4. The solution procedure

In this section, we introduce the solution algorithm and heuristic solutions. Since the formulations are not convex, we could not use gradient-based methods to solve them directly. After several trail, we found the geometric programming method would be a possible way to solve this problem effectively. First, by geometric programming method, constraints can be relaxed by transforming the primal problem into the dual problem. Second, the dual problem in geometric programming method is convex and can be solved by gradient-based methods.

We can apply the geometric programming method, however, our formulation is still too large to deal with effectively. Therefore, we divide the total formulation into smaller sub-problems to reduce the implementation complexity. The objective function of (IP1) and its constraints can be separated into two parts: 1) a "Steiner tree problem", and 2) a "posynomial geometric programming problem".

1) Steiner tree problem: As we need to connect the headend and the aggregate end users, but not the other nodes in the network, we have to solve an NP-complete problem called the steiner tree problem. Our steiner tree heuristic algorithm is called the Minimum Cost Path Heuristic (MPH) [4].

2) Posynomial geometric programming problem: After constructing a Steiner tree, we have all the OD pairs of users, which satisfy Constraints (1.7) to (1.12). The other constraints are mainly performance requirements, including how to ensure that the components (including the equipment and links) in the present steiner tree network meet CNR, X-MOD, CTB, and CSO requirements.

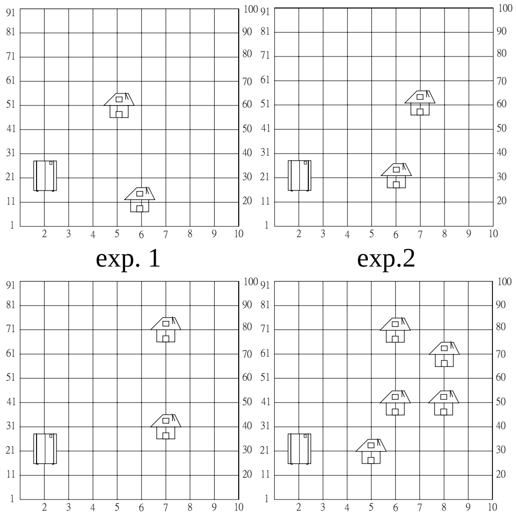
With regard to the geometric form of the constraints, the primal problem can be transformed into a geometric dual problem [5], which is a convex problem. We can then use a gradient-based method, such as the steepest descent method [4], to optimally solve the dual problem.

5. Numerical Experiments

In this section, we report on the experiments conducted to test the algorithm proposed above. The experiment parameters are described in TABLE II. To determine the difference between the cost using this algorithm and the optimal allocation and configuration, we test four network topologies, as shown in Figure 4.

TABLE II
EXPERIMENT PARAMETERS

The length of a cable segment	100 meters
The lower bound of the cable attenuation factor	0.01 or -20dB (100m)
The upper bound of the cable attenuation factor	0.5 or -3dB (100m)
The lower bound of amplifier full gain	20 dB
The upper bound of amplifier full gain	40 dB
The lower bound of amplifier NF	6.3 or 8 dB
The lower bound of amplifier NF	31.6 or 15 dB
The lower bound of amplifier XMOD	10^7 or 70dB
The upper bound of amplifier XMOD	10^9 or 90 dB
Input signal strength	35 dBmv
The O-D pair CNR constraint	43 dB
The O-D pair XMOD constraint	-46 dB
The O-D pair signal strength constraint	10dbmv
The attenuation factor of a splitter or directional coupler	-1 dB
The attenuation factor of a user tap	-1 dB
The O-D pair upstream CNR constraint	20 dB



exp. 1 exp.2

exp. 3 exp. 4

Fig. 4. The topology of experiments

The experiment results are shown in Table III. The minimum costs are found by exhaustive searching and compared with our geometric programming-based algorithm. The cost differences range from 0 to 2.4%. We have also tested some topologies with more users by our algorithm; however, the minimum cost could not get by using exhaustive search method. Therefore, we do not list here.

TABLE III
EXPERIMENT RESULTS

Exp.	GP-based	Min. Cost	Diff.(%)
1	4974	4974	0.00%
2	6840	6840	0.00%
3	10002	10000	0.02%
4	12775	12475	2.40%

6. Conclusion

In this paper, we have presented a mathematical model and solution procedure for CATV networks. By applying many mathematical programming techniques, we have successfully developed an algorithm for solving the planning problem of CATV networks.

In the examples we tested, the results generated by our algorithm are close to the optimal solutions found by exhaustive searching. It also shows how effectively our algorithm done in CATV network planning.

The other advantage of our algorithm is easy to expand. Since we use geometric programming method, a new constraint added to the primal problem would also be a new term added to the respective dual problem. In our algorithm, only few modifications are needed when new requirements are added or removed.

Finally, the mathematical model and algorithm provide many possibilities for CATV networks. Designers can use the algorithm in different ways. For example, they can use it to generate a feasible CATV network design, and improve it further through their experience. We believe that our mathematical model and algorithm will become increasingly important in the design process of CATV networks with versatile services.

7. References

[1] P.P. Yermolov, B. N. Shelkovnlkov, “Software for CATV Networks Design”, *International Crimean Conference on Microwave and Telecommunication Technology*, Sep., 2000.

[2] Dymarsky Ya.S., Nourmiyeva M.V., “Optimization Task of Calculation of Multiservice CATV Network Having Minimum Cost,” *The 11th International Conference on Microwave and Telecommunication Technology*, Sevastopal, Ukraine, 2001

[3] T. Kos, B. Zovko-Cihlar, S. Grgic, “New Services over CATV Network,” *EUROCON'2001, Trends in Communications, International Conference on. ,Volume: 2* , 4-7 July 2001

[4] Hamdy A. Taha, *Operations Research - An Introduction* (Department of Industrial Engineering/University of Arkansas, Fayetteville, 3rd Edition, pp. 780-784).

[5] Richard J. Duffin, Elmor L. Peterson, and Clarence Zener, *Geometric Programming - Theory and Application* (John Wiley & Sons Inc., 1967).

[6] Ciciora, Walter S.*, Cable Television in the United States: An Overview, Louisville, CO:* (Cable Television Laboratories, 1995).

[7] 12. R.P.C. Wolters, “Characteristics of Upstream Channel Noise in CATV-networks,”, *IEEE Transactions on Broadcasting*, *Vol. 42*, Issue: 4 , Dec. 1996, Pages:328 – 332

[8] Sharon Eisner Gillete, “*Connecting Homes to the Internet: An Engineering Cost Model of Cable vs. ISDN*,” Master’s thesis, Massachusetts Institute of Technology, 1995.

[9] H. J. Hsieh, Y. S. Lin, “Planning and Management of CATV Networks,” *ISCOM*, 1999