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A Lagrangean relaxation based sensor deployment algorithm to optimize quality of service for target positioning

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ABSTRACT

The target positioning service is one of useful applications for wireless sensor networks. So far, most papers considered traditional *uniform quality of services (QoS) for target positioning* in sensing fields. However, it is possible that all regions in a sensing field have different requirements for target positioning accuracy. We also concern the terrain of sensing fields might have some limitations for placing sensors. Therefore, this paper proposes a generic framework for the sensor deployment problem supporting *differential quality of services (QoS) for target positioning* to all regions in a sensing field. We define *weighted error distance* as metric of quality of positioning services. This problem is to optimize the *QoS level* for target positioning under the limitations of budget and *discrimination priorities* of regions, where locations and sensing radiuses of all sensors should be determined. We formulate the problem as a nonlinear integer programming problem where the objective function is to minimize of the maximum *weighted error distance* subject to the complete coverage, deployment budget, and discrimination priority constraints. A Lagrangean relaxation (LR) based heuristic is developed to solve the NP-hard problem. Experimental results reveal that the proposed framework can provide better quality of services for positioning than the previous researches, which only handles uniform QoS requirements. Moreover we evaluate the performance of proposed algorithm. As well as we adopt the previous algorithm, ID-CODE, as the benchmark to examine the proposed heuristic. The results show the proposed algorithm is very effective in terms of deployment cost.

1. Introduction

In a wireless sensor network (WSN), numbers of tiny sensor nodes collect physical information, process and forward the local information to the sink nodes. Hence, the back-ends can obtain global views and make appropriate decisions according to the information provided by the sensors (Akyildiz, Su, Sankarasubramaniam, & Cayirci, 2002). Obviously, the quality of the information is important, which dominates the final decisions of back-end in WSN. The quality of information can be improved by constructing a WSN providing high quality data through careful planning in the sensor deployment phase (Maleki & Pedram, 2005; Yan, He, & Stankovic, 2003).

One of useful applications for sensor network is the target location (Zou & Chakrabarty, 2003), i.e. target positioning, which decides the position of targets by cooperation of sensors in a sensor network (Chakrabarty, Iyengar, Qi, & Cho, 2002; Li, Xu, Pan, & Pan, 2005). Hence, the sensors must be deployed reasonably. Besides, the sensor network coverage has to cover the whole sensing field, if the coverage areas of multiple sensors overlap, they may all report a target in their respective zones. Based on the reports, location of the target can be determined by back-ends. If the target in a zone (i.e. region) can be detected by an unique set of sensors, the zone is denoted by distinguishable zone. The diameter of a distinguishable zone determines the granularity of the target positioning. Some papers use *service points* as reference points to replace zone/region for positioning services (Ray, Starobinski, Trachtenberg, & Ungrangsi, 2004; Ray, Ungrangsi, Pellegrini, Trachtenberg, & Starobinski, 2003). When sensor networks decide a target being at a certain service point, it means the target might occur on the zone/region including the service point.

Wang et al. define quality of service (QoS) parameters for OSI function layers (Wang, Liu, & Yin, 2006). In this paper, we take account of *positioning accuracy* as the QoS parameters, which is one of kind of data qualities for application layer. We also define the *error distance* as metric of positioning accuracy. In our previous paper, the uniform (i.e. fixed) sensing radius of all sensor nodes and uniform positioning quality in a sensing field are taken account in the

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problem, where the all locations of sensors are decision variables (Lin & Chiu, 2005). Nevertheless, several papers adopt various (i.e. adjustable) sensing radiuses to cope the sensor deployment problem under coverage and cost limitations (Chakrabarty et al., 2002; Dhawan, Vu, Zelikovsky, Li, & Prasad, 2006; Dhillon & Chakrabarty, 2003). In addition, given the locations of all sensors, the positioning accuracy can be improved by adjusting sensing radius of each sensor (Chiu, 2007; Chiu, Lee, & Peng, 2008). It motivated us to take account sensing radius as one of decision variables in this paper.

So far, most papers consider traditional uniform quality of positioning services in a sensing field. However, it is possible that all regions in a sensing field require different accuracy for target positioning services. For addressing this issue, we define the *discrimination priority* for each region of interest and redefine *weighted error distance* as the metric to measure positioning accuracy. In addition, we also concern the terrain of sensing fields might have some limitations for placing sensors. Therefore, this paper proposes a generic framework for the sensor deployment problem to support differential QoS for target positioning.

This problem is to optimize the QoS level for target positioning under the limitations of budget and *discrimination priority* of each region, where the number of sensors, location and sensing radius of each sensor should be determined. We formulate the problem as a nonlinear integer programming problem where the objective function is to minimize the maximum *weighted error distance* subject to the complete coverage, deployment budget, and discrimination priority constraints.

The sensor deployment problem where subject to coverage, can be corresponded to the set-cover problem, as well as the sensor deployment for target positioning problem also can correspond to the identifying code problem. Both the two classical difficult problems were proven to be NP-complete (Charon, Hudry, & Lobstein, 2002; Etcheberry, 1977; Slijepcevic & Potkonjak, 2001). Identifying code problems have been studied widely since 1998 (Karpovsky, Chakrabarty, & Levitin, 1998). A lot of research groups focused on information theoretical investigations, simultaneously several researchers made efforts in solving application oriented problems. As well as the target positioning problem belongs to the latter. From a theoretical perspective, the proposed sensor deployment problem is a variation of classical identifying code problems. Surveillance and target positioning ability are adopted as the QoS parameters of the sensor deployment problem in this paper; we formulate the sensor deployment problem as a mathematical optimization model. Hence, the proposed optimization problem is NP-hard by the definition (Charon, Hudry, & Lobstein, 2003; Neapolitan & Naimipour, 2004). To our knowledge, so far using the weighted error distance as a metric to measure positioning quality and allowing differential positioning quality requirements for all regions are not handled in previous papers.

Afterwards, a Lagrangean relaxation (LR) based heuristic is developed to cope with the NP-hard problem (Fisher, 1981, 1985). In sensor deployment phase, the proposed efficient algorithm can be executed at powerful computers or back-ends of WSNs to decide topology of sensor networks under the complete coverage, deployment budget, and discrimination priority constraints; it should not run at any sensors after deployment. This paper adopts the previous algorithm, ID-CODE, as the benchmark to examine the proposed algorithm.

The rest of this paper is organized as follows: Section 2 presents related work. In Section 3, the problem is described. Section 4 formulates the problem as a mathematical optimization model. Sections 5 and 6 present the algorithms and computational results. In Section 7, the performance of the proposed Lagrangean relaxation based algorithm is evaluated. Finally, we conclude the paper in Section 8.

2. Related work

In this section, we review the related positioning techniques and the previous papers from practical and theoretical perspectives.

2.1. Positioning systems

Several positioning systems (i.e. location systems) have been proposed and realized. For instance, the satellite-based global positioning system (GPS) is a common outdoor location system. However, GPS is not useful in indoor, dense, or harsh environments (Ray et al., 2004).

Hightower and Borriello presented three main techniques for location-sensing, including: triangulation, scene analysis, and proximity, which may be engaged in location systems individually or in mixing (Hightower & Borriello, 2001). Due to wave frequency, the indoor location systems were classified into three categories: infrared, ultrasound, and radio (Ray et al., 2004). These systems determine distance according to the signal strength and a known signal-to-noise ratio (SNR). Hence, the sensitivity for environmental conditions is very significant; quality of positioning is injured by interference frequently.

Bulusu, Heidemann, and Estrin (2000) suggested placing multiple beacons in a positioned field with overlapping regions of coverage and transmitting periodic beacon signals. Targets can be localized to the centroid of their proximate reference points. The beacons in positioning fields can be analogous to sensors of sensing fields in this paper.

So far the existing techniques for positioning have its characteristics and weaknesses. The location system designers can choose appropriate approaches according to situations and requirements of real-world. Therefore, sensor networks supporting positioning services can provide another low-cost choice for them.

2.2. Identifying codes

In Karpovsky et al. (1998), *identifying codes* first are proposed as a means for uniquely identifying malfunctioning processors in multiprocessor systems. Such systems can be modeled as a graph $G = (V,E)$, where V is the set of processors and E is the set of links between processors. Several researchers adopted concept of identifying codes (i.e. *power vectors* in this paper) to construct location systems. Given an undirected graph G and integer r (r-cover for each vertex), Charon et al. (2003) have proved the decision problem of the existence of an r-identifying code of size at most k codeword in G, is NP-complete for any r.

The goal of the identifying code problem is to find an identifying code with minimum cardinality for a given directed or undirected graph. Laifenfeld and Trachtenberg surveyed identifying code researches from applications, theoretical connections, and approximating optimal solutions perspectives (Laifenfeld & Trachtenberg, 2008); clearly classifying the research field. Both theoretical investigations and practical applications, the identifying code problem has been studied widely. As well as the target positioning problem belongs to the latter. From a theoretical perspective, the proposed sensor deployment problem is a variation of classical identifying code problems.

2.3. CIQ approach

Chakrabarty et al. (2002) investigated target location problem in sensor networks. The sensing field is presented as a (two or three dimensional) grid of points. If the coverage areas of multiple sensors overlap, they may all report detecting a target, then the location of the target can be determined by overlap of these sensor's detection zones. If every grid point in the sensing field is covered by an unique subset of sensors, location systems can easily determine the target occurring and its location by the set of reporting sensors.

Chakrabarty et al. solve the problem of placing sensors for unique target identification by the theory of identifying codes; we denote it by *CIQ approach* in this paper. The authors first build a primitive block which is completely discriminable by sensors on the block. Then, a larger sensing field can be constructed by tiling primitive blocks on the sensing field. However, this placement manner can use in regular sensing fields and fixed sensor detection radius. In additional, the grid points in boundary of sensing fields do not take into consideration by *CIQ approach* directly. Therefore, additional placement methods are necessary for achieving a completely distinguishable sensing field.

2.4. ID-CODE

Ray et al. apply identifying code theory to design a location system in sensor networks (Ray et al., 2004). They divide a continuous sensing field into a finite set of locatable regions represented by a designated point (a *service point* in this paper). Each service point can be identified unambiguously.

Ray et al. propose ID-CODE algorithm to deploy sensors in some of service points and build an identifying code for the given graph. First, every vertex is deployed a sensor, as a codeword. In each loop of ID-CODE algorithm, one of codeword is checked whether it can be deleted and results in identifying code. The authors suggest three predetermined sequences to visit vertices: random, descending, and ascending orders. The performance of ID-CODE algorithm depends on the sequence of vertices. Therefore, the authors propose a hybrid heuristic for ordering. When the average degree of graph is greater than half the number of vertices, the authors propose the descending order of degree is used. Otherwise, the ascending order is used in ID-CODE algorithm.

Both CIQ and ID-CODE approach are sophisticated; the papers significantly contribute to target positioning research field. Hence, CIQ approach is difficult by various sensing radius, we only adopt ID-CODE algorithm as one of benchmarks to examine the proposed algorithm in this paper.

3. Problem description

A number of papers investigate the sensor placement problems with grid based sensing field (Chakrabarty et al., 2002; Dhillon & Chakrabarty, 2003; Dhillon, Chakrabarty, & Iyengar, 2002). A grid based sensing field can be represented as a collection of two- or three-dimensional *grid points*. In this paper, we adopt grid based placement method and organize the sensing field as a set of grid points. The grid point, which requires the surveillance or positioning service, is also called *service point* in this paper. The distance between two adjacent grid points is used as the length unit. The granularity of the grid points is determined by the requirement for the positioning accuracy of application systems. A set of sensors can be deployed on the grid points to monitor the sensing field. The forms of sensing fields are not limited in the paper; any irregular forms should be pre-plotted to grid based fields, as well as the proposed approach can handle them. But, we still use rectangular sensing fields as a case for discussion in this paper.

If each *grid point/service point* in a sensing field can be detected by at least one sensor, we call the field is *completely covered*, as shown in Fig. 1. In this context, a target can be detected at any place in the field (Lin & Chiu, 2005). A *power vector*, which is analog to identifying code, is defined for each service point to indicate whether sensors can cover the service point in a field. As shown

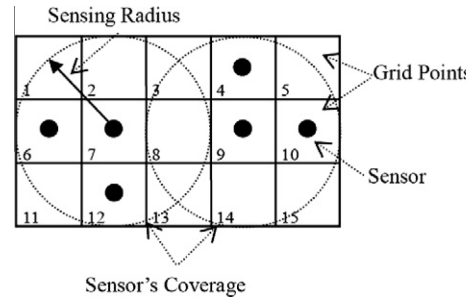

Fig. 1. Grid-based sensing field and power codes.

in Fig. 1, the power vector of service point 8 is (0,0,1,1,0,0) corresponding to sensor 4, 6, 7, 9, 10, and 12. In a *completely covered sensing field*, when each service point has a unique power vector, we note the sensing field is *completely discriminated*, as shown in Fig. 1. In this case, as soon as a target occurs in a grid of the sensing field, it can be located by the back-end according to the power vector of the service point.

Sometimes, due to resource (i.e. number of sensor nodes) limitations, a completely discriminated sensing field cannot be constructed. Consequently, these may lead to wrong determinations, whenever a target occurs at any one of the service points. Positioning accuracy, therefore, becomes a major consideration in solving the problem. The *error distance*, which presents the *Euclidean distance* between the actual and determined locations for one target, is one of the most natural criteria to measure the positioning accuracy. Hence, when complete discrimination is impossible, the goal of this problem is to minimize the maximum error distance, that is, to optimize the positioning accuracy of the sensor network. Besides, the differential quality of positioning accuracy requirements is considered in the paper, we naturally adopt the *weighted error distance* as the metric to measure positioning accuracy.

3.1. The framework

This study assumes that the terrain of sensing field is predetermined, and that the sensor deployment problem is addressed by the controlled approach. In other words, sensors are placed by a prior planning to satisfy a particular QoS requirement. As shown in Fig. 2, the sensing field can be represented as a collection of two-dimensional grid points, which are the candidate locations for sensors as well as the service points for the positioning service. Sensing fields also can be presented as a set of disjoint *regions of interest* (ROIs), each of them requires a different type of QoS for target positioning. In other words, all ROIs have different priorities for discrimination. A ROI is an irregular region, which comprises a set of adjacent service points.

As the previous papers (Chakrabarty et al., 2002; Ray et al., 2004), this study also applies the 0/1 detection model for sensors. In this model, the coverage indicator bit of the sensor for a service

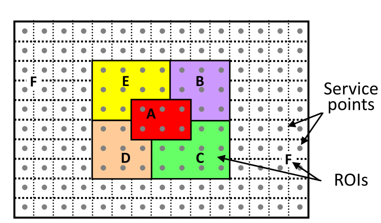

Fig. 2. A sensing field with 150 service points and 6 ROIs.

point is set to 1 if its sensing radius can cover the service point and 0 otherwise. Then, a *power code*, which is constructed by all coverage indicator bits of sensors, can be used to represent each service point. A service point with an unique power code is exactly positioning. Otherwise, the error distance of positioning is the maximum distance between those service points with the same power code.

Generally, a terrain of sensing field could have some placement limitations. That is, for all of the positions in the sensing field, the suitability for each placing sensor is unlikely. In most cases, sensors are expensive to place at most locations, and impossible in others, e.g. lakes or wetlands. Additionally, some locations might require surveillance and positioning services, but not be suitable for placing sensors.

Intuitively, we can adjust sensing radius of some sensors to overcome the placement limitation. Nevertheless, several papers adopt various sensing radiuses to cope the sensor deployment problem. In addition, given the locations of all sensors, the positioning accuracy can be improved by adjusting sensing radius of each sensor (Chiu et al., 2008). Therefore, we take account of sensing radius as one of decision variables for deploying a WSN.

This study provides a differential QoS instead of a traditional uniform QoS for target positioning. Three types of QoS for target positioning are provided for the sensor deployment problem as follows:

1. Completely discriminable: each service point in an ROI can be positioned individually. This is the best QoS for positioning provided by a WSN.
2. Discriminable: a service point can be positioned in an ROI but with a constant error distance. In this type of QoS, a lower error distance indicates a better QoS of positioning.
3. Surveillance-only: all service points can be sensed by sensors, but cannot be discriminated. In this paper, it is the basic QoS type.

Assume the resource is limited; the most important ROIs need to have the highest priorities to achieve their QoS requirements. Therefore, except for the QoS type, each ROI can specify its *discrimination priority* in pre-deployment phase. The QoS requirement of ROIs with the highest-level priority can be satisfied first if resources are limited. If the resources are still not exhausted, then the requirement of the ROIs with the second-level priority can be satisfied, and so on. The QoS requirement of ROIs with lower-level priority is degraded if the WSN lacks resources. However, all ROIs have to support the surveillance-only service.

The proposed sensor placement framework can be stated briefly as follow. In a sensing field with placement limitations, a WSN is constructed to support the differential QoS for positioning to all ROIs in the field. The WSN is constructed by deploying finite sensors at candidate locations, and adjusting the sensing radius of each sensor. The goal of the framework is either to satisfy QoS requirement for all ROIs, or to minimize QoS degradation for each ROI based on its level of priority.

3.2. The scenario

The sensing field in Fig. 2 includes six ROIs, denoted by ROIs A, B, C, D, E, and F. When an object enters ROI F, the WSN has to obtain the information rapidly, i.e. the surveillance service. Moreover, while objects reach the other ROIs, the WSN responds to locate the position of the objects.

Moreover, three *discrimination priority* classes, high, medium, and low, are assigned to each ROI to denote its importance. First, ROI A requires the highest level of QoS for positioning, so it has a high discrimination priority. Next, ROIs B, C, D, and E, have medium discrimination priorities, as well as ROI F requires a low discrimination priority.

Table 1
The levels of QoS in the scenario.

Level of QoS	QoS of positioning supported for ROIs
1	Completely discriminable: None Discriminable: ROI A Surveillance-only: ROIs B, C, D, E, and F
2	Completely discriminable: ROI A Discriminable: ROIs B, C, D, and E Surveillance-only: ROI F
3	Completely discriminable: ROIs A, B, C, D, and E Discriminable: ROI F
4	Completely discriminable: ROIs A, B, C, D, E, and F

According to the above settings, the QoS is separated into four levels, 1–4, as shown on Table 1. The best positioning quality, level 4 QoS, is satisfied if the given resource is adequate. Conversely, if the given resource is scarce, the QoS guaranteed for the ROI with lower service priority will be degraded to lower level of QoS, levels 1–3 QoS, according to its service priority. In this scenario, level 4 QoS is also called uniform QoS of positioning because that all of the service points are discriminated; they have the same QoS.

4. Problem formulation

This section presents a mathematical model for the proposed sensor placement problem. Since the proposed problem supports the differentiated quality of positioning services as well as the prioritized service, the mathematical model of the problem becomes quite intractable. This study introduces a parameter, *discrimination weight*, which is a positive real number, represents the priority of discrimination between service points i and j. An ROI with a larger discrimination weight has a higher priority to obtain guaranteed QoS for positioning. In this context, the objective of the proposed problem is to minimize the maximum weighted error distance for all pairs of service points.

4.1. Given parameters and decision parameters

The notations used to model the problem are listed as follows:

Given parameters

A	index set of the service points in the sensing field
B	index set of the sensor's candidate locations; $B \subseteq A$
C	set of the kinds of cost for sensor
W	set of the discrimination weight
R	set of candidate detection radiuses for sensor
d_{ij}	Euclidean distance between location i and j; $i, j \in A$
c_k	the cost of sensor located at position k; $k \in B$, $c_k \in C$
c_{min}	the minimum cost of sensors
G	the budget limitation for sensors
N	the maximum number of sensors; $N = \frac{G}{c_{min}}$
w_{ij}	discrimination weight; $i, j \in A$, $w_{ij} \in W$
$\overline{K}$	a larger number

Decision variables

y_k	1, if a sensor is allocated at position, k and 0 otherwise, $k \in B$
v_i	a power vector of location i. $v_i = (v_{i1}, v_{i2}, \ldots, v_{ik})$ where v_{ik} is 1 if the target at location i can be detected by the sensor at position k and 0 otherwise, $i \in A$, $k \in B$
r_k	detection radius of sensor located at k, $k \in B$

4.2. Mathematical model

The original problem (IP1) is presented as follows:

$$Z_{IP1} = \min_{v} \max_{\substack{\forall i,j \in A, \\ i \neq j}} \left(w_{ij} d_{ij} \Big/ 1 + \overline{K} \sum_{\forall k \in B} (v_{ik} - v_{jk})^2 \right) \tag{IP1}$$

subject to:

$$v_{ik} d_{ik} \leqslant y_k r_k, \quad \forall i \in A,\ k \in B,\ i \neq k, \tag{1}$$
$$d_{ik}/r_k > y_k - v_{ik}, \quad \forall i \in A,\ k \in B,\ i \neq k, \tag{2}$$
$$v_{ik} = y_k, \quad \forall i \in A, k \in B, i = k \tag{3}$$
$$um_{\forall k \in B} c_k y_k < G, \tag{4}$$
$$\sum_{\forall k \in B} v_{ik} \geqslant 1, \quad \forall i \in A, \tag{5}$$
$$\sum_{\forall k \in B} v_{ik} \leqslant N, \quad \forall i \in A, \tag{6}$$
$$r_k \in R, \quad \forall k \in B, \tag{7}$$
$$v_{ik} = 0 \text{ or } 1, \quad \forall i \in A,\ k \in B, \tag{8}$$
$$y_k = 0 \text{ or } 1, \quad \forall k \in B. \tag{9}$$

The objective of problem (IP1) is to minimize the maximum weighted error distance for any pair of service points. Suppose that $b = \sum_{\forall k \in B} (v_{ik} - v_{jk})^2$ presents the Hamming distance of two power vectors belonging to two service points i and j, respectively. If the power vectors are distinct, then the weighted error distance between service points i and j, i.e. $(w_{ij} d_{ij}/(1 + \overline{K} b))$ approaches zero. In contrast, if the power vectors are the same, then the weighted error distance between service points i and j is $w_{ij} d_{ij}$, which is greater than or equal to w_{ij}. Constraint (1) requires the power vector (v_{ik}) of a service point which locates on the outside of the sensor coverage to be zero. Constraint (2) requires that the power vector of service points located on the interior of sensor detection range is 1. Constraint (3) requires the coverage to be full for the service point on which sensor is located. Constraint (4) requires that the budget to be limited. Constraint (5) is the completed coverage requirement. Constraint (6) requires the amount of sensors to monitor service point i. Constraint (7) requires that the sensing radius of sensors belong to set R. Constraints (8) and (9) are integer constraints.

Subsequently, we discuss how to determine the values of weights and constant $\overline{K}$, two propositions are obtained and presented as follows. As well as the proofs are presented on Appendix A.

Proposition 1. *If the diameter of the sensing field is D, and the discrimination weights are* $w_1, w_2, \ldots, w_h$ *and* $w_1 < w_2 < \cdots < w_h$*. Then* $w_{i+1} > D_{W_i}$ *for any two adjacent weights* w_i *and* w_{i+1}*.*

Proposition 2. *If the diameter of sensing field is D; the detection range is r, and the discrimination weights are* $w_1, w_2, \ldots, w_h$*, and* $w_1 < w_2 < \cdots < w_h$*, then the constant* $\overline{K}$ *must satisfy constraints as follows:*

If $2r \geqslant D$, then $w_1 > (w_h D)/(1 + \overline{K})$.
If $2r < D$, then $w_1 > max\{(w_h \cdot 2r)/(1 + \overline{K}), (w_h \cdot D)/(1 + 2\overline{K})\}$.

5. Solution procedures

This section presents the algorithm for solving the proposed problem. An approach based upon Lagrangean relaxation is adopted. Lagrangean relaxation is a method for obtaining lower bounds (for minimization problems) as well as good primal solutions in integer programming problem (Fisher, 1981, 1985). A Lagrangean relaxation is obtained by identifying in the primal problem a set of complicated constraints whose removal will simplify the solution of the primal problem. Each of the complicated constraints is multiplied by a multiplier and added to the objective function. This mechanism is known as dualizing the complicating constraints (Geoffrion, 1974; Held, Wolfe, & Crowder, 1974).

The solution procedure presented in this section includes four steps: 1. equivalent model, 2. transformation, 3. relaxation, and 4. getting primal feasible solutions.

5.1. Equivalent model

The original model is a nonlinear combinatorial problem, and is therefore hard to solve directly. Hence, an equivalent model is developed. The transformation method is also presented in this section.

An equivalent formulation of problem (IP1) is given by (IP2) below. Define

$$S_v = \max_{\forall i,j \in A, i \neq j} \left(w_{ij} d_{ij} \Big/ \left(1 + \overline{K} \sum_{\forall k \in B} (v_{ik} - v_{jk})^2 \right) \right), \tag{10}$$

and rewrite the objective function as follows:

$$Z_{IP2} = \min_{v} S_v \tag{IP2}$$

subject to:

$$1/S_v \leqslant \left(1 + K \sum_{\forall k \in B} (v_{ik} - v_{jk})^2 \right) \Big/ w_{ij} d_{ij}, \quad \forall i,j \in A,\ i \neq j, \tag{11}$$

$$\overline{\overline{S}} \leqslant S_v \leqslant \overline{S}, \quad \begin{array}{l} \overline{\overline{S}} \text{ is lower bound of } S_v, \\ \overline{S} \text{ is upper bound of } S_v, \end{array} \tag{12}$$

$$v_{ik} d_{ik} \leqslant y_k r_k, \quad \forall i \in A,\ k \in B,\ i \neq k, \tag{1}$$
$$d_{ik}/r_k > y_k - v_{ik}, \quad \forall i \in A,\ k \in B,\ i \neq k, \tag{2}$$
$$v_{ik} = y_k, \quad \forall i \in A, k \in B, i = k \tag{3}$$
$$\sum_{\forall k \in B} c_k y_k \leqslant G, \tag{4}$$
$$\sum_{\forall k \in B} v_{ik} \geqslant 1, \quad \forall i \in A, \tag{5}$$
$$\sum_{\forall k \in B} v_{ik} \leqslant N, \quad \forall i \in A, \tag{6}$$
$$r_k \in R, \quad \forall k \in B, \tag{7}$$
$$v_{ik} = 0 \text{ or } 1, \quad \forall i \in A,\ k \in B, \tag{8}$$
$$y_k = 0 \text{ or } 1, \quad \forall k \in B. \tag{9}$$

Constraint (11) is added to stipulate the upper and lower bound of the maximum weighted error distance S_v. The theoretical upper and lower bounds are then proposed as the follows. As well as the proofs are presented on Appendix A.

Proposition 3. *Theoretical upper bound of* S_v *denoted by* $\overline{S}$ *is* $w_h \tilde{d}$*, where* w_h *denotes the highest discrimination weight, and* $\tilde{d}$ *(i.e.* $\tilde{d} = \min\{D, 2r\}$*) is the minimal value of the diameter of a sensor field (D) and the maximum sensing range of sensors (2r).*

Proposition 4. *Theoretical lower bound of* S_v *denoted by* $\overline{\overline{S}}$ *is* $w_l/(1 + \hat{d} \cdot \overline{K})$*, where* w_l *is the lowest discrimination weight, and* $\hat{d}$ *represents the maximal possible Hamming distance of power vectors for all service points. If* $\bar{n}$ *is the maximal number of sensors that can cover a service point; as well as N denotes the number of sensor nodes then* $\hat{d} = min\{2\bar{n}, N\}$*.*

Table 2
Truth table for variables v_{ik}, v_{jk}, and t_{ijk}. $p_1 \sim p_4$ are vertices.

v_{ik}	v_{jk}	t_{ijk}	Vertices
0	0	0	p_1
0	1	0	p_2
1	0	0	p_3
1	1	1	p_4

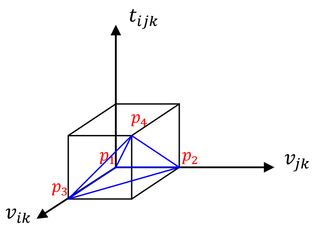

Fig. 3. The relationship of v_{ik}, v_{jk}, t_{ijk} and vertices $p_1 \sim p_4$.

5.2. Transformation

Problem (IP2) is still very hard to solve, since Constraint (11) is nonlinear. Instead of solving problem (IP2) directly, the cutting plane method is applied to transform Constraint (11) to a linear Constraint (17).

An auxiliary variable, t_{ijk}, is introduced, where $t_{ijk} = v_{ik}v_{jk}$. Table 2 shows the truth table for variables v_{ik}, v_{jk}, and t_{ijk}. The possible values for the three variables only exist in four integer vertices, p_1, p_2, p_3, and p_4, of the polyhedron, depicted in Fig. 3. The four planes constructing the polyhedron are presented as following:

$$v_{ik} - t_{ijk} \geqslant 0, \quad \forall i,j \in A,\ i \neq j,\ k \in B, \tag{13}$$

$$v_{jk} - t_{ijk} \geqslant 0, \quad \forall i,j \in A,\ i \neq j,\ k \in B, \tag{14}$$

$$v_{ik} + v_{jk} - t_{ijk} \leqslant 1, \quad \forall i,j \in A,\ i \neq j,\ k \in B, \tag{15}$$

$$t_{ijk} \geqslant 0, \quad \forall i,j \in A,\ i \neq j,\ k \in B. \tag{16}$$

The auxiliary variable t_{ijk} is employed to replace $v_{ik} \cdot v_{jk}$. Constraint (11) can thus be transformed to linear Inequality (17). The transformation is described as follows:

$$(v_{ik} - v_{jk})^2 = v_{ik}^2 - 2v_{ik}v_{jk} + v_{jk}^2 = v_{ik} - 2v_{ik}v_{jk} + v_{jk}.$$

Let $t_{ijk} = v_{ik}v_{jk}$, then $(v_{ik} - v_{jk})^2 = v_{ik} + v_{jk} - 2t_{ijk}$.

According to the cutting plane method, Constraints (13)–(15), (18) and (19) must be added to require relationship of v_{ik}, v_{jk}, and t_{ijk}. Constraint (18) requires that the number of sensors that can cover both points (i and j) cannot be over the total number of sensors. Hence, the nonlinear combinatorial problem (IP1) is transformed to an equivalent linear combinatorial problem (IP3)

$$Z_{IP3} = \min_{v} S_v \tag{IP3}$$

subject to:

$$1/S_v \leqslant \left(1 + \overline{K}\sum_{\forall k \epsilon B}(v_{ik} + v_{jk} - 2t_{ijk})\right) \Big/ w_{ij}d_{ij}, \quad \forall i,j \epsilon A,\ i \neq j \tag{17}$$

$$\bar{\bar{S}} \leqslant S_v \leqslant \bar{S}, \quad \begin{array}{l}\bar{\bar{S}} \text{ is lower bound of } S_v,\\ \bar{S} \text{ is upper bound of } S_v,\end{array} \tag{12}$$

$$v_{ik}d_{ik} \leqslant y_k r_k, \quad \forall i \epsilon A,\ k \epsilon B,\ i \neq k, \tag{1}$$
$$d_{ik}/r_k > y_k - v_{ik}, \quad \forall i \epsilon A,\ k \epsilon B,\ i \neq k, \tag{2}$$
$$v_{ik} = y_k, \quad \forall i \epsilon A, k \epsilon B, i = k \tag{3}$$
$$\sum_{\forall k \epsilon B} c_k y_k \leqslant G, \tag{4}$$
$$\sum_{\forall k \epsilon B} v_{ik} \geqslant 1, \quad \forall i \epsilon A, \tag{5}$$
$$\sum_{\forall k \epsilon B} v_{ik} \leqslant N, \quad \forall i \epsilon A, \tag{6}$$
$$r_k \epsilon R, \quad \forall k \epsilon B, \tag{7}$$
$$v_{ik} = 0 \text{ or } 1, \quad \forall i \epsilon A,\ k \epsilon B, \tag{8}$$
$$y_k = 0 \text{ or } 1, \quad \forall k \epsilon B, \tag{9}$$

$$v_{ik} - t_{ijk} \geqslant 0, \quad \forall i,j \epsilon A,\ i \neq j,\ k \epsilon B, \tag{13}$$
$$v_{jk} - t_{ijk} \geqslant 0, \quad \forall i,j \epsilon A,\ i \neq j,\ k \epsilon B, \tag{14}$$
$$v_{ik} + v_{jk} - t_{ijk} \leqslant 1, \quad \forall i,j \epsilon A,\ i \neq j,\ k \epsilon B, \tag{15}$$

$$\sum_{\forall k \epsilon B} t_{ijk} \leqslant N, \quad \forall i,j \epsilon A,\ i \neq j, \tag{18}$$
$$t_{ijk} = 0 \text{ or } 1, \quad \forall i,j \epsilon A,\ i \neq j,\ k \epsilon B. \tag{19}$$

5.3. Relaxation

This section presents the Lagrangean relaxation procedure for the proposed problem (IP3), relaxation and decomposition, as well as the lower bound should be discussion.

5.3.1. Relaxation

By Lagrangean relaxation, we dualize Constraints (17), (1), (2), (3), (13), (14) and (15) of problem (IP3), as well as get the following Lagrangean relaxation problem.

Problem (LR1):

$$\begin{aligned} &Z_D(u^1,u^2,u^3,u^4,u^5,u^6,u^7) \\ &= \min_v \Bigg\{ S_v + \sum_{\forall i \in A}\sum_{\forall j \in A, i \neq j} u_{ij}^1\Big(1/S_v - \big(1+\overline{K}\sum_{\forall k \in B}(v_{ik}+v_{jk}-2t_{ijk})\big)/w_{ij}d_{ij}\Big) \\ &\quad + \sum_{\forall i \in A}\sum_{\forall k \in B, i \neq k} u_{ik}^2(v_{ik}d_{ik} - y_k r_k) + \sum_{\forall i \in A}\sum_{\forall k \in B, i \neq k} u_{ik}^3(y_k - v_{ik} - d_{ik}/y_k) \\ &\quad + \sum_{\forall i \in A}\sum_{\forall k \in B, i = k} u_{ik}^4(v_{ik} - y_k) + \sum_{\forall i \in A}\sum_{\forall j \in A, i \neq j}\sum_{\forall k \in B} u_{ijk}^5(t_{ijk} - v_{ik}) \\ &\quad + \sum_{\forall i \in A}\sum_{\forall j \in A, i \neq j}\sum_{\forall k \in B} u_{ijk}^6(t_{ijk} - v_{jk}) \\ &\quad + \sum_{\forall i \in A}\sum_{\forall j \in A, i \neq j}\sum_{\forall k \in B} u_{ijk}^7(v_{ik} + v_{jk} - t_{ijk} - 1) \Bigg\} \end{aligned} \tag{LR1}$$

subject to:

$$\bar{\bar{S}} \leqslant S_v \leqslant \bar{S}, \quad \begin{array}{l}\bar{\bar{S}} \text{ is lower bound of } S_v,\\ \bar{S} \text{ is upper bound of } S_v,\end{array} \tag{12}$$

$$\sum_{\forall k \in B} c_k y_k \leqslant G, \tag{4}$$
$$\sum_{\forall k \in B} v_{ik} \geqslant 1, \quad \forall i \epsilon A, \tag{5}$$
$$\sum_{\forall k \in B} v_{ik} \leqslant N, \quad \forall i \epsilon A, \tag{6}$$
$$r_k \epsilon R, \quad \forall k \epsilon B, \tag{7}$$
$$v_{ik} = 0 \text{ or } 1, \quad \forall i \epsilon A,\ k \epsilon B, \tag{8}$$
$$y_k = 0 \text{ or } 1, \quad \forall k \epsilon B, \tag{9}$$

$$\sum_{\forall k \in B} t_{ijk} \leqslant N, \quad \forall i,j \in A,\ i \neq j, \tag{18}$$

$$t_{ijk} = 0 \text{ or } 1, \quad \forall i,j \in A,\ i \neq j,\ k \in B. \tag{19}$$

The multipliers $u^1, u^2, \ldots, u^7$ are the vectors of $\left\{u^1_{ij}\right\}, \left\{u^2_{ij}\right\}, \ldots, \left\{u^7_{ijk}\right\}$, respectively. Besides Constraint (3) with multiplier $\{u^4_{ik}\}$, the other constraints are dulized such that the corresponding multipliers, u^1, u^2, u^3, u^5, u^6 and u^7 are nonnegative.

The dual problem (LR1) is rewritten to Eq. (LR2), where the constant terms are omitted.

$$\begin{aligned}
&Z_D(u^1,u^2,u^3,u^4,u^6,u^7) \\
&= \min_v \Bigg\{ S_v + \sum_{\forall i \in A} \sum_{\substack{\forall j \in A \\ i \neq j}} \left(u^1_{ij}/S_\tau\right) - \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \left(u^1_{ij}/W_{ij}d_{ij}\right) \\
&- \overline{K} \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \left(u^1_{ij} \sum_{\forall k \in B} v_{ik}/w_{ij}d_{ij}\right) - \overline{K} \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \left(u^1_{ij} \sum_{\forall k \in B} v_{jk}/w_{ij}d_{ij}\right) \\
&+ 2\overline{K} \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \left(u^1_{ij} \sum_{\forall k \in B} t_{ijk}/w_{ij}d_{ij}\right) + \sum_{\forall i \in A} \sum_{\substack{\forall k \in B, \\ j \neq k}} u^2_{ik} d_{ik} v_{ik} \\
&- \sum_{\forall i \in A} \sum_{\substack{\forall k \in B, \\ j \neq k}} u^2_{ik} r_k y_k + \sum_{\forall i \in A} \sum_{\substack{\forall k \in B, \\ i \neq k}} u^3_{ik} y_k - \sum_{\forall i \in A} \sum_{\substack{\forall k \in B, \\ i \neq k}} u^3_{ik} v_{ik} \\
&- \sum_{\forall i \in A} \sum_{\substack{\forall k \in B, \\ i \neq k}} (u^3_{ik} d_{ik}/r_k) + \sum_{\forall i \in A} \sum_{\substack{\forall k \in B, \\ i = k}} u^4_{ik} v_{ik} - \sum_{\forall i \in A} \sum_{\substack{\forall k \in B, \\ i = k}} u^4_{ik} y_k \\
&+ \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \sum_{\forall k \in B} u^5_{ijk} t_{ijk} - \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \sum_{\forall k \in B} u^5_{ijk} v_{ik} \\
&+ \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \sum_{\forall k \in B} u^6_{ijk} t_{ijk} - \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \sum_{\forall k \in B} u^6_{ijk} v_{ik} \\
&+ \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \sum_{\forall k \in B} u^7_{ijk} v_{ik} + \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \sum_{\forall k \in B} u^7_{ijk} v_{jk} \\
&- \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \sum_{\forall k \in B} u^7_{ijk} t_{ijk} - \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \sum_{\forall k \in B} u^7_{ijk} \Bigg\}
\end{aligned} \tag{LR2}$$

where the constant term is

$$\left(- \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \left(u^1_{ij} \big/ w_{ij} d_{ij}\right) - \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \sum_{\forall k \in B} u^7_{ijk} \right).$$

5.3.2. Decomposition

According to Lagrangean relaxation approach, problem (LR2) will be decomposed into four mutually independent and easily solvable subproblems. Each sub-problem only involves one or two decision variables and must be optimal solved. Note that, the constant part is excluded from the objective function in the subproblems but will be considered in the lower bound computation.

Subproblem 1. For S_v

$$Z_{SUB1}(u^1) = \min_{S_v} \left(S_v + \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \left(u^1_{ij}/S_v\right) \right) \tag{SUB1}$$

subject to:

$$\overline{\overline{S}} \leqslant S_v \leqslant \overline{S}, \quad \begin{array}{l} \overline{\overline{S}} \text{ is lower bound of } S_v, \\ \overline{S} \text{ is upper bound of } S_v. \end{array}$$

To optimal solve the subproblem, the right hand side of Eq. (SUB1) will be differentiated respected to variable S_v. Let new equation equals to zero and get the optimal solution of variable S_v, $S_{opt} = \sqrt{\sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} u^1_{ij}}$.

If $\overline{\overline{S}} \leqslant S_{opt} \leqslant \overline{S}$ then let $Z_{SUB1} = 2\sqrt{\sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} u^1_{ij}}$. Otherwise, $\overline{\overline{S}}$ and $\overline{S}$ are substituted for S_v to get the upper and lower, $\left(\text{i.e. } Z^U_{SUB1}\left(u^1_{ij}\right) \text{ and } Z^L_{SUB1}\left(u^1_{ij}\right)\right)$, of Z_{SUB1}. We can get optimal solution such that

$$Z_{SUB1}(u^1) = \min \left\{ \left(\overline{S} + \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \left(u^1_{ij} \big/ \overline{S}\right) \right), \left(\overline{\overline{S}} + \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \left(u^1_{ij} \big/ \overline{\overline{S}}\right) \right) \right\}.$$

Subproblem 2. For y_k and r_k

$$\begin{aligned}
Z_{SUB2}(u^2,u^3,u^4) &= \min_{yk,rk} \left(- \sum_{\forall i \in A} \sum_{\substack{\forall k \in B, \\ i \neq k}} u^2_{ik} r_k y_k + \sum_{\forall i \in A} \sum_{\substack{\forall k \in B, \\ i \neq k}} u^3_{ik} y_k - \sum_{\forall i \in A} \sum_{\substack{\forall k \in B, \\ i \neq k}} (u^3_{ik} d_{ik}/r_k) - \sum_{\forall i \in A} \sum_{\substack{\forall k \in B, \\ i = k}} u^4_{ik} y_k \right) \\
&= \min_{yk,rk} \sum_{\forall k \in B} \left(\sum_{\substack{\forall i \in A, \\ i \neq k}} ((-u^2_{ik} r_k + u^3_{ik}) y_k - (u^3_{ik} d_{ik}/r_k)) - \sum_{\substack{\forall i \in A, \\ i = k}} u^4_{ik} y_k \right)
\end{aligned} \tag{SUB2}$$

subject to:

$$\sum_{\forall k \in B} c_k y_k \leqslant G, \tag{4}$$

$$r_k \in R, \quad \forall k \in B, \tag{7}$$

$$y_k = 0 \text{ or } 1, \quad \forall i \in A,\ k \in B. \tag{9}$$

Subproblem 2 comprises $|B|$ problems. For each sensor k, we let $b_k(r_k)$ represent the function (SUB2) while $y_k = 1$.

$$b_k(r_k) = \sum_{\substack{\forall i \in A, \\ i \neq k}} ((-u^2_{ik} r_k + u^3_{ik}) - (u^3_{ik} d_{ik}/r_k)) - \sum_{\substack{\forall i \in A, \\ i = k}} u^4_{ik}.$$

Then, we calculate b_k for each r_k, which belongs to set R, as well as find the optimal r_k such that b_k is the minimum denoted by $(b_k)_{min}$. Next, from the set of unallocated sensors, we iteratively choose sensor k and r_k with the minimal $(b_k)_{min}$ to be set. The cost of sensor k must be accumulated. While adding the budget of sensor k will exceed the total deployment cost G, the procedure must be stopped.

Subproblem 3. For v_{ik}

$$\begin{aligned}
&Z_{SUB3}(u^1,u^2,u^3,u^4,u^5,u^6,u^7) \\
&= \min_{v_{ik}} \Bigg(- \overline{K} \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \left(u^1_{ij} \sum_{\forall k \in B} v_{ik} \Big/ w_{ij} d_{ij}\right) \\
&- \overline{K} \sum_{\forall i \in A} \sum_{\forall j \in A, i \neq j} \left(u^1_{ij} \sum_{\forall k \in B} v_{jk} \Big/ w_{ij} d_{ij}\right) + \sum_{\forall i \in A} \sum_{\forall k \in B, i \neq k} u^2_{ik} d_{ik} v_{ik} \\
&- \sum_{\forall i \in A} \sum_{\forall k \in B, i \neq k} u^3_{ik} v_{ik} + \sum_{\forall i \in A} \sum_{\substack{\forall k \in B, \\ i = k}} u^4_{ik} v_{ik} - \sum_{\forall i \in A} \sum_{\substack{\forall j \in A, \\ i \neq j}} \sum_{\forall k \in B} u^5_{ijk} v_{ik} \\
&- \sum_{\forall i \in A} \sum_{\forall j \in A, i \neq j} \sum_{\forall k \in B} u^6_{ijk} v_{jk} + \sum_{\forall i \in A} \sum_{\forall j \in A, i \neq j} \sum_{\forall k \in B} u^7_{ijk} v_{ik} + \sum_{\forall i \in A} \sum_{\forall j \in A, i \neq j} \sum_{\forall k \in B} u^7_{ijk} v_{jk} \Bigg).
\end{aligned} \tag{SUB3}$$

To simplify Eq. (SUB3), variable v_{jk} should be eliminated from Eq. (SUB3). For each term with the variable v_{jk}, index j substitutes for i contrariwise. As well as the values of parameters w_{ji}, d_{ji}, u^1_{ji}, u^5_{jik}, u^6_{jik} are the same as w_{ji}, d_{ji}, u^1_{ji}, u^5_{jik}, u^6_{jik}. Consequently, the equivalent subproblem (SUB3a) replaces Eq. (SUB3).

$$Z_{SUB3}(u^1,u^2,u^3,u^4,u^5,u^6,u^7)$$
$$= \min_{v_{ik}} \sum_{\forall i\in A}\left(\sum_{\forall k\in B}\sum_{\substack{\forall j\in A,\\ i\neq j}}(-2\overline{K}u^1_{ij}/w_{ij}d_{ij} - u^5_{ijk} - u^6_{ijk} + 2u^7_{ijk}) + \sum_{\substack{\forall k\in B,\\ i\neq k}}(u^2_{ik}d_{ik} - u^3_{ik}) + \sum_{\substack{\forall k\in B,\\ i=k}} u^4_{ik}\right) v_{ik} \tag{SUB3a}$$

subject to:

$$\sum_{\forall k\in B} v_{ik} \geqslant 1, \quad \forall i\in A, \tag{5}$$

$$\sum_{\forall k\in B} v_{ik} \leqslant N, \quad \forall i\in A, \tag{6}$$

$$v_{ik} = 0 \text{ or } 1, \quad \forall i\in A,\ k\in B. \tag{8}$$

Subproblem 3 comprises $|A\times B|$ problems. For each service point i, we calculate the coefficient of each variable v_{ik}, and sort them in non-decreasing order. Iteratively, if the minimal one of the coefficient for v_{ik} is a positive number, we set the corresponding v_{ik} to be zero. Otherwise, the corresponding v_{ik} is assigned to 1 under the number of sensors constraint. Additionally, for each service point, the coverage constraint must be satisfied also. If no any v_{ik} is 1 for service point i, the v_{ik} with the minimum coefficient will be set.

Subproblem 4. For t_{ijk}

$$Z_{SUB4}(u^1,u^5,u^6,u^7)$$
$$= \min_{t_{ijk}}\left(2\overline{K}\sum_{\forall i\in A}\sum_{\substack{\forall j\in A,\\ i\neq j}}\left(u^1_{ij}\sum_{\forall k\in B}t_{ijk}\Big/w_{ij}d_{ij}\right) + \sum_{\forall i\in A}\sum_{\substack{\forall j\in A,\\ i\neq j}}\sum_{\forall k\in B}u^5_{ijk}t_{ijk} + \sum_{\forall i\in A}\sum_{\substack{\forall j\in A,\\ i\neq j}}\sum_{\forall k\in B}u^6_{ijk}t_{ijk} - \sum_{\forall i\in A}\sum_{\substack{\forall j\in A,\\ i\neq j}}\sum_{\forall k\in B}u^7_{ijk}t_{ijk}\right)$$
$$= \min_{t_{ijk}}\sum_{\forall i\in A}\sum_{\substack{\forall j\in A,\\ i\neq j}}\sum_{\forall k\in B}\left(2\overline{K}u^1_{ij}\Big/w_{ij}d_{ij} + u^5_{ijk} + u^6_{ijk} - u^7_{ijk}\right)t_{ijk} \tag{SUB4}$$

subject to:

$$\sum_{\forall k\in B} t_{ijk} \leqslant N, \quad \forall i,j\in A,\ i\neq j, \tag{18}$$

$$t_{ijk} = 0 \text{ or } 1, \quad \forall i,j\in A,\ i\neq j,\ k\in B. \tag{19}$$

Subproblem (SUB4) comprises $|A\times A\times B|$ problems. Subproblem (SUB4) can be solved easily. First, the coefficient is calculated for each t_{ijk}. Then we sort the coefficients in non-decreasing order. If the coefficient for t_{ijk} is non-positive, t_{ijk} is assigned to 1. Otherwise, t_{ijk} is zero. However, the number of t_{ijk} with one cannot exceed the maximal number of sensors.

5.3.3. Lower bound

In each iteration, after every subproblem is optimally solved, the objective value of the dual problem, Z_D, is a lower bound of original problem. It can be obtained by summarizing the objective values of all subproblems and the constant part as the following equation:

$$Z_D(u^1,u^2,u^3,u^4,u^5,u^6,u^7) = Z_{SUB1} + Z_{SUB2} + Z_{SUB3} + Z_{SUB4} + \left(-\sum_{\forall i\in A}\sum_{\substack{\forall j\in A,\\ i\neq j}}\left(u^1_{ij}\Big/w_{ij}d_{ij}\right) - \sum_{\forall i\in A}\sum_{\substack{\forall j\in A,\\ i\neq j}}\sum_{\forall k\in B}u^7_{ijk}\right).$$

According to the weak Lagrangean duality theorem (Fisher, 1981, 1985), the optimal objective value of the dual problem (LR), $Z_D(u^1,u^2,u^3,u^4,u^5,u^6,u^7)$, is a lower bound on primal problem (IP3), where Z_{IP3} is subject to $(u^1,u^2,u^3,u^4,u^5,u^6,u^7) \geqslant 0$. Therefore, we can obtain the lower bound by

$$Z_D = \max_{(u^1,u^2,u^3,u^4,u^5,u^6,u^7)\geqslant 0} Z_D(u^1,u^2,u^3,u^4,u^5,u^6,u^7). \tag{D1}$$

Several methods can be used for solving Eq. (D1) to get the highest lower bound. One of the most popular methods is the subgradient method. Let a $(|A|^2 + |A||B| + |A||B| + |A||B| + |A|^2|B| + |A|^2|B| + |A|^2|B|)$ vector, g represents a subgradient of $Z_D(u^1,u^2,u^3,u^4,u^5,u^6,u^7)$. We denote $\pi = \left(u^1_{ij}, u^2_{ik}, u^3_{ik}, u^4_{ik}, u^5_{ijk}, u^6_{ijk}, u^7_{ijk}\right)$ as the vector of Lagrangean multipliers with respect to relaxed constraints. In iteration m of subgradient optimization procedure, the multiplier π^m is updated by $\pi^{m+1} = \pi^m + \xi^m g^m$.

The step size is ξ^m determined by $\xi^m = \lambda(Z^*_{IP3} - Z_D(\pi^m))/\|g^m\|^2$, where Z^*_{IP3} represents an upper bound on the primal objective value, obtained by applying a heuristic to (IP3), and λ is a scalar satisfying $0 \leqslant \lambda \leqslant 2$.

5.4. Getting primal feasible solutions

After optimally solving each Lagrangean relaxation problem, a set of decision variables can be found. Since some constraints are relaxed, the solutions of Lagrangean relaxation might be infeasible for the primal problem. Hence, an efficient heuristic algorithm which adjusts the dual solutions to obtain the feasible solutions for the primal problem (IP3) be developed as follows. By increasing the number of iterations, the better primal feasible solution is an upper bound (UB) on the primal problem (IP3), while the dual problem provides the lower bound (LB) of the primal problem (IP3).

Step 1: Initialize the decision variables, y_k, v_{ik}, and r_k.>
- Step 1.1: Before the fifth iteration, initial decision variables y_k are determined by sub-problem (SUB2) on Lagrangean relaxation problem. For each sensor, the five recent history solutions are recorded. After the fifth iteration, we can randomly determine whether decision variables y_k should be one by the placement probability for the sensor k in history record.
- Step 1.2: Check Constraints (1)–(3), for each sensor k. For each service point i, let $v_{ik} = 0$ if d_{ik} is more than the maximal candidate radius. Add sensor k if $v_{ik} = 1$ and d_{ik} is less than the maximal candidate radius, for each service point i.
- Step 1.3: Determine the radius r_k if sensor k is allocated. For each sensor k, find the farthest distance between sensor k and the service points with $v_{ik} = 1$ to determine the radius r_k.
- Step 1.4: The decision variables v_{ik} can be obtained by the decision variables y_k.

Step 2: To satisfy the coverage and cost constraint, the sensors might be added, deleted or changed the radius.

Step 2.1: If the coverage constraint is violated, "Change Radius" or "Add Sensor" procedure will be executed. Randomly select a sensor if the increase of radius for the sensor can improve coverage, the radius will be changed. If the operation of change radius is not suitable, the other operation, "Add Sensor", can be tried. The sensor that can cover the most uncovered service points will be added with a proper radius until the coverage constraint is satisfied.

Step 2.2: If the budget constraint is violated, "Delete Sensor" procedure can be applied. Remove a sensor away from the sensor field randomly, if the coverage of the sensor field is not changed. The operation is executed until the budget constraint is satisfied.

Step 2.3: Running the previous two steps until both the coverage and budget constraint are satisfied.

Step 3: For each sensor, the operations "Change Position" and "Modify Radius" are tried in order to improve the discrimination resolution.

6. Computational results

Three sets of experiments were conducted to evaluate the performance of the proposed algorithm under various settings for the amount of resources, fixed/adjustable sensing radius, differential/uniform QoS of positioning, the placement limitations, and size of sensing field. The proposed LR heuristic was coded in C in the Microsoft Visual C++ 6.0 development environment. All the experiments were performed on a Pentium IV-3.0 GHz PC running Microsoft Windows XP Pro. As well as the performance metrics were assessed in terms of the solution quality and computation time.

6.1. Experiment I

The first experiment was designed to observe the solution qualities of the proposed algorithm. In this experiment, the amount of resources, the sensing radiuses were variables; as well as the placement limitation and topography of sensing field are fixed.

The sensing field was a rectangular field divided into 10×15 service points and seven different ROIs shown as Fig. 2. The level of QoS and discrimination priority for these ROIs are presented in Section 3.2. The candidates of sensing-radius are between 1 and 8 units of length. The parameters about LR based algorithm include: $0 \leqslant \lambda \leqslant 2$, improvement counter is 40, and number of iterations is 1500.

For obtaining the differential positioning accuracy according to the requirements of ROIs, we let the set of *discrimination weights* be $\{0.1, 5, 100\}$ based on Proposition 1. The discrimination weights between any two service points have to be the highest weight, $w_h = 100$, while both service points are on ROI A or they are on different ROIs. For any two service points which both on ROIs B, C, D, or E, their discrimination weights should be medium, $w_m = 5$. The

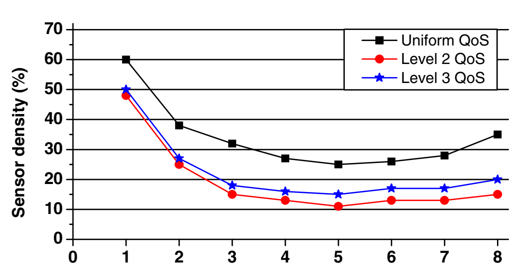

Fig. 4. The sensor density versus various fixed sensing radiuses.

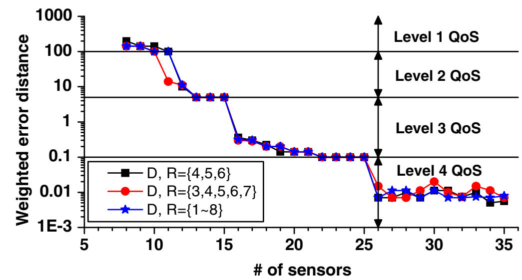

Fig. 5. The best-found objective values for various set of sensing radius and differential QoS.

weights between any two service points, which both are on ROI F are set to the low weight, $w_l = 0.1$.

Based on Proposition 2, we let parameter $\overline{K}$ be 20000. Additionally, the diameter of the sensor field D, is 16 units of length.

Assume the sensor deployment budget of each location, c_k, is one, then the number of sensors corresponds to the deployment budget, G. We define the *sensor density* $\left(\sum_{\forall k \in B} y_k / G\right) * 100\%$ and it replaces the amount of sensors to be observed in Fig. 4.

Fig. 4 shows the relationships between sensing radius and the resource requirements (i.e. sensor density) for different levels of QoS. The range of candidate sensing radiuses is from 1 to 8 units. The marks, "Level 2 QoS" and "Level 3 QoS" represent Level 2 and Level 3 QoS requirements for the sensing field are satisfied, as listed in Table 1. And the traditional scheme without differential positioning quality is marked by "Uniform QoS", which means that all service points in the sensing field have the same positioning priority. Fig. 4 demonstrates that the sensor density requirements for the sensor network deployment depend strongly on the sensing radius of sensors. The results indicate that adopting sensors with medium size of sensing radius (i.e. about 5 units in the case) yields the lowest deployment density.

Fig. 5 shows that the best-found objective values of the differential QoS for various sets of sensing radius and the number of sensors. The candidate sets of the sensing radius for sensors in the experiment were $\{4,5,6\}$, $\{3,4,5,6,7\}$, and $\{1,2,3,4,5,6,7,8\}$. Table 3 lists the ranges of objective values, Z_{IP3}, for the differential QoS. These objective values are the weighted error distance and

Table 3
The levels of QoS and their ranges of Z_{IP3} in the scenario for differential QoS.

Level of QoS	Z_{IP3}	Notes
Level 1	$Z_{IP3} \geqslant 100$	$Z_{IP3} \geqslant w_k$
Level 2	$100 > Z_{IP3} \geqslant 5$	$w_h > Z_{IP3} \geqslant w_m$
Level 3	$5 > Z_{IP3} \geqslant 0.1$	$w_m > Z_{IP3} \geqslant w_l$
Level 4	$Z_{IP3} < 0.1$	$Z_{IP3} < w_1$

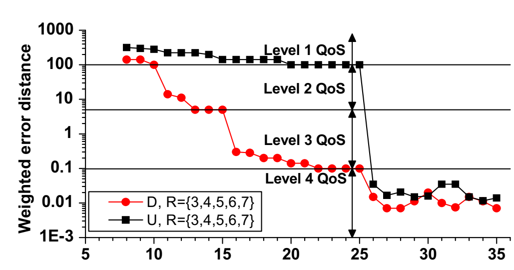

Fig. 6. Performance comparison between the uniform (U) and differential (D) QoS services with adjustable radius, $R = \{r | 3 \leqslant r \leqslant 7\}$.

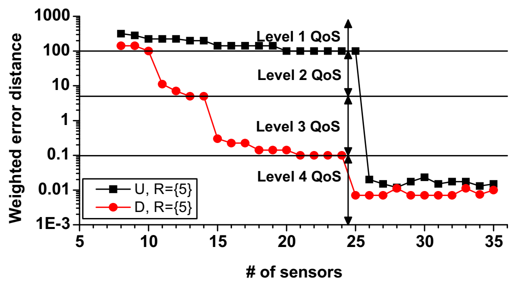

Fig. 7. Performance comparison between the uniform (U) and differential (D) QoS services with fixed radius, $R = \{5\}$.

their corresponding level of positioning service providing by the sensor network. Fig. 5 shows that the objective values of the three candidate sets are very close. In spite of using any one of these candidate sets of sensing radius, while the amount of sensor nodes increase to 26, the completely discriminated sensing field always can be obtained.

For contrasting the differential QoS with traditional uniform QoS of positioning, we design another experiment for uniform QoS. Assume all of service points in the sensing field require the same discrimination priority, so we let $w_{ij} = 100$ for all i, j. When the objective values are less than 100, the sensing field is completely discriminated. Otherwise, the sensing field is discriminated but the weighted error distances are $(Z_{IP3}/100)$ units of length.

Figs. 6 and 7 show the performance comparison between the traditional uniform and differential QoS using fixed and adjustable sensing radiuses, i.e. $R = \{5\}$ and $R = \{r|3 \leqslant r \leqslant 7\}$. Both Figs. 6 and 7 indicate that the differential QoS has better objective values, i.e. the lower weighted error distance, than traditional uniform QoS while the number of deployed sensors was less than 26. It means we need more sensor nodes to achieve Level 1 to Level 3 QoS without the differential QoS deployment method. Since the traditional uniform QoS deployment method cannot guarantee effectively positioning priority for ROIs. In other words, the proposed differential QoS deployment method got higher or equal level of QoS than traditional uniform QoS under the same number of sensor nodes. The results confirm the effectiveness of the proposed framework and algorithm.

6.2. Experiment II

The second experiment was designed to observe the solution qualities of the proposed algorithm under placement limitations. As Fig. 8 shown, sensors could not be placed at the 35 (gray) grid points on the right-upper corner of the topography. So, sensing radiuses should be larger than 4 to satisfy the coverage constraint while we adopt fixed uniform sensing radiuses. Let the deployment cost of the location with placement limitation be infinitive and the other grid points still be one. Besides, the scenario for this experiment was the same as that in Experiment I.

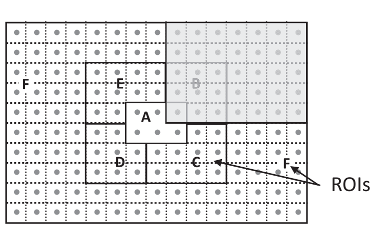

Fig. 8. The scenario with placement limitations: locations in the gray region are very difficult for placing sensors.

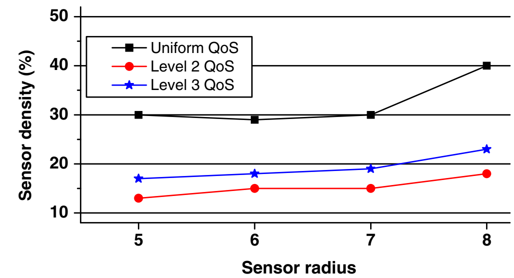

Fig. 9. The minimal sensor densities requirements for various sensing radius. (Fixed uniform radius, placement limitations.)

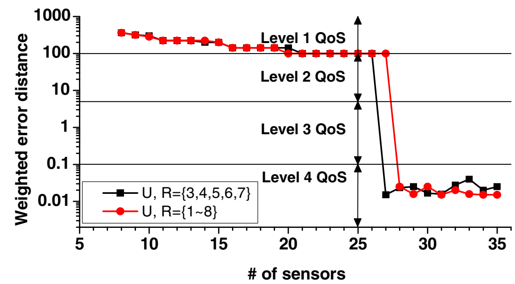

Fig. 10. The best-found objective values for various set of adjustable sensing radius. (Uniform QoS, placement limitations.)

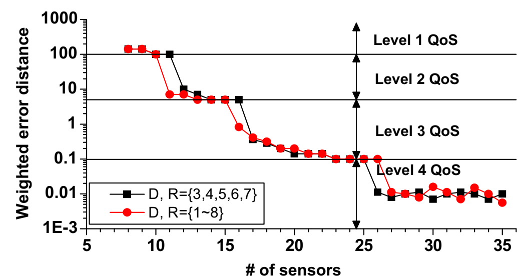

Fig. 11. The best-found objective values for various set of adjustable sensing radius. (Differential QoS, placement limitations.)

In Figs. 9–11, we have two observations for differential QoS for positioning with placement limitations. As Fig. 9 shown, in spite of what level of QoS is considered, the minimal required sensor densities are depend strongly on the selection of sensing radiuses when all sensors have the same sensing radius. And the larger sensing radiuses still donot have advantages in this sensor deployment problem. From Figs. 4 and 9, we found the decisions

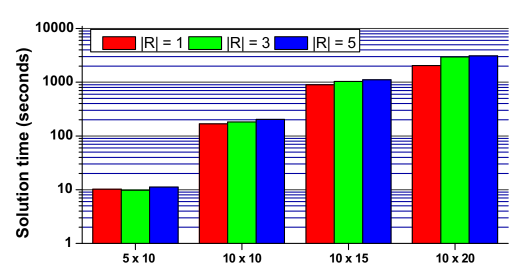

Fig. 12. The computation time of the LR based algorithm.

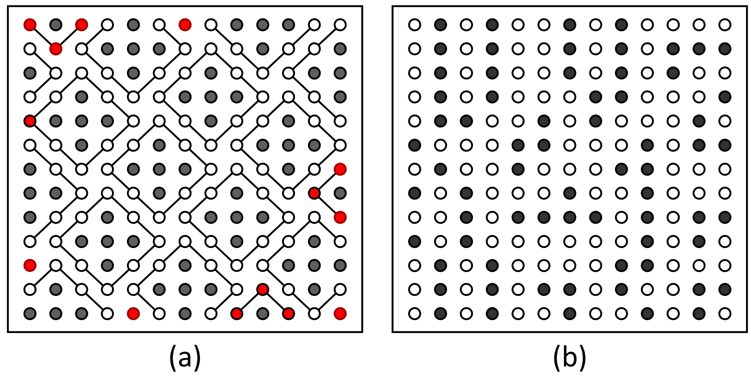

Fig. 13. The sensor deployment in a 13 × 13 sensor field. (a) By CIQ approach (79 sensors). (b) By the proposed LR approach (68 sensors).

of sensing radiuses are depend on placement limitations while all sensors have uniform sensing radius. Moreover, Figs. 10 and 11 shown either traditional uniform or differential QoS, the different sets of candidate sensing radius should obtain almost the same objective values in the case of sensors with adjustable radiuses. So, we can claim the proposed adjustable sensing radius and LR based algorithm is effective to address the sensor deployment problem with placement limitations.

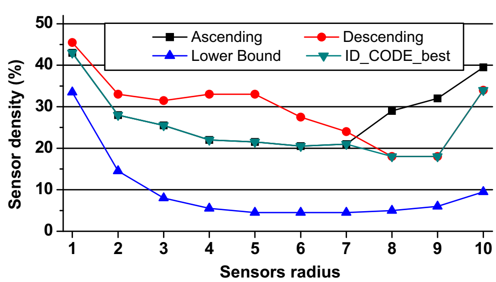

Fig. 14. Performance of the ID-CODE algorithm in the sensing field with 150 service points.

6.3. Experiment III

In Experiment III, sensor fields 50, 100, 150, and 200 service points were used to evaluate the scalability of the proposed algorithm. The solution space increase exponentially as the sensor field size increased linearly. Therefore, this study observes the variation of computation time and solution quality while the problem size increases.

The parameters about LR based algorithm include: $0 < \lambda \leqslant 2$ and improvement counter is 40. The number of iteration for field size 50, 100, 150, and 200 are 500, 1000, 1500, and 1500, respectively.

Fig. 12 presents the computation time of the proposed algorithm, where $|R|$ is cardinality of set of candidate sensing radius. The solution space of the proposed problem exhibited steep growth when $|R|$ increased slightly. However, as shown in Fig. 12, the computation time did not increase significantly when $|R|$ was increased. These findings clearly indicate that the proposed algorithm is scalable in terms of the candidate sensing-radius.

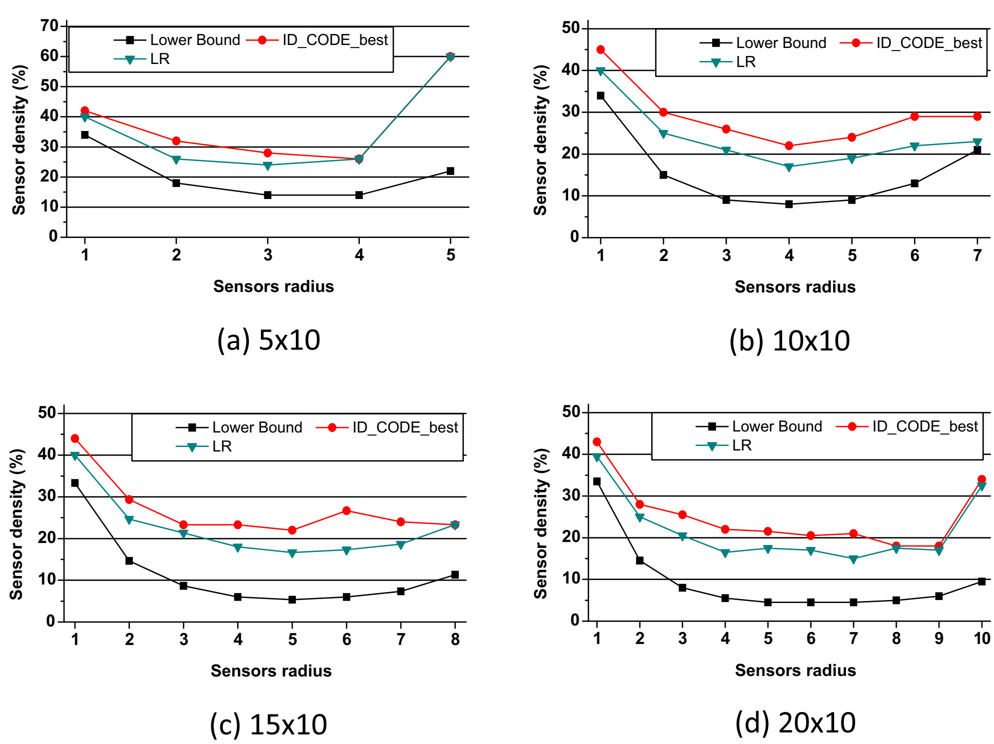

Fig. 15. Performance comparisons between ID_CODE_best and LR based algorithms under various size of the sensing fields.

In contrast, the computation time only increased by a factor of about 17 when the number of service points grew from 50 to 100, and by about 15 times as the number of service point increased from 100 to 200. Results of this experiment indicate that the computation time does not increase exponentially as the solution space grows exponentially. Therefore, the proposed algorithm is also scalable in computation time.

7. Performance comparisons

In this section, we compare the proposed algorithms with previous approaches. Chakrabarty et al. applied the coding theory to solve the target location problem in sensor networks (Chakrabarty et al., 2002). In the paper, we denote the placement method proposed by Chakrabarty et al. as "CIQ approach". Both simplicity and quickness are main advantages of CIQ deployment method.

Ray et al. applied the identifying code to solve the target location problem in sensor networks (Ray et al., 2004; Ray et al., 2003). They proposed the "ID-CODE" sensor placement algorithm, and designed three visiting orders: random, ascending, and descending orders.

However, CIQ approach is difficult for irregular sensing fields; as well as it ignores the sensor field boundary effect for deployments. In addition, both CIQ and ID-CODE approaches cannot directly use adjustable sensing radiuses to deploy sensors for the proposed target positioning problem. These approaches donot also take differential positioning qualities into account. Hence, the proposed LR based approach addresses these difficulties.

Fig. 13(a) shows a 13×13 sensor field which is deployed 65 sensors by CIQ approach (Chakrabarty et al., 2002). To solve the boundary problem, we deploy 14 extra sensors (total 79 sensors) for satisfying the completely discrimination constraint. In the same scenario, i.e. 13×13 sensor field and traditional uniform positioning priority, we also use sensors with sensing radius one to deploy a completely discriminated sensing field by LR algorithm. The results are illustrated as Fig. 13(b); only 68 sensors are needed. We can declare that our approach is more flexible for applications and effective in terms of deployment cost.

For comparing the proposed LR based algorithms with ID-CODE approach, we design a set of experiments considering traditional uniform QoS. Due to the results that were obtained by the random order of the ID-CODE approach highly depend on probability. It should result in either a larger deviation or too much time consumption to obtain a statistical or the best result. Hence, as Fig. 14 shown, in each experiment, we only adopt the ascending and descending orders of the ID-CODE algorithm. Furthermore, we take the best results of these two visiting orders and denote it by "ID_CODE_best" as a benchmark. The curve denoted "Lower Bound" is a modified version of lower bound according to Theorem 1-(2) and 1-(3) in Karpovsky et al. (1998).

Fig. 15(a)–(d) illustrate performance comparisons between ID-CODE_best and LR_1 algorithms under various size of sensing fields, i.e. 50, 100, 150, and 200 service points, respectively. We find that LR based algorithm outperforms ID_CODE_best in terms of deployment cost, i.e. sensor density.

8. Conclusions

In this paper, we propose a generic framework for the sensor deployment problem to support the differential quality of positioning services for different ROIs in sensing fields. We use weighted error distance as the criteria to measure positioning accuracy, i.e. quality of positioning.

Under the budget, complete coverage, and QoS priority constraints, the problem is modeled as a nonlinear integer programming problem where the objective function is to minimize the maximum weighted error distance. Besides the sensor's locations, we consider the sensing radius of each sensor as decision variables. Next, we develop LR based algorithm to cope with the NP-complete problem. Moreover, due to the results of the performance evaluations and comparisons, we make a summary as follows.

First, the proposed framework has flexibility to support differential QoS for positioning. The *level of QoS* obtained by the proposed framework always is higher than or equivalent to traditional uniform QoS using the same number of sensor nodes.

Second, deciding sensing radiuses depends on topographies and placement limitations of sensing fields while all sensors have uniform sensing radius. But, by adjustable sensing radius, the proposed algorithm effectively gets well solution quality. The performance of the proposed algorithm is almost independent of the set of candidate radius.

Third, the proposed algorithms are scalable in terms of the cardinality of set for candidate sensing radius and the number of service points in sensing fields.

Hence we state the proposed framework is effective for deploying a sensor network to support differential QoS for target positioning with placement limitations. Moreover, the proposed Lagrangean based algorithm is efficient and scalable. Obviously, the paper contributes to sensor deployment problem for target positioning services.

Acknowledgments

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Appendix A. The proofs of the propositions

Proposition 1. *If the diameter of the sensing field is D, and the discrimination weights are* $w_1, w_2, \ldots, w_h$ *and* $w_1 < w_2 < \cdots < w_h$. *Then* $w_{i+1} > D_{w_i}$ *for any two adjacent weights* w_i *and* w_{i+1}.

Proof. Some groups of service points all have the same power code. Among these groups, the pair of service points with the highest discrimination weight and the furthest distance has the maximum weighted error distance (the worst positioning accuracy). In this sensor network, the weighted error distance S of any pair of service points, which has discrimination weight w_{i+1} and one unit length apart, should be smaller than DW_i. □

Proposition 2. *If the diameter of sensing field is D; the detection range is r, and the discrimination weights are* $w_1, w_2, \ldots, w_h$, *and* $w_1 < w_2 < \cdots < w_h$, *then the constant* $\overline{K}$ *must satisfy constraints as follows:*

$$\text{If } 2r \geqslant D, \quad \text{then } w_1 > w_h D/(1+\overline{K}).$$
$$\text{If } 2r < D, \quad \text{then } w_1 > max\{(w_h \cdot 2r)/(1+\overline{K}), (w_h \cdot D)/(1+2\overline{K})\}.$$

Proof. The minimum value of weighted error distance for indistinguishable service points is w_1, which must be greater than the weighted error distance for each pair of discriminated service points, $w_{ij}d_{ij}\Big/\Big(1+\overline{K}\sum_k(v_{ik}-v_{jk})^2\Big)$. We first discuss the maximum of $w_{ij}d_{ij}\Big/\Big(1+\overline{K}\sum_k(v_{ik}-v_{jk})^2\Big)$.

For $2r \geqslant D$:

The furthest distance between any pair of discriminated service points is D. The minimal Hamming distance between two discriminated power codes is 1. Hence, the maximum value of $w_{ij}d_{ij}\Big/\left(1+\overline{K}\sum_k(v_{ik}-v_{jk})^2\right)$ is $w_{ij}D/(1+\overline{K})$. Therefore, the constraint, $w_1 > w_hD/(1+\overline{K})$, must be satisfied for $\overline{K}$.

For $2r < D$:

The two discriminated service points with furthest distance D cannot be covered with the same sensors while the detection radius of the sensor is less than the diameter of the sensing field. Hence, the minimal Hamming distance between their power codes is 2, and $w_{ij}d_{ij}\Big/\left(1+\overline{K}\sum_k(v_{ik}-v_{jk})^2\right)$ is $w_hD/(1+2\overline{K})$.

Conversely, to consider the case of two discriminated service points covered by at least one sensor, the further distance between them is $2r$, and the minimal Hamming distance between their power codes is 1. In this case, the value of $w_{ij}d_{ij}/\left(1+\overline{K}\sum_k(v_{ik}-v_{jk})^2\right)$. is $w_h \cdot 2r/(1+\overline{K})$.

Therefore, both $w_hD/(1+2\overline{K})$ and $w_h \cdot 2r/(1+\overline{K})$ have to be less than w_1. □

Proposition 3. *Theoretical upper bound of S_v denoted by $\overline{S}$ is $w_h\tilde{d}$, where denotes the highest discrimination weight, and $\tilde{d}$ (i.e. $\tilde{d} = min\{D, 2r\}$) is the minimal value of the diameter of a sensor field (D) and the maximum sensing range of sensors (2r).*

Proof. If the sensor field is not completely discriminable, then a pair of service points on the field that has the farthest distance $\tilde{d}$ and the same power vectors can be found. The value of $\tilde{d}$ is not greater than the diameter of the field and the maximum detection range of the sensors. Hence, $\overline{S}$ is bounded by $w_h\tilde{d}$. □

Proposition 4. *Theoretical lower bound of S_v denoted by $\overline{\overline{S}}$ is $w_l/(1+\widehat{d}\cdot\overline{K})$, where w_l is the lowest discrimination weight, and $\widehat{d}$ represents the maximal possible Hamming distance of power vectors for all service points. If $\overline{n}$ is the maximal number of sensors that can cover a service point; as well as N denotes the number of sensor nodes then $\widehat{d} = min\{2\overline{n}, N\}$.*

Proof. If $w_{ij} = w_l$ and $d_{ij} = 1$, $i \neq j$, the numerator of Eq. (10) is the minimum as well as $\sum_{\forall k\in B}(v_{ik}-v_{jk})^2 = \widehat{d}, i\neq j$, the denominator of Eq. (10) is the maximum, then the theoretical lower bound of S_v can be obtained. It occurs when the sensor field is completely discriminable. There are the service points i and j with the lowest discrimination weight w_l. Concurrently, the power vectors of service points i and j have the maximal possible Hamming distance, $\widehat{d}$. Hence, the lower bound $\overline{\overline{S}}$ is $w_l/(1+\widehat{d}\cdot\overline{K})$. □

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