# Optical WDM Network Planning Using Heterogeneous Multi-granularity OXCs 

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#### Abstract

Multigranular optical WDM network aims to reduce network cost by grouping multiple wavelengths and then switching those wavelengths together at waveband or fiber levels. To configure such multi-granular network, we have two choices- homogeneous network and heterogeneous network. The former applies only a single type of optical cross connect (OXC) while the latter allows different types of OXCs. Due to the demands varies and change asymmetrically geographically, networks with heterogeneous OXC nodes is especially suitable for placing best switching types at different location. In this paper, we aim at the design of an algorithm to solve the network planning problem in optical network with heterogeneous multigranularity OXCs. The planning program determines not only the switching granularity for each node but also determine the routing and wavelength assignments to satisfy the given demands. The contributions of the paper are four folded. First, we propose a graph model to represent the OXC node. The transformed graph simplifies the representation of node structure and helps to model the problem. Secondly, we propose a mathematical formulation to model the network planning problem as an ILP problem. Thirdly, a Lagrangean relaxation based heuristic algorithm is proposed to obtain a near optimal solution. Fourthly, we studied the impact of waveband size on network cost. This work reveals that the waveband size plays a crucial factor. The proposed algorithms can determine the optimal number of waveband size.


Keywords- Optical Network, WDM, Multi-granular OXC, Waveband Switching, Asymmetric Traffic, Network Planning, Lagrangean Relaxation.

## 1. Introduction

WDM technology provides high transmission capacity by multiplexing different wavelengths together in a single fiber. Nowadays, a fiber could support more than 200 wavelengths, and each wavelength channel can carry data with speed higher than 10 Gbps. In an optical network, network topology is usually very irregular and traffic demands are highly asymmetric among nodes. Network planning needs to accurately take these factors into account to determine network structure with lowest cost.
To maintain high scalability and flexibility at low cost, WDM networks often include switching devices with multigranularity switching capability [5, 6]. In particular, examples of multi-granularity switching include switching on a single lambda (LSC), a waveband (WBSC), i.e., multiple lambdas, or an entire fiber (FSC) basis. Although switching by using fine-grained switching capable devices holds higher flexibility,
it incurs higher device costs as well. There are two ways to construct a multi-granular optical network. One is to use a single structured OXC for each node, that contains wavelength, waveband, and fiber switching capabilities in an OXC. Network constructed by such single type multi-granular OXC is called homogeneous network because of the uniform structure in each node. The other way is to construct a heterogeneous network with different structured switch in different node. Each OXC can be any single granularity switch or a combination of them.

Homogeneous network with MG-OXC has advantage on simpler planning and management due to the uniform of switch types. On the other hand, by taking the flexibility of OXC switching capability, heterogeneous network is especially suitable on taking asymmetric traffic distribution into consideration. However, this network planning problem is more challenging than homogeneous networks. Problem for Heterogeneous Optical Network Planning is referred to HtONP problem is this paper.

HtONP problem contains OXC node structure selection and Routing and Wavelength Assignment (RWA) problem. In this work, we assume there is no wavelength conversion capability in each node. Therefore, the end-to-end demand is enforced to follow the same wavelength in the routing path from the source node to the destination node. To the best of our knowledge, there is no prior research address the HtONP problem.

In Fig. 1, we illustrate an example to show the basic idea of HtONP. There are three OD pairs $(1 \rightarrow 5,1 \rightarrow 6$, and $1 \rightarrow 7)$ and the traffic demands are all 10 lambdas. Assume each fiber can carry 15 lambdas and 3 wavebands implying that each waveband includes 5 lambdas. We assume that the cost of OXC is linear dependent on the ports. Then cost of $\mathrm{LSC}=5 \times \mathrm{WBSC}=$ $15 \times$ FSC. Hence, the minimum cost is to have every OXC to be FSC. However, when all OXCs are FSC, we could not locate feasible RWA to meet the traffic requirements. When all OXCs are LSC, we could easily find feasible RWA to meet the traffic requirements. However, a more cost effective solution is to facilitate node 4 as WBSC and the other nodes to be FSC. That is routing 10 lambdas on path $1 \rightarrow 2 \rightarrow 3 \rightarrow 5$, routing 5 lambdas on path $1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6$, routing 10 lambdas on path $1 \rightarrow 4 \rightarrow 7$ and routing 5 lambdas on path $1 \rightarrow 4 \rightarrow 6$.

RWA in a single granularity optical network has been extensively studied (e.g. [1, 9]). In recent years, multi-granular homogeneous network based on MG-OXC has attracted new research interest. Some research works address the RWA problem in homogeneous optical network. In [3], the RWA problem for MG-OXC network has been proposed to minimize
the usage of number of ports. In [4] the RWA for dynamic routing to minimize the call blocking probability is proposed.

In [6], RWA in a heterogeneous optical network with FSC and LSC is first studied. In [11], the work is extended to further include waveband switch. In these two works, the OXC switching granularity is given. But in this paper, the OXC switching granularity is a decision variable. Hence, the planning problem of HtONP determines not only the switching granularity for each node but also determine the routing and wavelength assignments to satisfy the given demands.

We first propose a graph model to represent the OXC node. The transformed OXC graph simplifies the representation of node structure and helps to model the problem. According to the transformed graph, we propose a mathematical formulation to model the HtONP problem as an integer linear programming problem. HtONP problem is an NP-hard problem since the RWA problem embedded and that has been proved to be NP-hard. Instead finding optimal solution, a Lagrangean relaxation based heuristic algorithm is proposed to obtain a near optimal solution. We will show that our proposed algorithm is very close to the optimal solution. Furthermore, we studied the impact of waveband size on network cost. Simulation reveals that the waveband size plays a crucial factor. The proposed algorithms can determine the optimal number of waveband size.

The remainder of this paper is organized as follows. In Section 2, we propose the graph transformation technique for the OXC node and the mathematical formulation. In Section 3, the solution approach to this problem is presented. The numerical experiment is described in Section 4. Finally, in Section 5, concluding remarks are presented.


Figure 1. Example for switch sizing problem

## 2. Problem Formulation

In [11], a bipartite graph has been used to represent a waveband switching OXC. In this work, we extend the graph transform technique to simplify the selection and the modeling of the three switching capabilities in a single OXC. An example for $3 \times 3$ OXC is shown in Fig. 2. In the transformed graph, edges are associated with a binary decision variable to represent the selection of switching capability. For example, if $E_{1, n}^{\text {WBSC }}$ is selected, then node $n$ is a WBSC capable OXC. Bipartite subgraphs are used to represent the configuration of FSC or WBSC switch. Only bipartite matching is a candidate solution.

As shown in Fig. 2, each input fiber link connects to a first stage vertex. First stage edges connect first stage vertices with second stage vertices. Without loss of generality, we assume that each OXC node can facilitate only one of the three switching capabilities, i.e., only one kind - FSC, WBSC, LSC.


Figure 2 Transformed Graph for OXC architecture
In addition, we also assume that there is no wavelength conversion inside each OXC node.

If FSC is selected, in the second stage bipartite FSC subgraph, then selected second stage edges are used to represent the internal switch configuration. The difference between FSC and WBSC is that more than one bipartite subgraph are used in WBSC to represent multiple waveband switching. There is no bipartite transform for LSC due to the capability to switching each wavelength independently.
The HtONP problem is formulated as a linear integer problem stated as follows. Given a physical topology and available wavelengths on each link, and requested lightpath demands between all source-destination pairs, determine the OXC type and routes and wavelengths of lightpaths, such that network building cost is minimized, subject to the wavelength continuity constraint. For ease of illustration, we assume in the sequel that the number of available wavelengths on each link is the same.

We summarize the notation used in the formulation as follows:
Input values:
$N$ : set of switch nodes in the network;
$L$ : set of fiber links in the network;
$E_{1, n}^{F S C}, E_{1, n}^{W B S C}, E_{1, n}^{L S C}$ : set of first stage FSC, WBSC, LSC edge
in node $n$;
$E_{2, n}^{F S C}, E_{2, n}^{F S C}$ : set of second stage FSC, WBSC edges;
$V_{1, n}$ : set of first stage vertices of node $n$;
$V_{2, n}^{\text {in, }{ }^{\text {PSC }}}, V_{2, n}^{\text {out }, F S C}$ : set of FSC's input, output vertices in the
bipartite graph for node $n$;
$V_{2, n}^{\text {in,WBSC }}, V_{2, n}^{\text {out,WBSC }}$ : set of WBSC's input, output vertices of
bipartite graph for node $n$;
$J$ : set of wavelengths on each link;
$W$ : set of OD pairs requesting lightpath set-up;
$B_{n}$ : set of wavebands at node $n$;
$P_{w}$ : candidate path set for OD pair $w$;
$\delta_{p e} \quad:=1$, if path $p$ includes edge $e ;=0$, otherwise;
$\tau_{e j}:=1$, if wavelength $j$ belongs to the waveband edge $e ;=0$, otherwise;
$\theta_{e b}:=1$, if edge $e$ is belong to waveband $b ;=0$, otherwise;
$\sigma_{e v}:=1$, if edge e is incident to node $v ;=0$, otherwise;
$\Psi_{n}^{F S C}, \Psi_{n}^{\text {WBSC }}, \Psi_{n}^{L S C}$ : cost function of installing FSC, WBSC, LSC at node $n$;
Decision variables:
$x_{p j}:=1$, if lightpath $p$ uses wavelength $j ;=0$, otherwise;
$y_{e} \quad:=1$, if edge e is selected; $=0$, otherwise;
$z_{n}^{F S C}, z_{n}^{\text {WBSC }}, z_{n}^{L S C}:=1$ if FSC, WBSC, LSC is installed at node $n ;=0$, otherwise
Problem (P):
$\min \sum_{n \in N} \Psi_{n}^{F S C}\left(z_{n}^{F S C}\right)+\Psi_{n}^{W B S C}\left(z_{n}^{W B S C}\right)+\Psi_{n}^{L S C}\left(z_{n}^{L S C}\right)$
subject to:
$\sum_{p \in P_{w}} \sum_{j \in J} x_{p j}=\lambda_{w} \quad \forall w \in W$
$\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l} \leq 1$
$\forall j \in J, l \in L$
$x_{p j}=0$ or 1
$\forall j \in J, p \in P_{w}, w \in W$
$z_{n}^{F S C}+z_{n}^{W B S C}+z_{n}^{L S C}=1 \quad \forall n \in N$
$z_{n}^{F S C}=0$ or 1
$\forall n \in N$
$z_{n}^{\text {WBSC }}=0$ or $1 \quad \forall n \in N$
$z_{n}^{L S C}=0$ or 1
$\forall n \in N$
$y_{e}=z_{n}^{F S C}$
$\forall e \in E_{1, n}^{F S C}, n \in N$
$y_{e}=z_{n}^{\text {WBSC }}$
$\forall e \in E_{1, n}^{\text {WBSC }}, n \in N$
$y_{e}=z_{n}^{L S C}$
$\forall e \in E_{1, n}^{L S C}, n \in N$
$\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p e} \leq y_{e} \quad \forall j \in J, e \in \bigcup_{n \in N} E_{1, n}^{F S C}$
$\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p e} \leq y_{e} \quad \forall j \in J, e \in \bigcup_{n \in N} E_{1, n}^{W B S C}$
$\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p e} \leq y_{e} \quad \forall j \in J, e \in \bigcup_{n \in N} E_{1, n}^{L S C}$

$$
\begin{array}{ll}
y_{e}=0 \text { or } 1, \forall e \in E_{1, n}^{F S C} \cup E_{1, n}^{W B S C} \cup E_{1, n}^{L S C} \cup E_{2, n}^{F S C} \cup E_{2, n}^{W B S C}, n \in N \\
\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p e} \leq y_{e} & \forall j \in J, e \in \bigcup_{n \in N} E_{2, n}^{F S C} \\
\sum_{e \in E_{2, n}^{F S C}} y_{e} \sigma_{e v}=z_{n}^{F S C} & \forall v \in V_{2, n}^{i n, F S C}, n \in N \\
\sum_{e \in E_{2, n}^{F F C}} y_{e} \sigma_{e v}=z_{n}^{F S C} & \forall v \in V_{2, n}^{o u t, F S C}, n \in N \\
\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p e} \leq y_{e} \tau_{e j} & \forall j \in J, e \in \bigcup_{n \in N} E_{2, n}^{W B S C} \\
\sum_{e \in E_{2, n}^{W S S C}} y_{e} \theta_{e b} \sigma_{e v}=z_{n}^{W B S C} & \forall b \in B_{n}, v \in V_{2, n}^{i n, W B S C}, n \in N \\
\sum_{e \in E_{2, n}^{W B S C}} y_{e} \theta_{e b} \sigma_{e v}=z_{n}^{W B S C} & \forall b \in B_{n}, v \in V_{2, n}^{o u t, W B S C}, n \in N  \tag{20}\\
\sum_{p \in P_{w}} x_{p j} \delta_{p e} \leq 1, \forall w \in W, j \in J, e \in E_{1, n}^{F S C} \cup E_{1, n}^{W B S Q} \cup E_{1, n}^{L S C}, E_{2, n}^{F S C} \cup E_{2, n}^{W B S q} \cup L
\end{array}
$$

The objective function is to minimize the total deployment cost of the switch nodes. Constraint (1) enforces the lightpaths demands of all OD pairs to be satisfied. Constraint (2) indicates that for each fiber link, there can be at most one lightpath using each wavelength, and with Constraint (3) jointly correspond to the wavelength continuity constraint. Constraints (4)~(7) require one kind of switching capability be selected. Constraints (8) $\sim(10)$ require that when OXC node is equipped with a certain kind of switching capability, the associated first stage edge is selected. Constraints (11)~(13) indicate that the wavelength continuity constraints should also be enforced at the first stage edges. Constraints (16), (17), (19), (20) enforce the matching in FSC and WBSC bipartite graph. Constraints (15) and (18) indicate that the wavelength continuity constraints should also be enforced at the second stage edges. Finally, Constraint (21) is a redundant constraint to Constraint (2), (11)~(13), (15) and (18), which is added for optimization purpose.

## 3. Lagrangean Relaxation with Heuristics (LRH)

The Lagrangean relaxation (LR) method [8, 9] has been successfully employed to solve complex mathematical problems by means of constraint relaxation and problem decomposition. Particularly for solving linear integer problem, unlike the traditional linear programming approach that relaxes integer into non-integer constraints, the LR method generally leaves integer constraints in the constraint sets while relaxing complex constraints such that the relaxed problem can be decomposed into independent manageable subproblems. Through such relaxation and decomposition, the LR method as will be shown provides tighter bounds and shorter computation time on the optimal values of objective functions than those provided by the linear programming relaxation approach in many instances [8].

The original primal problem is first simplified and transformed into a dual problem after some constraints are relaxed. If the objective of the primal problem is a minimization (maximization) function, the solution to the dual problem is a lower (upper) bound to the original problem. Such Lagrangean lower bound is a useful by-product in resolving the Lagrangean relaxation problem. Next, due to constraint relaxation, the lower bound solutions generated during the computation might be infeasible for the original primal problem. However, these solutions and the generated Lagrangean multipliers can serve as a base to develop efficient primal heuristic algorithms for achieving a near-optimal upper-bound solution to the original problem. Based on LR, the work reported in [10], [6], and [9] resolved the RWA problems for multi-fiber WDM networks, for multigranularity WDM network, and WDM networks with limited-range wavelength converters, respectively. To the best of our knowledge, the LR approach is first time used in this paper to resolve a network planning problem for multi-granularity WDM networks.
In the sequel, we first give the transformed dual problem and the derivation of the lower bound. We then present the primal heuristic algorithm for obtaining the upper-bound solution.

### 3.1 The Dual Problem and the subproblems

In the relaxation process, Constraints (2), (11), (12), (13), (15), and (18) are first relaxed from the constraint set. The six expressions corresponding to the six constraints, are respectively multiplied by Lagrangean multipliers $q, r, s, t, u$, and $v$ respectively, and then summed with the original objective function. Problem ( P ) is thus transformed into a dual problem, called Dual_P, given as follows:

$$
\begin{align*}
& Z_{\text {dual }}(q, r, S, t, u, v)= \\
& \min \sum_{n \in N}\left(\Psi_{n}^{F S C}\left(z_{n}^{F S C}\right)+\Psi_{n}^{W B S C}\left(z_{n}^{W B S C}\right)+\Psi_{n}^{L S C}\left(z_{n}^{L S C}\right)\right)+ \\
& \sum_{l \in L} \sum_{j \in J} q_{l j}\left(\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p l}-1\right)+\sum_{e \in \bigcup_{E_{1, n}^{F S C}} \sum_{j \in J} r_{e j}\left(\sum_{w \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p e}-y_{e}\right)+} \begin{array}{l}
\sum_{e \in \bigcup_{n \in N} E_{1, n}^{W B S C}} \sum_{j \in J} S_{e j}\left(\sum_{n \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p e}-y_{e}\right)+\sum_{e \in \bigcup_{n \in N} E_{1, n}^{L S C}} \sum_{j \in J} t_{e j}\left(\sum_{n \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p e}-y_{e}\right)+ \\
\sum_{e \in \bigcup_{n \in N} E_{2, n}^{F S C}} \sum_{j \in J} u_{e j}\left(\sum_{n \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p e}-y_{e}\right)+\sum_{e \in \bigcup_{n \in N} E_{2, n}^{W B S C}} \sum_{j \in J} v_{e j}\left(\sum_{n \in W} \sum_{p \in P_{w}} x_{p j} \delta_{p e}-y_{e} \tau_{e j}\right)
\end{array}, l
\end{align*}
$$

Subject to: (1), (3)~(10), (14), (16), (17), (19), (20), and (21).
We can divide Dual_P into two independent subproblems, S1 and S2. Specifically, we have

$$
\begin{equation*}
Z_{d u a l}(q, r, s, t, u, v)=Z_{s 1}+Z_{s 2}-\sum_{l \in L} \sum_{j \in J} q_{l j} \tag{23}
\end{equation*}
$$

where subproblem S 1 is given by $Z_{S 1}(q, r, s, t, u, v)=\min$

$\left.\sum_{e \in \bigcup_{n \in N} E_{1, n}^{L S C}} t_{e j} \delta_{p e}+\sum_{e \in \bigcup_{n \in N} E_{2, n}^{F S C}} u_{e j} \delta_{p e}+\sum_{e \in \bigcup_{n \in N} E_{2, n}^{W B S C}} v_{e j} \delta_{p e}\right) x_{p j}$,
subject to Constraints (1), (3), (21);

Subproblem S 2 is given by $Z_{S 2}(r, s, t, u, v)=$ $\min \sum_{n \in N}\left(\Psi_{n}^{F S C}\left(z_{n}^{F S C}\right)+\Psi_{n}^{W B S C}\left(z_{n}^{W B S C}\right)+\Psi_{n}^{L S C}\left(z_{n}^{L S C}\right)\right)-\sum_{e \in \bigcup_{E \in, n}^{F S C}} \sum_{j \in J} r_{e j} y_{e}$
 subject to Constraints (4)~(10), (14), (16), (17), (19), and (20).

In Subproblem S1, for $x_{p ;}$, it can be decomposed into $|\mathrm{W}|$ independent subproblems, and each of which can be optimally solved by the shortest path algorithm and the link disjoint K-shortest path algorithm. The Modified Successive Shortest Path (MSSP) algorithm [6] can be applied to solve the problem withtime complexity $O(a(m+n \log n))$ for each OD-pair $w \in W$, where $m=|L|+\sum_{n \in N}\left(\left(\text { Inc }_{n}\right)^{2} \times B\right), \quad n=|N|, \quad a=$ $\max \left\{\lambda_{w},|J|\right\}$ and Inc $_{n}$ is the number of incident fibers to node $n$.

In Subproblem S 2 , for $z$ and $y$, they can be decomposed into $|\mathrm{N}|$ independent subproblems, each of which can be optimally solved by an exhaustive search (only three possibilities) on $z_{n}^{F S C}, z_{n}^{\text {WBSC }}, z_{n}^{L S C}$ to determine $y_{e}$. When Constraint (4) is removed for MG-OXC node, there are only eight possibilities. While determining $y_{e}$, Bipartite Weighted Matching algorithm [8] is performed to select the second stage edges. The time complexity of for each node $n \in N$ is $O\left(\left(|J| /\left|B_{n}\right|\right) \times\left(\text { Inc }_{n}\right)^{3}\right)$.

According to the weak Lagrangean duality theorem [8], $Z_{\text {dual }}$ in Equation (23) is a lower bound of the original Problem (P) for any non-negative Lagrangean multiplier vector $\rho=(\boldsymbol{q}, \boldsymbol{r}, \boldsymbol{s}, \boldsymbol{t}, \boldsymbol{u}, \boldsymbol{v})$ $\geq 0$. Clearly, we are to determine the greatest lower bound. Equation (22) can be solved by the subgradient method, as shown as a part of the LRH approach in Fig. 4. As shown in Fig. 4, the algorithm run for a fixed number of iterations (i.e., Max_Iteration_Number). In every iteration, the two sub-problems (S1-S2) are solved (described above), resulting in the generation of a new Lagrangean multiplier vector value. Then, according to Equation (22), a new lower bound is generated. If the new lower bound is tighter (greater) than the current best achievable lower bound (LB), the new lower bound is designated as the LB. Otherwise, the LB value remains unchanged. Significantly, if the LB value remains unimproved for a number of iterations that exceeds a threshold, called Quiescence_Threshold (QT), the step size coefficient ( $\lambda$ ) of the subgradient method is halved, in an attempt to reduce oscillation possibility. Specifically, in the "update-step-size" and "update-multiplier" procedures in Fig. 4, the Lagrangean multiplier vector $\rho$ is updated as $\rho_{k+1}=\rho_{k}+\theta_{k} b_{k}$, where $\theta_{k}$ is the step size, determined by $\theta_{k}=\lambda_{k}\left(U B-Z_{\text {dual }}(\rho)\right) /\left\|b_{k}\right\|^{2}$, in which $\lambda_{k}$ is the step size coefficient, $U B$ is the current achievable least upper bound obtained from the Primal Heuristic Algorithm described next, and $b_{k}$ is a subgradient of $Z_{\text {dual }}(\rho)$.


Figure 3. Example for Reusable Lambdas

### 3.2. The Primal Heuristic Algorithm and Upper Bound

Decision variables in subproblem S2 (for switching node assignment) are adopted as the initial configuration of OXC nodes. Then we solve the RWA problem to satisfy the traffic demands. The multipliers in $\mathrm{S} 2(r, s, t, u, v)$ are assigned as the link arc weight. Based on these link arc weights, Dijkstra's shortest path algorithm is performed on each wavelength to identify the shortest lightpath. Then the arc weight of the selected wavelength along this lightpath is set to infinite to prevent from selecting again. This process is repeated until all traffic demands are all satisfied. If any traffic demand could not be satisfied (i.e. we could not find a lightpath), we choose most promising coarse grained OXC to upgrade its switching capability (i.e., replacing FSC with WBSC, and WBSC with LSC). The most promising OXC is selected based on the following criteria, "number of reusable lambda".

For example, in Fig. 3, we have a FSC OXC with two incoming fiber links (F1, F2) and two outgoing fiber links (F3, F4). Assume the wavelengths in F1 and F2 are dropped at this node and the wavelengths in F4 are added at this node. Assume each fiber carry 8 lambdas and 2 wavebands. Table 1 shows the lambdas in each fiber link that are currently in use.

In Table 1, this OXC node could not switch any lambdas from input fiber links to output fiber links. If we upgrade this OXC node to WBSC OXC and waveband in each fiber link is switching as follows ( $1 W 1$ dropped, $1 W 2->3 W 2,2 W 1->4 W 1$, $2 W 2$ dropped). $1 W 1$ dropped means that waveband 1 (lambdas $1 \sim 4)$ in F1 is dropped at this node. $1 W 2->3 W 2$ means waveband 2 (lambdas $5 \sim 8$ ) in F1 is switching to waveband 2 in F3. By this waveband assignment, there are total eight reusable lambdas ( $1 W 2->3 W 2$ and $2 W 1->4 W 1$ ) for switching at this node. After calculating the number of reusable lambdas for every OXC, we select the one with highest value to be the most promising OXC to upgrade. Then reroute all the traffic demands transmit via this OXC.

When all traffic demands are satisfied, we apply the "downgrading" step (LSC->WBSC, WBSC->FSC) to minimize the deployment cost. We downgrade one OXC at a time to see if it is still a feasible solution with all the RWA assignments remain unchanged. If it is a feasible solution, this OXC node is downgraded, otherwise examine the other OXC

TABLE I. UsED LAMBDAS

|  | F1 | F2 | F3 | F4 |
| :---: | :---: | :---: | :---: | :---: |
| Used <br> Lambdas | $1 \sim 4$ | $5 \sim 8$ | None | $5 \sim 8$ |

node. After the downgrading step, the deployment cost could be reduced significantly. The complexity of the above getting primal heuristic algorithm is $\mathrm{O}\left(|J| n k(p h n)^{2}\right)$ where $k$ is the number of lighpaths, $n$ is the number of nodes, and $p h n$ is the number of vertexes. Note that the number of vertexes is linearly proportional to the number of physical links.

The proposed Lagrangean relaxation and getting primal heuristic algorithms to solve Problem ( P ) is shown in Fig. 4. The complexity of LRH for each iteration is also $\mathrm{O}\left(|J| n k(p h n)^{2}\right)$.

```
Algorithm LRH
begin
    initialize the Lagrangean multiplier vector
        \(q:=0, r:=0, s:=0, t:=0, u:=0, v:=0 ;\)
    \(U B:=\) cost of LSC at every node and \(L B:=0 ; / *\) upper
        bound and lower bound on network cost*/
    quiescence_age: \(=0\);
    step size coefficient \(\lambda_{k}:=2\);
    for each \(k:=1\) to Max_Iteration_Number do
    begin
        solve sub-problem S 1 ;
        solve sub-problem S 2 ;
        \(Z_{\text {dual }}(q, r, s, t, u, v)=Z_{s 1}+Z_{s 2}-\sum_{l \in L} \sum_{j \in J} q_{l j} ;\)
        if \(Z_{\text {dual }}>\) LB then \(L B:=Z_{L R} ;\) quiescence_age \(:=0\);
            else quiescence_age \(:=\) quiescence_age +1 ;
        if quiescence_age \(\geq\) Quiesceince_Threshold then
                    \(\lambda_{k}:=\lambda_{\star} / 2 ;\) quiescence_age \(:=0 ;\)
        run Primal Heuristic Algorithm
        If \(u b<U B\) then \(U B:=u b\);
        /* \(u b\) is the newly computed upper bound */
        update the step size and the multiplier vector
    end;
```

end.

Figure 4. Algorithm LRH
TABLE II. EXPERIMENTAL PARAMETERS

| Parameters | NET 1 | GTE <br> Network | USA <br> Network |
| :---: | :---: | :---: | :---: |
| Nodes | 7 | 12 | 28 |
| Links | 28 | 50 | 90 |
| Wavelengths | 16 | 16 | 16 |
| Wavebands | 4 | 4 | 4 |
| Connectivity | 0.667 | 0.379 | 0.12 |
| Node degree | 4 | 4.167 | 3.214 |
| OD pairs | 28 | $40 \sim 70$ | $30 \sim 50$ |
| Lightpaths | $130 \sim 168$ | $160 \sim 280$ | $120 \sim 200$ |
| Lightpaths/OD pair | $4.6 \sim 6.0$ | 4 | 4 |

## 4. Computational Experiments

We conduct several computational experiments to test the solution quality and effectiveness of our solution approach (LRH). In the computation using LRH, Max_Iteration_Number and Quiesceince_Threshold were set to 1000 and 30 respectively. The step_size_coefficient $(\lambda)$ was initialized to be


Figure 5. Network topology.

2 and became halved when the objective function value of the dual problem did not improve for iterations up to Quiesceince_Threshold. The program was running in a PC with PIV 1.8 GHz CPU. The experimental results of LRH were obtained within one hour of computational time. The accuracy of the algorithms is measured in terms of the duality gap (\%), which is defined as the ratio of the difference of the upper bound (UB) and lower bound (LB) values to the LB value in percentage. In order to show the solution quality of LRH, we have also developed another heuristic algorithm (Lee), which is a modified version of the heuristic algorithm in [6]. The basic idea is that at initial stage, all OXCs are FSC interface, and the RWA is determined from the algorithms from [6]. If any traffic demand could not be satisfied, we upgrade most promising OXCs based on the number of reusable lambda criteria proposed in Sec. 3.2. The upgrading procedure is repeated until all traffic demands are satisfied or stops at infeasibility when all OXC nodes are upgraded to LSC interface.

Several network topologies (NET1, GTE, USA shown in Fig. 5) are tested. Input parameters are shown in Table 2. In realistic network, traffic distribution is asymmetric. Therefore, in Table 2 , we assume there are $20 \%$ of the nodes are large nodes and $80 \%$ of the nodes are small nodes. And the traffic distribution between large nodes to large nodes is $40 \%$, large nodes to small nodes is $40 \%$ and small nodes to small nodes is $20 \%$ [12]. Hence, the ODs pairs and lightpaths demands in Table 2 are generated based on these asymmetric assumptions. And we assume that the cost for OXC node is proportional to the number of ports. Each plotted point in Figs. 6-10 is a mean value over 10 simulation results.

### 4.1. Solution Quality Comparison

In Figs. 6, 7 and 8, we show the OXC deployment cost under different traffic demands. LR1 and LR2 are the LRH algorithms proposed in Fig. 4, where LR1 does not perform downgrading procedure. Lee1 and Lee2 are the heuristic algorithms modified from [6] to include the OXC node structure selection. Lee1 does not perform downgrading procedure. There are three important observations.

First, the results show that the algorithms with downgrading procedure are better than the others (i.e. Lee2 is better than Lee1 and LR2 is better than LR1).

Second, the proposed LRH algorithms (LR2 and LR1) are better than the heuristic algorithms (Lee2 and Lee1). Moreover, when in heavy traffic demands, LRH algorithms are significantly superior to the Lee heuristic algorithms. As shown
in Figure 6 and 7, as lightpath demand larger than 5.6 and number of OD pairs larger than 70, Lee's algorithm cannot find any feasible solution.

Third, the cost is almost linear to traffic demands even in asymmetric traffic distributions. This indicates that the proposed algorithms (LR2) could select the best locations to upgrade the OXC switching capability. In addition, the solutions in LR2 are very close to problem lower bounds. The duality gap between lower bound and upper bound for LRH algorithms are all within $10 \%$ for these experiments.

### 4.2. Optimal Number of Wavebands per Fiber

Since switch cost is mainly determined by the number of ports, in a WBSC switch, the switch with small number of wavelengths per waveband (i.e., small waveband size) owns better switching capability but at high device cost. On the contrary, large waveband size has the advantage of low cost but suffer from coarse granularity. To obtain optimal waveband size, we conduct simulation study for both GTE and USA Networks under heavy lightpath demands.

Based on the results displayed in Fig. 9 and Fig. 10, the deployment costs are expensive for both large and small waveband number. In the former case, the coarse granular WBSC is not able to satisfy the traffic demands such that most of them are upgraded to LSC OXC. In the latter case, the high cost directly comes from the high cost of fine granularity WBSC. The optimal number of waveband is the same as 4 for both networks. Such findings provide a design guideline for deciding number of wavebands per fiber for WBSC switch.

## 5. Conclusions

Mutigranular heterogeneous optical network is able to adapt to network with asymmetric traffic and geographically irregular topology. To resolve the HtONP problem, we first transform the network into a transformed augmented graph that has simplified the modeling of the HtONP problem. The integer linear programming problem is resolved using the LRH method, which is a Lagrangean relaxation based approach with an efficient primal heuristic algorithm. According to the computational experiments on three benchmark network topologies, the primal heuristic algorithm of LRH achieves near optimal solutions that are all within $10 \%$ difference to the problem lower bound values. Furthermore, we studied the relationship of waveband size to the network cost. Experimental results demonstrate that four wavebands per fiber is the best choice for those benchmark networks.

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Figure 6. Solution quality of NET1 network


Figure 7. Solution quality of GTE network


Figure 8. Solution quality of USA network
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Figure 9. Cost structure in GTE network


Figure 10. Cost structure in USA network

