Relay Station Placement Strategy in IEEE 802.16j WiMAX Networks

Hsiao-Chen Lu, *Student Member, IEEE,* Wanjiun Liao, *Fellow, IEEE,* and Frank Yeong-Sung Lin

***Abstract*—In this paper, we study the relay station (RS) placement strategy in IEEE 802.16j WiMAX networks. Specifically, the impact of RS placement on IEEE 802.16j network performance is analyzed. A throughput maximization RS placement problem is mathematically formulated as a binary integer programming problem. We prove the NP-hardness of the formulated problem. To find the sub-optimal solution to the problem with huge input size, we propose an efficient near-optimal placement solution for IEEE 802.16j WiMAX networks. Simulations on the IEEE 802.16j network performance with our RS placement strategy are conducted. The throughput performance shows that with the deployment strategy we proposed, the IEEE 802.16j network capacity can be tremendously enhanced, especially when hotspots are present in the network.**

***Index Terms*—IEEE 802.16j, wireless relay network, placement.**

I. INTRODUCTION

IEEE 802.16 WiMAX is an emerging broadband wireless access technology to the Internet [1][2][3][4][5]. The typical IEEE 802.16 network components include base stations (BSs) and subscriber stations (SSs). Within the coverage of each WiMAX cell, BS is the central entity that controls the channel usage and allocates resource to SSs in both downlink and uplink directions. In WiMAX, adaptive coding and modulation is supported. Each SS negotiates its burst profile with the associated BS before the connection is established. Here the burst profile refers to the set of coding and modulation settings for the transmission between BS and SS to reflect the location-dependent or time-varying link conditions. Different burst profiles lead to different levels of robustness and transmission rates. The worse the channel condition (or the farther the link distance), the more robust the burst profile, and hence the lower the transmission rate.

In the newly defined IEEE 802.16j standard [6], relay stations (RS) were introduced to enhance the coverage, throughput, and system capacity. RSs are placed between BS and SSs. Two types of RSs are defined for the IEEE 802.16j: Transparent Relay Stations (TRSs) and Non Transparent Relay Stations (NTRS). Transparent RSs have no control on the channel usage and resource allocation. They simply decode and forward received data packets to the destinations on the same frequency band as that used by the serving BSs. In this way, BS-TRS links and TRS-MS links have to contend for the resources of the same frequency band. TRSs appear transparent to each MS. When connecting to a TRS, the MS regards it as being connected to a BS. Non-transparent RSs, on the other hand, operate on separate frequency bands from those of their serving BSs. They are able to control resource allocations of their downstream links. Since NTRSs have their dedicated frequency bands, there are no contentions between NTRS-MS links and BS-NTRS links. In general, an NTRS is more expensive than a TRS due to its much more sophisticated capability. In any case, placing an RS between BS and SS may 1) shorten the distance for transmissions (i.e., one long BS-SS link becomes two short BS-RS and RS-SS links), thus allowing a less robust burst profile and a higher transmission rate, and 2) increase the coverage of each BS, thereby enhancing the network capacity. For these reasons, placing RSs in the network is believed to be a cost-effective option for 4G network deployment [7][8]. Moreover, a good placement strategy of relay stations can further enhance the system capacity.

Most research on wireless relay networks focuses on the resource allocation for relay stations or on the design of relay-based MAC protocols [9][10][11]. Existing research results for node placement are mainly for wireless access points or sensor nodes [12][13], aiming at placing homogeneous nodes in the target area to maximize the throughput or connectivity. The work in [14] is among very few that considers placing a given number of relay nodes in a multi-rate WLAN cell with uniformly distributed mobile hosts. The authors then formulate this problem as an optimization problem and solve it with a Lagrangian-relaxation-based iterative algorithm. The results show that an evenly distributed multiple ring structure in the cell is best suited for the scenario with mobile stations uniformly distributed in the network. That work, however, assumes that MSs are uniformly distributed. In IEEE 802.16j, MSs are free to move and thus hotspots may be formed dynamically. This renders an evenly distributed multiple ring structure inappropriate. It is essential to discuss the number and the placement of RSs under a non-uniform MS/traffic distribution. Moreover, the network environment in [14] consists of homogeneous WiFi relay points. As a result, it is not applicable to IEEE 802.16j WiMAX networks in which BS, TRSs and NTRSs are included.

In this paper, we study the RS placement problem in

Paper approved by B. Sikdar, the Editor for Wireless Packet Access and Cross-Layer Design of the IEEE Communications Society. Manuscript received September 23, 2009; revised April 12, 2010.

This work was supported in part by the Excellent Research Projects of National Taiwan University, under Grant Number 97R0062-06, and in part by the National Science Council (NSC), Taiwan, under Grant Number NSC96-2628-E-002-004-MY3.

H.-C. Lu is with the Graduate Institute of Communication Engineering, National Taiwan University, Taipei, Taiwan (e-mail: f95942045@ntu.edu.tw).

W. Liao (corresponding author) is with the Department of Electrical Engineering and the Graduate Institute of Communication Engineering, National Taiwan University, Taipei, Taiwan (e-mail: wjliao@ntu.edu.tw).

Frank Y.-S. Lin is with the Department of Information Management, National Taiwan University, Taipei, Taiwan (e-mail: yslin@im.ntu.edu.tw).

Digital Object Identifier 10.1109/TCOMM.2010.110310.090558

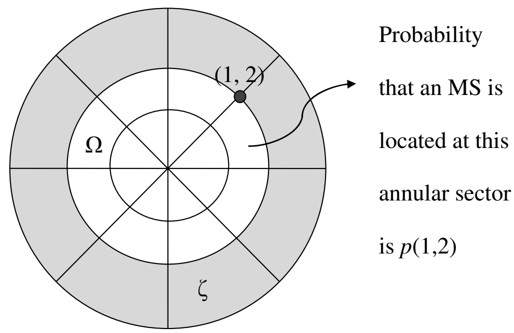


Fig. 1. The coverage and division of an IEEE 802.16j cell.

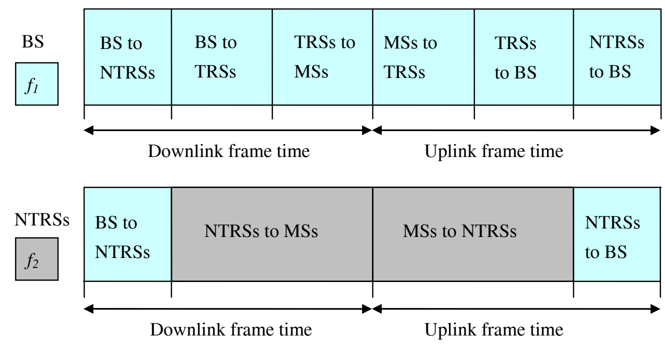


Fig. 2. The resource allocation scheme of the IEEE 802.16j network.

IEEE 802.16j WiMAX network. We consider 1) the resource allocation schemes for a BS to TRSs and to NTRSs, 2) the impact of the location of an intermediate RS on the end-to-end transmission time, and 3) the deployment costs of TRSs and NTRSs. The deployment cost for RSs is a key network planning issue for operators. More specifically, the deployment cost should not exceed the budget determined by operators according to their business considerations. We formulate the RS placement problem via an Integer Linear Programming (ILP) model. The objective of our work is expressed as follows. Given an arbitrary distribution of MSs, we would like to determine the number of TRSs and NTRSs required and where to deploy them such that the network capacity can be maximized under the deployment budget constraint. We prove that this problem is NP-hard, and propose a greedy heuristic to provide a sub-optimal solution to this problem. We then conduct simulations to verify the effectiveness of the proposed algorithm. The results show that the proposed algorithm can achieve significant throughput gain for IEEE 802.16j WiMAX networks.

The rest of this paper is organized as follows. The system model and problem description are included in Section II. The throughput maximization RS placement problem is mathematically formulated and its complexity analyzed in Section III. In Section IV, a greedy heuristic is proposed to find a sub-optimal solution to the RS placement problem. The simulation results are shown and discussed in Section V. Finally, this paper is concluded in Section VI.

II. System Model and Problem Description

A. System Model

We adopt the fixed infrastructure usage model proposed in [15], where fixed relay stations are deployed to extend coverage and enhance user throughput. A BS is located at the center of the target deployment region and the coverage area of the BS is represented by Ω. When RSs are deployed at the boundary of the BS coverage area, the possible coverage region of the BS extended by the RSs is denoted by ζ. The MSs located in ζ are out of the transmission range of the BS. Therefore, they can only access BS via intermediate RSs.

We further divide the extended BS cell (consisting of both Ω and ζ) to a set of small areas as shown in Fig. 1. All the areas in Ω are considered as the candidate locations to deploy RSs. Each small annular sector area is represented by a polar coordinate (i, j), which means that this area is located at the i^{th} sector and j^{th} ring of the cell, as in [14]. Since the BS can approximately locate an MS by detecting the strength and the direction of arrival of the signal transmitted by the MS, dividing the cell into annular sector areas will allow the BS to easily monitor traffic distribution in the network. According to the traffic distribution, we denote the probability of an MS existing in area (i, j) by $p(i, j)$.

In the IEEE 802.16j network, both BS and NTRSs will transmit their own preambles and MAPs to reflect their schedules of downstream transmissions. Therefore, BS and NTRSs need to operate on separate frequency bands to avoid co-channel interference. On the other hand, TRSs simply forward the messages from the associated BS, so they need not transmit their own MAPs. TRSs thus share the same frequency band with the BS. NTRSs may either share the same frequency band among them or have a dedicated frequency band for each of them. If NTRSs share the same frequency band, the coverage area for each should not overlap so as to avoid interference. Each station is assumed to be equipped with only one antenna. As a result, switching between frequency bands is needed if that station (such as an NTRS in this case) has to operate on two frequency bands. To simplify the system model, we assume that MSs can access the network by only one of the following three schemes: 1) connect directly to the BS, 2) connect to the BS via an intermediate TRS, and 3) connect to the BS via an intermediate NTRS. Note that we do not consider three or more hops of relaying in the following discussion due to practical considerations.

Under the non-transparent relaying scheme, only BS-NTRS links will occupy the frame time of the BS. Suppose that both BS and TRSs operate on frequency f_1. The BS needs to sequentially send packets to each NTRS and TRS on f_1. Whenever no packets are coming to an NTRS from the BS, the NTRS can forward the received packets to its downstream MSs on its own frequency band, f_2, without interfering with the on-going transmissions of the BS to other stations. Clearly, the amount of data that one NTRS can transmit to its downstream MSs in a superframe is limited by the portion of the BS frame time allocated to that NTRS. Therefore, an NTRS is not able to support too many MSs (i.e., the number of associated MSs for an NTRS is bounded by a given constant); otherwise, it will be overloaded and unstable. On the other hand, under the transparent relaying scheme, both

$$T_{total} = \sum_{(i,j)\in S} \left\{ \sum_{(\delta,\tau)\in B} \frac{k}{\psi(\delta,\tau;0,0)} y_{(\delta,\tau)}^{(i,j)} + \sum_{(\delta,\tau)\in B} \left[\frac{k}{\psi(\delta,\tau;0,0)} + \frac{k}{\psi(i,j;\delta,\tau)} \right] x_{(\delta,\tau)}^{(i,j)} + \frac{k}{\psi(i,j;0,0)} z^{(i,j)} \right\} \quad (1)$$

$$\begin{aligned} T'_{total} &= \sum_{(i,j)\in S} p(i,j) \left\{ \begin{array}{l} \sum_{(\delta,\tau)\in B} \frac{k}{\psi(\delta,\tau;0,0)} \cdot y_{(\delta,\tau)}^{(i,j)} + \sum_{(\delta,\tau)\in B} \left[\frac{k}{\psi(\delta,\tau;0,0)} + \frac{k}{\psi(i,j;\delta,\tau)} \right] x_{(\delta,\tau)}^{(i,j)} \\ + \left[1 - \sum_{(\delta,\tau)\in B} \left(x_{(\delta,\tau)}^{(i,j)} + y_{(\delta,\tau)}^{(i,j)} \right) \right] \frac{k}{\psi(i,j;0,0)} \end{array} \right\} \\ &= \sum_{(i,j)\in S} p(i,j) \cdot k \left\{ \frac{1}{\psi(i,j;0,0)} - \sum_{(\delta,\tau)\in B} \left[\begin{array}{l} \left[\frac{1}{\psi(i,j;0,0)} - \left(\frac{1}{\psi(\delta,\tau;0,0)} + \frac{1}{\psi(i,j;\delta,\tau)} \right) \right] x_{(\delta,\tau)}^{(i,j)} \\ + \left(\frac{1}{\psi(i,j;0,0)} - \frac{1}{\psi(\delta,\tau;0,0)} \right) y_{(\delta,\tau)}^{(i,j)} \end{array} \right] \right\} \end{aligned} \quad (2)$$

$$\max \sum_{(i,j)\in S} \sum_{(\delta,\tau)\in B} \{ G_{trs}(i,j;\delta,\tau) \cdot x_{(\delta,\tau)}^{(i,j)} + G_{ntrs}(i,j;\delta,\tau) \cdot y_{(\delta,\tau)}^{(i,j)} \} \quad (3)$$

BS-TRS and TRS-MS transmissions are scheduled by BS on freuquency f_1. To avoid interference, only one among BS and all the TRSs can transmit data packets at each time slot. From the discussions above, we summarize the resource allocation scheme for the IEEE 802.16j network in Fig. 2. Note that in Fig. 2 we assume all NTRSs share the same frequency band (i.e. f_2).

The transmission powers of each MS, NTRS, TRS and BS are denoted as P_{ms}, P_{ntrs}, P_{trs}, and P_{bs}, respectively. We use the free space propagation model to calculate the signal to noise ratio (SNR) of each link as follows.

$$SNR = 10 \cdot \log_{10} \left(\frac{P_t}{P_n} \cdot \left(\frac{c}{4\pi f d} \right)^2 \right), \quad (4)$$

where P_t is the transmission power, P_n is the thermal noise power, f is the center frequency, c is the speed of light, and d is the distance. In this way, the achievable data rate can be calculated by $W \cdot \ln(1 + SNR)$, where W is the available bandwidth.

B. Problem Description

Denote by S the set of all coordinates in the extended BS cell and, by B, the set of all candidate coordinates for RS placement in the cell (i.e., the coordinates in Ω). B is a subset of S. Assume that each MS has a minimum traffic demand, e.g., to transmit (or receive) k bytes of data in a superframe. The total transmission time required to fulfill the minimum traffic requirement of all MSs within this network, denoted by T_{total}, can be expressed by (1), where $x_{(\delta,\tau)}^{(i,j)} = 1$ indicates that MS at (i,j) is served by TRS at location (δ,τ), $y_{(\delta,\tau)}^{(i,j)} = 1$ indicates that MS at (i,j) is served by NTRS at (δ,τ), and $z^{(i,j)} = 1$ indicates that MS at (i,j) connects directly to the BS. $\psi(i,j;\delta,\tau)$ is the function which calculates the achievable transmission rate from location (i,j) to (δ,τ). The problem of maximizing the network throughput is then equivalent to minimizing the total transmission time T_{total}.

If an MS is neither connected to a TRS nor to an NTRS, it must be directly connected to the BS. Thus, $z^{(i,j)}$ can be replaced by $1 - \sum_{(\delta,\tau)\in B}(x_{(\delta,\tau)}^{(i,j)} + y_{(\delta,\tau)}^{(i,j)})$ Furthermore, since MSs are mobile, we use the probability mass function $p(i,j)$ to indicate the probability of an MS in area (i,j), instead of fixed MS locations. As such, the total transmission time can be converted to an expected total transmission time, denoted by T'_{total}, as in (2).

Equation (2) shows that the expected total transmission time consists of two components: the total expected transmission time when each MS is directly connected to BS minus the total expected transmission time saved from connecting to intermediate RSs. The first term is constant, so the objective that minimizes the total expected transmission time is equivalent to maximizing the total expected transmission time saved from connecting to RSs.

We further define the performance gains of relaying packets via TRS and NTRS as follows:

$G_{trs}(i,j;\delta,\tau) = p(i,j) \cdot \{\frac{1}{\psi(i,j;0,0)} - (\frac{1}{\psi(i,j;\delta,\tau)} + \frac{1}{\psi(\delta,\tau;0,0)})\}$: the expected transmission time saved if MS at (i,j) connects to TRS at (δ,τ) instead of connecting directly to the BS.

$G_{ntrs}(i,j;\delta,\tau) = p(i,j) \cdot \{\frac{1}{\psi(i,j;0,0)} - \frac{1}{\psi(\delta,\tau;0,0)}\}$: the expected transmission time saved if MS at (i,j) connects to NTRS at (δ,τ) instead of connecting directly to the BS.

In this way, the objective of our problem can be expressed by (3).

III. Problem Formulation and Proof of NP-Hardness

In this section, we formulate the throughput maximization relay station placement (TM-RSP) problem via a binary integer linear programming model. The given parameters and the constraints of the TM-RSP problem are listed in the following. We further prove that the TM-RSP problem is NP-hard.

The given parameters of the TM-RSP problem are:

S: the set of all coordinates in the deployment region;

B: the set of all candidate coordinates for RS placement. B is a subset of S;

r: the transmission range of TRS/NTRS[1];

$p(i,j)$: the probability of an MS in area (i,j);

$\psi(i,j;\delta,\tau)$: the function calculating the achievable transmission rate from location (i,j) to (δ,τ);

$\varphi(i,j;\delta,\tau)$: the function that is set to 1 if and only if the distance between (i,j) and (δ,τ) are no greater than r, the transmission range of RSs; otherwise, it is 0;

[1]Here we let the transmission ranges of TRSs and NTRSs be equal to simplify the expression. It is straightforward to use different transmission ranges for TRS and NTRS.

C_{trs}: the cost of a TRS;

C_{ntrs}: the cost of an NTRS;

C_B: the deployment budget for RS placement;

$NTRS_{cap}$: the maximum number of MSs an NTRS can serve.

The decision variables of the TM-RSP problem are:

$x^{(i,j)}_{(\delta,\tau)}$: equals 1 if MS at (i,j) is served by TRS at (δ,τ), and 0 otherwise.

$y^{(i,j)}_{(\delta,\tau)}$: equals 1 if MS at (i,j) is served by NTRS at (δ,τ), and 0 otherwise.

$t_{(\delta,\tau)} = 1 - \prod_{(i,j)\in S}(1 - x^{(i,j)}_{(\delta,\tau)})$: equals 1 if a TRS must be deployed at (δ,τ), and 0 otherwise. $(\delta,\tau) \in B$.

$n_{(\delta,\tau)} = 1 - \prod_{(i,j)\in S}(1 - y^{(i,j)}_{(\delta,\tau)})$: equals 1 if an NTRS must be deployed at (δ,τ), and 0 otherwise. $(\delta,\tau) \in B$.

The objective of the TM-RSP problem is already shown in (3). Moreover, there are the following constraints for this problem.

$$\sum_{(\delta,\tau)\in B} (x^{(i,j)}_{(\delta,\tau)} + y^{(i,j)}_{(\delta,\tau)}) \leq 1, (i,j) \in S, p(i,j) > 0 \quad (5)$$

$$x^{(i,j)}_{(\delta,\tau)} \leq \varphi(i,j;\delta,\tau), (i,j) \in S, (\delta,\tau) \in B \quad (6)$$

$$y^{(i,j)}_{(\delta,\tau)} \leq \varphi(i,j;\delta,\tau), (i,j) \in S, (\delta,\tau) \in B \quad (7)$$

$$t_{(\delta,\tau)} + n_{(\delta,\tau)} \leq 1, (\delta,\tau) \in B \quad (8)$$

$$\sum_{(i,j)\in S} y^{(i,j)}_{(\delta,\tau)} \leq NTRS_{cap}, (\delta,\tau) \in B \quad (9)$$

$$\sum_{(\delta,\tau)\in B} (C_{trs} \cdot t_{(\delta,\tau)} + C_{ntrs} \cdot n_{(\delta,\tau)}) \leq C_B \quad (10)$$

$$\frac{1}{\psi(i,j;0,0)} - [\frac{1}{\psi(\delta,\tau;0,0)} + \frac{1}{\psi(i,j;\delta,\tau)}] \geq 0, \forall y^{(i,j)}_{(\delta,\tau)} = 1 \quad (11)$$

Constraint (5) indicates that each MS can only connect to one serving station (i.e., a BS, a TRS or an NTRS). Constraints (6) and (7) state that an MS can connect to an RS only when it is in the transmission range of the RS. Constraint (8) ensures that at each possible deployment location (δ,τ), only one RS (either a TRS or an NTRS) can be deployed. Constraint (9) limits the number of MSs each NTRS can serve so as to prevent the NTRS from being overloaded. Constraint (10) guarantees the total deployment cost not to exceed the deployment budget. Constraint (11) excludes the cases that MSs connecting to NTRSs suffer from longer end-to-end delay than connecting directly to the BS.

With these formulations, in the following, we prove the NP-hardness of the TM-RSP problem.

Theorem 1: The TM-RSP problem is NP-hard.

Proof: Consider a special case of the TM-RSP problem in which there is only one type of RS, the TRSs, to be deployed and all MSs access BS via an intermediate TRS. Besides, the transmission time gain for MS at (i,j) connecting to RS at (δ,τ) is independent of (i,j) and (δ,τ) (i.e., it can be denoted as a constant G). In this way, the objective of this new problem P' becomes:

$$P' : \max \sum_{(i,j)\in S} \sum_{(\delta,\tau)\in B} G \cdot x^{(i,j)}_{(\delta,\tau)}. \quad (12)$$

Furthermore, constraints (7) to (9) and (11) can be removed in this case. The remaining constraints are expressed as follows:

$$\sum_{(\delta,\tau)\in B} x^{(i,j)}_{(\delta,\tau)} = 1, (i,j) \in S \quad (13)$$

$$x^{(i,j)}_{(\delta,\tau)} \leq \varphi(i,j;\delta,\tau), (i,j) \in S, (\delta,\tau) \in B \quad (14)$$

$$\sum_{(\delta,\tau)\in B} t_{(\delta,\tau)} \leq \lfloor \frac{C_B}{C_{trs}} \rfloor = K \quad (15)$$

Note that (14) is identical to (6). We can observe that by applying (13), the objective function of P' can be reduced to $|S| \cdot G$, where $|S|$ is the number of elements in set S and G is the constant transmission time gain. Therefore, $|S| \cdot G$ is the optimal objective function value in this special case, and every feasible solution is the optimal solution. Thus, this problem can be described as follows: given the candidate RS deployment location set B, the MS set S, and coverable MSs of each RS location in B (derived from (14)), we want to find a subset of B such that every MS is covered (satisfying (13)) and the total number of RSs will not exceed K.

Clearly, the special case of the TM-RSP problem, P', can be reduced from the set cover problem [16], a well-known NP-complete problem. The set cover decision problem is described as follows. Given a universe U, a set V consisting of subsets of U, and a constant L. A set Q is said to cover U if it is a subset of V and the set union of its elements is equal to U. The problem is to determine if there exists such a set Q of size L or less to cover U.

Now we introduce the mapping from the set cover problem to the special case of the TM-RSP problem. The universe U is mapped to S. Each element in V, i.e., a subset of U, is mapped to a subset of S which represents the MSs that a particular RS can cover. The constant L is mapped to K. In this way, every instance of the set cover problem can be mapped to an instance of P' in polynomial time. Moreover, P' has an optimal solution if and only if the set cover problem has a cover. From the discussion above, the special case of the TM-RSP problem can be reduced in polynomial time from a set cover problem. Since the set cover problem is NP-complete, we conclude that the TM-RSP problem is NP-hard. ■

IV. Solution Approach

A. Proposed Heuristic

The input to TM-RSP are N coordinates and N MS probabilities , where $N = |S|$. In order to accurately approximate the real-world situation in which every location in the BS cell could possibly be a candidate RS deployment location, we need to divide the BS cell into as many annular sectors as possible. However, this gives rise to huge input size and huge number of decision variables. Although some meta heuristics such as genetic algorithm and tabu search are often applied to solve computationally intractable problems, they are very inefficient for the TM-RSP problem due to the very large solution space resulting from the huge input size of the TM-RSP problem. Therefore, our goal here is to propose an efficient heuristic to tackle this difficult problem.

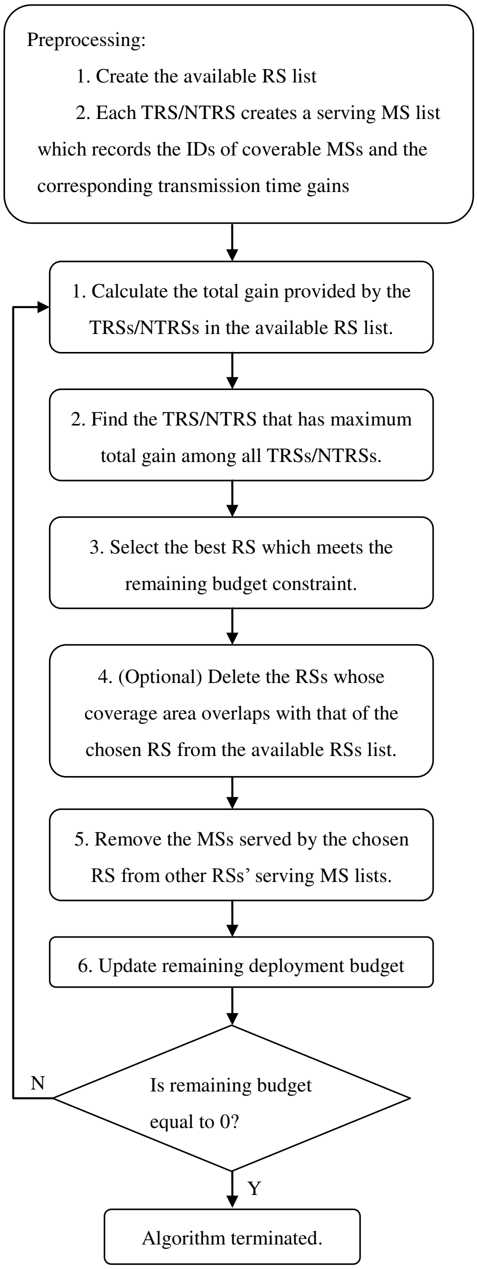


Fig. 3. The flow chart of the proposed algorithm.

The objective of the TM-RSP problem consists of the sum of transmission time gains provided by the deployed RSs to every MS. To approach the optimal solution, each RS to be deployed must be able to provide as much gain to the network as possible. In our algorithm, initially we generate a candidate TRS and a candidate NTRS for each location in B. All the generated candidate RSs are recorded by an available RS list. Also, each RS maintains its serving MS list, which records the set of MSs that is likely to be served by this RS. In Step 1, we calculate the total transmission time gains provided by an RS to the network, which is the summation of the transmission time gains provided by the RS to the MSs in its serving MS list. Then we find the TRS and NTRS with the maximum total transmission time gains in Step 2, and determine which one to be deployed without violating the budget constraint in Step 3. Two decision metrics are considered: the total gain value and the gain per unit cost value. To further reduce the deployment cost, in Step 4, we can delete the candidate RSs that are too close to the deployed RS. In Step 5, the MSs in the serving MS list of the deployed RS will be removed from the serving MS lists of other RSs. Also, the RSs on the chosen location will be removed from the available RS list to avoid duplicated deployment. Then we update the deployment budget in Step 6. If the remaining budget is not zero, we go back to Step 1, finding the next RS to be deployed, until the remaining budget equals zero. The proposed algorithm is summarized in Fig. 3.

B. Performance Analysis of the Proposed Heuristic

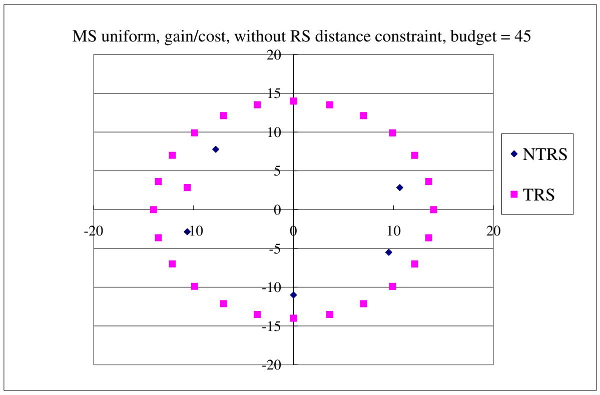
Let M be the number of coordinates in ζ. From our system model, M is a fraction of N. Thus, the number of coordinates in Ω is $N - M$, which is also the number of candidate RS locations. In the preprocessing phase of our algorithm, each candidate TRS and NTRS needs to create its serving MS list, which takes $O(N^2)$ time. In Step 1 and Step 5, the serving MS list of every candidate RS must be checked, which also takes $O(N^2)$ time. Other steps in our algorithm takes no more than $O(N)$ time. Since there can at most be $\lfloor \frac{C_B}{C_{trs}} \rfloor = K$ RSs deployed in the network, K is the upper bound of the number of iterations of our greedy heuristic. Thus, the time complexity of our algorithm is $K \cdot (O(N^2) + O(N)) = O(N^2)$.

Due to the integer nature of the decision variables, the proposed heuristic algorithm may only achieve sub-optimality of the original problem. Besides, the very large input size and the integer nature of our problem limits the possibility from directly finding the optimal solution with mathematical tools such as MATLAB, Lingo, etc. Consequently, we try to verify the effectiveness of the proposed heuristic by calculating the approximation ratio to a relaxed problem P'', in which the binary constraints of the decision variables are relaxed and the other constraints remain the same as those of the TM-RSP problem. Since we left other constraints of P'' the same as those of the TM-RSP problem, the optimal solution to P'' has similar performance to that of TM-RSP. Moreover, by relaxing the hard binary decision constraints, the optimal solution to P'' can be solved by commercial mathematical tools efficiently and it can serve as a performance index for TM-RSP.

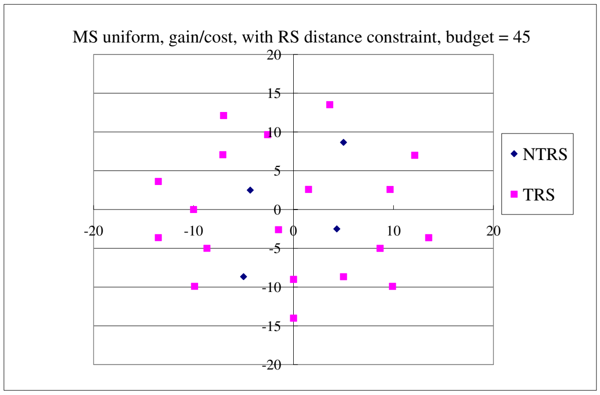
The feasible solution set of P'' will contain the feasible solution set of TM-RSP. If we search through the feasible solution set of P'' to find the optimal solution, the whole feasible solution set of TM-RSP will also be searched. Since the objective function of P'' and TM-RSP are the same, the optimal solution of P'', denoted as $s''*$, should be no worse than the optimal solution of TM-RSP, denoted as $s*$. In other words, the objective function value of P'', derived from $s''*$, is no less than the objective function value of TM-RSP, derived from $s*$. Thus, we have $f_{TM-RSP}(s*) \leq f_{P''}(s''*)$, where $f_P(s)$ is the value of the objective function of problem P given the solution s. We find the optimal solution of P'' (i.e., $s''*$) by Lingo and the sub-optimal solution to TM-RSP, $s_{heuristic}$, by our heuristic. From the discussion above, we have:

$$\frac{f_{TM-RSP}(s_{heuristic})}{f_{TM-RSP}(s*)} \geq \frac{f_{TM-RSP}(s_{heuristic})}{f_{P''}(s''*)} \qquad (16)$$

Although we cannot find $\frac{f_{TM-RSP}(s_{heuristic})}{f_{TM-RSP}(s*)}$, the approximation ratio of our heuristic, we can find $\frac{f_{TM-RSP}(s_{heuristic})}{f_{P''}(s''*)}$

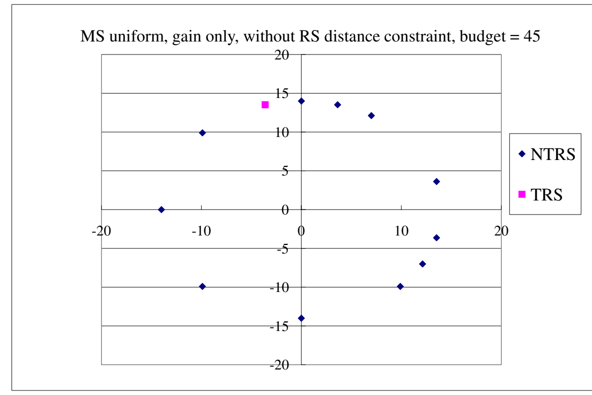


(a) Without RS distance constraint

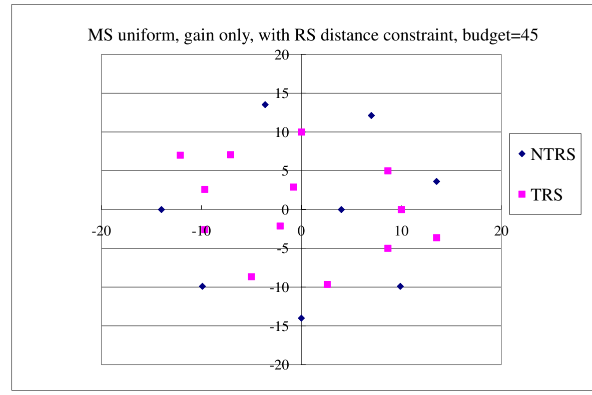


(b) With RS distance constraint

Fig. 4. The calculated RS locations for uniformly distributed MSs (decision metric is the gain per unit cost).



(a) Without RS distance constraint



(b) With RS distance constraint

Fig. 5. The calculated RS locations for uniformly distributed MSs (decision metric is the total gain).

and it serves as the lower bound of the approximation ratio of our heuristic.

V. Performance Evaluation

In this section, we evaluate the performance of our proposed heuristic by computer simulations. We consider one cell only, in which a BS is located at the center, and generate two different MS distribution probabilities: concentrated in one hotspot and uniform distribution. There are two types of RSs to be deployed: TRSs and NTRSs. There are two versions of our heuristic: with or without the RS distance constraint (i.e., Step 4 in Fig. 3). In the case with the RS distance constraint, the distance between NTRSs is no less than $2r$ and the distance among other RSs (e.g., between a NTRS and a TRS or between TRSs) is no less than r. The transmission range of a BS is 15 km and that of an RS is 5 km. The angle of a sector is 15 degrees and the width of a ring is 1 km. The capacity limitation of each NTRS is serving at most 25 MSs. The cost of one TRS is 1 unit and the cost of one NTRS is 4 unit.

The RS placements determined by the two versions of our heuristic for the uniform MS distribution scenario are shown in Fig. 4 and Fig. 5. Comparing Fig. 4 and Fig. 5, we observe that NTRSs are preferred if we use the total gain of RSs as our decision metric. On the contrary, TRSs are the better choice if we use the gain per unit cost as our decision metric. The effect of the RS distance constraint is also shown in both figures. The RS placements with the RS distance constraint tend to spread out across the cell while those without the RS distance constraint do not.

For each RS placement determined by our heuristic with two different MS distributions, we also calculate the lower bound of the approximation ratio. Fig. 6 depicts that for both MS distributions, the RS placements generated by our heuristic can achieve at least 90% of the optimal solution if there is no RS distance constraint and the transmission time gain is the only decision metric. This shows that our heuristic can efficiently find the near-optimal solution to TM-RSP. If we further consider the RS distance constraint and the gain per unit cost as our decision metric, the lower bound of the approximation ratio will decrease, which means that the objective value of the heuristic solution will possibly be farther (i.e., smaller) from that of the optimal solution. Nevertheless, in most cases, the approximation ratios are no worse than

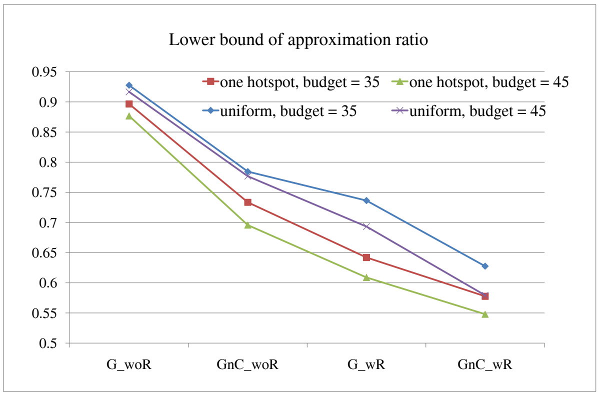
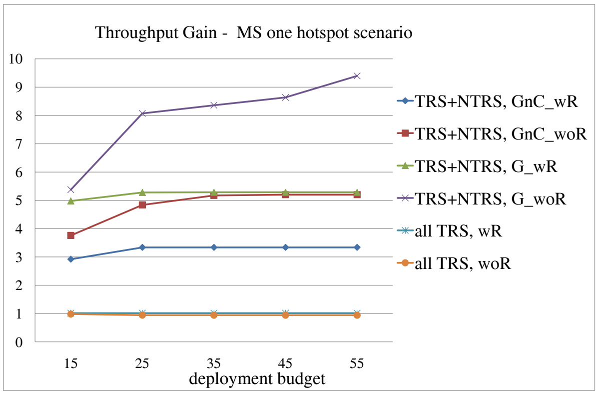
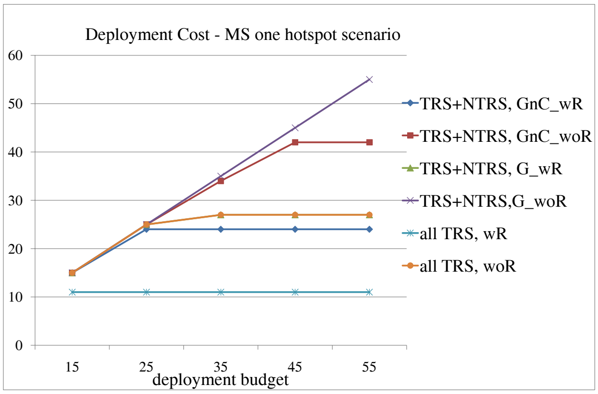


Fig. 6. The lower bound of the approximation ratios for two MS distributions.(G: decision metric is gain; GnC: decision metric is gain/cost. wR: with RS overlapping constraint, woR: without RS overlapping constraint).



(a) Throughput gain



(b) Deployment cost

Fig. 7. Simulation results for MS one hotspot distribution.(G: decision metric is gain; GnC: decision metric is gain/cost. wR: with RS overlapping constraint, woR: without RS overlapping constraint).

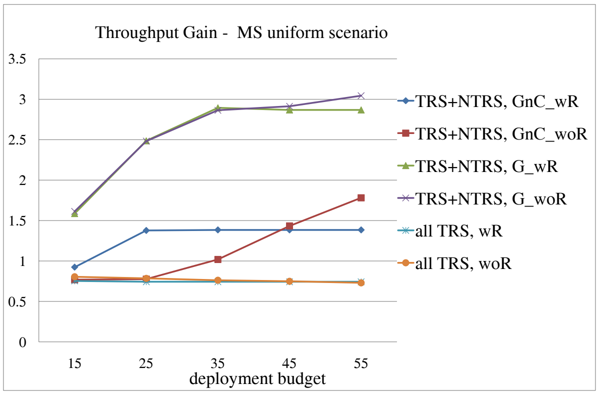
65%. Despite the fact that adding the RS distance constraints may reduce the network aggregate transmission time gain, it could also provide RSs the chance to reuse frequency band. The benefits brought by frequency reuse may balance the transmission time loss.

We next simulate the throughput gain and deployment cost with different RS placement strategies. For both MS probabilistic distributions (i.e., uniform and one hotspot), 100 sets of MS locations, each with 200 MSs, are generated according to the MS distribution probabilities. The RS placements generated by our heuristic for both MS distributions along with 100 sets of MS locations are parsed into an IEEE 802.16j network simulator. The cases with no RS deployed, i.e. only MSs in Ω are able to access BS, are also simulated. In all simulations, every MS will generate uplink CBR traffic at a rate of 16 kBps. Under different deployment budget, the actual deployment cost and the throughput gain (i.e., the network throughput with RSs deployed divided by the network throughput without any RSs) for both MS distributions are shown in Fig. 7 and Fig. 8. The total network throughput can be greatly improved when NTRSs are deployed. The performance can be further improved when MSs form hotspots. In fact, when MSs are more densely gathered, more MSs can benefit from one single RS. That is why there are less RSs to be deployed in the one-hotspot scenario than in the uniform scenario, given the same deployment budget. Comparing the RS placements with and without the RS distance constraint, we observe that for those placements without RS distance constraint, the deployment budget will be depleted in most cases and the throughput gain is higher than that of the placements with the RS distance constraint. When MSs are densely gathered, as in the one hotspot scenario, there are not many candidate RS locations with transmission time gain. Thus, it is not required to set the RS distance constraint to sacrifice throughput gain and therefore reduce the deployment cost. In Fig. 7(b), when the deployment budget reaches 45 units, the deployment cost will often saturate even without the RS distance constraint. However, as shown in Fig. 8(b), when the MSs are spread out all over the network, setting the RS distance constraint will further reduce the deployment cost while the throughput gain is still almost as high as that without the constraint.

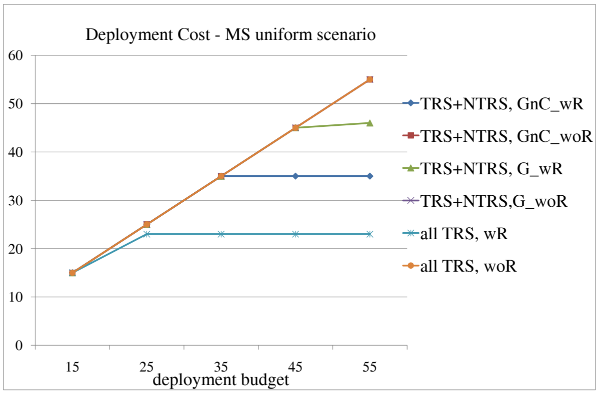
For both MS distributions, we can also observe that only deploying TRSs may not increase the total network throughput. Due to the coverage extension by TRSs, some MSs far from the BS can still enter the network and contend for the resource of the BS with MSs closer to the BS. The fact that the network has to accommodate more low-rate MSs makes the total network throughput lower than that when only high-rate MSs are present.

VI. Conclusion

In this paper, the impact of relay station placement on the network performance in the IEEE 802.16j network is studied. Specifically, the throughput maximization relay station placement problem is mathematically formulated and its complexity is analyzed. An efficient heuristic algorithm is proposed to determine the number and locations of relay station deployment, with a given MS distribution and deployment budget. The simulation results show the different impacts of deploying TRSs and NTRSs on the performance of the IEEE 802.16j network. Moreover, we verify that by properly planning the RS placements, the capacity of IEEE 802.16j networks can be greatly enhanced by RSs, especially when hotspots are present.



(a) Throughput gain



(b) Deployment cost

Fig. 8. Simulation results for MS uniform distribution.(G: decision metric is gain; GnC: decision metric is gain/cost. wR: with RS overlapping constraint, woR: without RS overlapping constraint)

References

[1] "IEEE standard for local and metropolitan area networks part 16: Air interface for fixed broadband wireless access systems," *IEEE Std 802.16-2004 (Revision of IEEE Std 802.16-2001)*, pp. 1-857, 2004.

[2] W.-H. Kuo, W. Liao, and T. Liu, "Adaptive resource allocation for layer-encoded IPTV multicasting in IEEE 802.16 WiMAX wireless networks," *IEEE Trans. Multimedia Special Issue Quality-Driven Cross-Layer Design Multimedia Commun.*, May 2009.

[3] C. Chiang, W. Liao, T. Liu, I. Chan, and H. Chao, "Adaptive downlink and uplink channel split ratio determination for TCP-based best effort traffic in TDD-based WiMAX networks," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 2, pp. 182-190, Feb. 2009.

[4] B. Rong, Y. Qian, K. Lu, H. Chen, and M. Guizani, "Call admission control optimization in WiMAX networks," *IEEE Trans. Veh. Technol.*, vol. 57, no. 4, pp. 2509-2522, July 2008.

[5] S. Ahmadi, "An overview of next-generation mobile WiMAX technology," *IEEE Commun. Mag.*, vol. 47, no. 6, pp. 84-98, June 2009.

[6] "IEEE standard for local and metropolitan area networks part 16: Air interface for broadband wireless access systems amendment 1: Multiple relay specification," *IEEE Std 802.16j-2009 (Amendment IEEE Std 802.16-2009)*, pp. c1-290, June 2009.

[7] R. Pabst, B. Walke, D. Schultz *et al.*, "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Commun. Mag.*, vol. 42, no. 9, pp. 80-89, Sept. 2004.

[8] D. Schultz and B. Walke, "Fixed relays for cost efficient 4G network deployments: An evaluation," in *IEEE 18th International Symp. Personal, Indoor Mobile Radio Commun. (PIMRC)*, Sept. 2007, pp. 1-5.

[9] Y. Liu, R. Hoshyar, X. Yang, and R. Tafazolli, "Integrated radio resource allocation for multihop cellular networks with fixed relay stations," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 11, pp. 2137-2146, Nov. 2006.

[10] C. Hong and A. Pang, "3-approximation algorithm for joint routing and link scheduling in wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 2, pp. 856-861, Feb. 2009.

[11] S. Zou, B. Li, H. Wu, Q. Zhang, W. Zhu, and S. Cheng, "A relay-aided media access (RAMA) protocol in multirate wireless networks," *IEEE Trans. Veh. Technol.*, vol. 55, no. 5, pp. 1657-1667, Sept. 2006.

[12] S. Misra, S. Hong, G. Xue, and J. Tang, "Constrained relay node placement in wireless sensor networks to meet connectivity and survivability requirements," in *INFOCOM 2008, IEEE 27th Conf. Computer Commun.*, Apr. 2008, pp. 281-285.

[13] X. Ling and K. L. Yeung, "Joint access point placement and channel assignment for 802.11 wireless LANs," *IEEE Trans. Wireless Commun.*, vol. 5, no. 10, pp. 2705-2711, Oct. 2006.

[14] A. So and B. Liang, "Enhancing WLAN capacity by strategic placement of tetherless relay points," *IEEE Trans. Mobile Comput.*, vol. 6, no. 5, pp. 522-535, May 2007.

[15] J. S. et al., "Harmonized contribution on 802.16j (mobile multihop relay) usage models," *IEEE 802.16j-06/015*, 2006.

[16] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to Algorithms*, 2nd ed. The MIT press, 2001.

Hsiao-Chen Lu Hsiao-Chen Lu received her BS degree in Electrical Engineering from National Taiwan University (NTU), Taipei, Taiwan. She is currently a PhD candidate in the Graduate Institute of Communication Engineering, NTU. Her research interests are in the field of network planning and performance optimization for relay-assisted wireless access networks, especially for the IEEE 802.16j WiMAX network and the 3GPP Long Term Evaluation (LTE) network. She is a student member of IEEE.

Wanjiun Liao Wanjiun Liao received her Ph.D. degree in Electrical Engineering from the University of Southern California, USA, in 1997. She is a Distinguished Professor of Electrical Engineering, National Taiwan University (NTU), Taipei, Taiwan, and an Adjunct Research Fellow of Research Center for Information Technology Innovation, Academia Sinica, Taiwan. Her research focuses on the design and analysis of wireless and multimedia networking protocols. Prof. Liao is currently an Associate Editor of IEEE Transactions on Wireless Communications, and she has served on the editorial board of IEEE Transactions on Multimedia. She has also served on the organizing committees of many international conferences, including serving as Tutorial Co-Chair of IEEE INFOCOM 2004, TPC Area Chair of IEEE International Conferences on Multimedia and Expo (ICME) 2004, TPC Vice Chair of IEEE GLOBECOM 2005 Symposium on Autonomous Networks, TPC Co-Chair of IEEE GLOBECOM 2007 General Symposium, TPC Co-Chair of IEEE VTC 2010 Spring and TPC Co-Chair of IEEE ICC 2010 Next Generation Networking and Internet Symposium. Prof. Liao has received many research awards and recognitions from different government and professional organizations, including IEEE and ACM. She was a recipient of the Republic of China (R.O.C.) Distinguished Women Medal in 2000. She is a Fellow of IEEE.

Frank Yeong-Sung Lin Prof. Yeong-Sung (Frank) Lin received his BS degree in electrical engineering from the Electrical Engineering Department, National Taiwan University in 1983, and his Ph.D. degree in electrical engineering from the Electrical Engineering Department, University of Southern California in 1991. After graduating from the USC, he joined Telcordia Technologies (formerly Bell Communications Research, abbreviated as Bellcore) in New Jersey, U.S.A., where he was responsible for developing network planning and capacity management algorithms for a wide range of advanced networks. In 1994, Prof. Lin joined the faculty of the Electronic Engineering Department, National Taiwan University of Science and Technology. Since 1996, he has been with the faculty of the Information Management Department, National Taiwan University. His research interests include network optimization, network planning, performance evaluation, high-speed networks, wireless communications systems, distributed algorithms, information security and biometrics.